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MASTER OF COMMERCE

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MANAGEMENT SCIENCE

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UNIT 1 MANAGEMENT SCIENCE

After French revolution,the most influential change took place in the form of management. Though management existed all along the era of human civilization, it is with the beginning of the work of Frederick W Taylor,that management evolved as a new dscipline, affecting businesses, industries and academics.

Two historical incidents have revolutionized industry, commerce and business – worldwide – invention of wheel, and introduction of scientific management.

In our society, critical work is performed by individuals with such title as principal, chairman, manager, president,captain etc.They practice one thing in common – management.

Prosperity of Japan, China, India, Hong Kong, Iran .... Is all due to efficiency in management,brought by scientific management. On the contrary, poverty of so many other countries has been largely due to poor managerial ability or lack of scientific management.

It is globally accepted that management is the most vital and strategic factor in the productive process.In the ultimate analysis, management promotes prosperity and the success or failure of business organizations largely depends on qualityof management. Thus emerged management science.

Because of humanity's desire to excel, new reforms are taking place. Business environment has become more challenging and complex, demanding more from managers. The art of management that was once being learnt through experience and handed down from generation to generation, now requires sound knowledge of scientific principles and systematic applications of appropriate methodology.

Management - definitions

The term management, which like philosophy and religion, is the most difficult to define.

CFL Brech defined management as 'social process entailing responsibility for effective planning , and regulation of operations of anenterprise."

FW Taylor defined management as "the art of knowing what you want to do and seeing that it is done in the best and cheapest way"

Henry Fayol stated " management is to forecast, to plan, to organize, to command,to co ordinate and to control."

Thus management is continuous organisation and control of people and resources for realizing goals of an organisation.

Management science

Management consists of problem solving and decision making. A scientific approach is necessary to attain organisation's goals and objective through the techniques of problem solving and decision making. Modern manager has to be a scientist with extensive

knowledge of the elements of decision making. This knowledge enables him to identify problems, build model, and to solve them effectively and efficiently. This approach to management, through problem solving and decision making is called management science.

Definitions

Management science is "approach of a manager, to solving managemen problems and decision making, by identifying, analyzing, modeling and setting options with the help of quantitative techniques and research methods."

Management science is application of scientific method to problems involving identification, analysis and interpretation, using models, relating to operations of business organizations."

Management science is a "systematic and analytical approach to decision making and problem solving through scientific methods of observation, experimentation and inference, relating to managerial issues"

The term management science is preferred by American academicians, while the British practitioners named this science operations research.

Evolution of management science

Application of scientific methods to solve industrial problems dates back to the days of Adam smith, when he described the advantages of division of labor in 1776, for the increase in the manufacture of pins. Early in nineteenth century, Charles Babbage, through his writing on 'Economy of machines and manufacturer' (1832) advocated the use of scientific principles in the analysis of business problems.

The growth of industrialization after industrial revolution of England brought forward serious problems related to organizational planning and control. However, the principles of managements started emerging in their useful forms since 1903, when F W Taylor presented a paper on 'shop management" before American Society for Mechanical Engineers. Through his professional career, Taylor kept impressed that the manager should accept special responsibilities of developing a science for man's work. According to him, the managers must gather and classify all traditional knowledge and transform this knowledge to laws, rules and formulae so that the workers and laymen are benefitted, by doing their routine jobs.

A K Erlang ,in 1909, published his most important work containing the development of formulae on waiting time, based upon lows of statistics. These are being widely used in practice. In the area of inventory control, Ford W Harris deduced a formula in 1905, on 'Economic Order Quantity' that constituted as a basis of inventory control for a long time, and still finds wide use today. Walter Shewhart used the principles of statistics to establish the concept of control charts in quality control of manufactured products. The control charts are considered as one of the important tools in statistical quality control.

Wassily Leontiff was first to develop and apply linear programming models in business problems. In 1947, George B Dantzig developed Simplex algorithm - an efficient

computational scheme to solve problems related to allocation of scarce resources. Since 1956, a large number of theories and models have been described to analyze the decision problems for their solutions. Some of them are mathematical programming ,simulation theory, decision theory, network models etc.

Characteristics

Definitions of management science bring out following characteristics:

1. System orientation

Business organization is considered as a system and an entity. Any part of organization has some effect on the activity of every other part. To evaluate any decision, one has to identify all possible interaction and determine the impact on the organization a whole.

2. Interdisciplinary

It is recognized that combined effort of different persons can produce more unique solutions with greater probability of success. The operations research team will look at the problem from many angles, to determine the best solution

3. Scientific method

Operations research team applies the methods of obs ervation, experimentation, inference and testing frequently, on all problems. When an issue is identified, its critical factors are observed, experiments are conducted on its behavior. Several inferences are arrived as to its causes and impacts, and finally testing and simple solutions are arrived at.

3. New problems

Solution to a managerial problem may uncover several related and unrelated problems. The result of management science study pertaining to a particular problem need not wait until all the conceived problems are solved.Integrated and scientific approach will lead to gradual solutionof newly uncovered problems.

4. Decision making

Decision making is inherent to management science and problem solution. Scientific decision making is a systematic process, consisting of diagnosis of the problem, identifying critical factors, revealing alternatives, and then selecting the best alternative.

6. Quantittive solution

Management science provides the decision makers with a quantitative basis for decision making. Forthis purpose, objectives of situation is identified, constraints are subjected to analysis,and various pay offs under each alternativeis evaluated. Allthese are performed in numerical terms.

7.Human factors

Human factors play a dominant role in managerial problems. In quantitative analysis, human factors cannot be considered and given due importance. However, management

science takes into account all human factors, besides material factors Management science will be incomplete without a study of such factors.

8. Objectivity

Management science approach seeks to obtain an optimalsolution to the pro0blem under analysis, consideringoverall objective and departmental goals. For this a measure of desirability or effectiveness is defined. A measure of desirability so defined is thenused to compare alternative courses of actionwith respect to their outcomes.

9.Modeling

Modeling is an important factor of management science. Modeling is a representation in the formof relationships between variables of a s situation. The model is solved to get an optimal solution tothe problem.

Scientific method in management science

Management science applys scientific methods of observation, experimentation and modification to managerial operations. The scientific method consists of following three phases - judgement phase, research phas and action phase.

Judgmenet phase

Judgement phase consists of determination of operations to be managed, determination of objectives, effectiveness of measures and formulatin of problem. Operation is a combination of different actions dealing with resources such as men and machines, which form a structure from which an action with regard to broader objectives is attained. In this phase, several decisionas are taken regarding objectives and values, measures of effectiveness, and problems to be formulated.

Research phase

Research phase includes data collection, formulatin of hypothesis, analysis of data and verification of hypotheses, prediction and generalisatin of results and consideration of alternative methods.Qualitative as well as quantitative methods may be used for this purpose.

Action phase

Action phase consists of making recommendations for remedial actin to those who first posed the problem and who control the operations directly. These recommendations consist of a clear statement of the assumptions made, scope and limitations of the information presented about the situaton, alternative courses of action, effects of each alternatives as well as the pereferred course of action.

Role of Management science in industry and commerce

Industry and commerce highly depends on management science for finding answers to many of its managerial and operatinsl problems.

1. Complexity

In a big indusrttry, the number of factors influencing a decision has increased. Situation has become big and complex because these factors interact with each other in complicated fashion. There is great uncertainy sbout the outcome of interactin of factors like technological, environmental, competitive etc. with the help of kmthematical models, complex problems can be split up into simple parts, each part can be analysed separately and then the results can be synthesised to give insights into the problem.

2. Responsibility

Responsibility and authority of decision making is scattered throughpout the organisation and thus the organisation ,may be following inconsistent goals. Mathematical quantification of decision situation is often necessitated. Management science overcomes this difficulty, to a great extent.

3. Uncertainty

There is great uncertainty about economic and general environment. With economic growth, uncertainty is also increasing. This makes each decision costlier and time consuming. Management science is thus quite essential from reliability point of view.

4. Knowledge

Knowledge is increasing at a very fst rat. Majority of the industries are not uptodate with thelatest knowledge and are at adisadvaantage. Mkanagement science teams collect thelatest information for analyusis pruposes which is quite useful for theindustries.

Role of management science in decision making

Very often quantification of the elements of decision making environment becomes necessary. The appropriate analysis of these quantitative elements can yield significant inpouts for the purpose of decision making. Many decision making problems are amenable to quantitative analysis and therefore effective decisions .The recommendations of these quantitative analysis may provide many inputs to the decision maker- not only quantitative inputs but also subjective inputs. The ability of the decision maker lies in evaluating and utilizing these inputs to make an effective decision.

Management science provides us many techniques which help in making quantitative analysis of the elements of decision making environment and to arrive at effective decision.

Several problems are faced by a manger while promoting effectiveness and efficiency in his organization. The manager selects and depends on several criteria to devise methods for solving such problems. Management science is one of such criteria-.

The first step in the use of management science is identifying a decision making role. It is followed by structuring of the problem after establishing relationship among various

elements of the operating system. Management science serves just these two basic purposes.

Any operating system is a model having four feature sknown as input, processor, output and control. The input may consist of men, materials,money etc. The process may be comprising of machines, tools, materials etc. All these are interacting to produce according to pre-laid down specifications. Data are tapped from the output and compared with the specifications. If the output data do not match with the predicted specifications, corrective measures are applied by the control. A continuous flow of information is maintained during the operation of the system. In this process, many simple and complex situations arise when decision making is a serous matter for the manager. The situations are related to the problems of scheduling, operations, control and modification.

Effective scheduling requires a systematic approach in application of forecasting techniques to have estimated data for planning the future. The manager has to employ careful planning and control on inventory, quality and finance, in respect of product. Use of several mathematical models, and digital computers are necessary to compile, process and present cutting edge data ,usefully, so that these may serve as essential ingredient in a decision making process.

Sometimes, the outcome associated with a particular approach might be quite difficult to obtain a solution. Another technique knownas simulation is employed to reach the decision, in such a situation. Thus management science plays a very important role in designing managerial decisions and solving of management n problems.,

Management science process

The process of science of management is complex and consists of following phases: defining the problem, developing the model, acquiring input data, developing a solution, analysing theresults and implementing the results

Defining the problem

In many cases defining the problem is the most important and the most difficult step. In analysing a situation, one may be related to other problems, simultaneously.

Attempting to solve one problem, without regard to other related problems, can make the entire situation worse. Thus it is important to analyse how the situation to one problem has impact on other problems.

Formulating a problem consists of identifying, defining and specifying the features of the components of a decision mode. This should yield a statement of the problem's element that includes the controllable variables, the uncontrollable parameters, the restrictions or constraints of the variables and the objectives for obtaining an improved solution.

Variables are measurable aspects that have a bearing on the problem. Variables may be controllable and uncontrollable. Controllable variables are those under the direct control of the decision maker. They are also called decision variables. Uncontrollable variables are

those which cannot be manipulated by the decision maker. For example, a controllable variable in an inventory problem, is re-order level , while the demand for an item in the inventory is uncontrollable variable.

Developing the model

Afterselecting the problemtobe analysed, thenextstep is to develop a mode. The models that are generally used are mathematical. Mathematical model is a set of mathematicalrelationships. The use of mathematics as a language for model representation has the advantage of being very compact and has the capability of using high mathematical techniques.

A mathematical model includes decision variables , constraints and objective function.Decision variables are the unknowns and are to be determined by solving themodel. Parameters are constraints which relate to the decision variables, the constraints and objective function. Constraints are the restrictions on the decision variables. Objective function defines the measure of effectiveness of the system. An optimal solution to the model is obtained when the values of the decision variables yield best value of the objective function. The model should be carefully developed. It should be solvable, realistic,easy to understand and modify.

Acquiring input data

Once the structure of a modelhas been determined,it is usually necessary to collectdatafor the modeling process. For a larger problem,collecting accurate data canbe one of the most difficult steps in performing quantitative analysis. Obtaining accurate data for the model is essential. Even if the model is perfect, improper data will result in misleading results.

The data collection step will be adequate to the extent to which an organization has developed its management information system. Data base management systems allowe decision makers great flexibility in accessing or manipulation of data. Company reports and documents , interviews with employees or other persons related to the firm etc can be used for gathering data ,useful for constructing models.

Developing solution

A solution tothemodel means thevalues of the decision variables that optimize themeasures of effectivenessina model. There are various methods for obtaining the solution likeanalytical method, numerical method, simulation method etc.

If the model fits into linear programming,an optimal solution may be obtained by using linear programmingtechniques. Ifthe mathematical relationships ofthemodel are too complex to permit analytical method, and then simulation approach may be appropriate for finding the solution.

Testing solution

Before a solution can be analysed and implemented,it needs to be completely tested. The solution depends on the input data.

Testing the input data and the model includes determining the accuracy and completeness of data used by the model. Inaccurate data will lead to an inaccurate solution. There are several ways inwhich the input data can be tested. One method of testing the data is to collect additional data from a different source. These additional data canbe compared with the original data and statistical tests can be employed to test whether there is any difference between original data and additional data

If there are significant differences, much effort is required to obtain accurate input data. If the data are accurate but the results are inconsistent with the problem, then model may not be appropriate Models are to be checked to see whether they are logicl and they represent real situation. A model is said to be valid if it can give a reliable prediction of the systems performance. A good model should have long life and must be a good representation of the system. In effect, performance of the model must be compared with policy or procedure that it is meant to replace.

Analyzing and interpreting results

Analyzing the results start with determining the implications of thesolution. Inmostcases,the solution of a problemwill result in some kind of action or change in the way an organization is operating. The implications of these actions or changes must be determined and anlysed before the results are implemented.

Because a model is only an approximation ofreality, the sensitivity of the solution tochanges in themodel and input data is a very important part of analyzing the results. This type ofanalysis is called sensitivity analysis. This analysis determines how much the solution would change if there were changes in the model for the input data. Whenthesolutionis sensitive tochanges in theinput data, and the model specification, additional tastings should be performedtomake sure that the modeland input data are accurate and valid. It may be noted that if the modelor data are wrong, the solutionwould be wrong.

When one ormorevariables change significantly, the solution goes out of control. Insuch situation, the model should be modified accordingly.

Implementingresults

The final step in post modeling is to implement the results.The solution obtained and its realities must be carefully examined at this stage. This is the stage of incorporating the solution into the organization. The experts of management science and those who are responsible for managing the organization must show mutual cooperation for implementing the results derived.

Applications of Management Science

Managementscience is widely implemented in different areas of human activity,such as, project planning, capital budgeting, production planning , inventory analysis,accounting, market planning, qualitycontrol, plantlocation, personnel management

etc. The various studies conducted, show that management science activities are being conducted by a large percentage of corporate organizations and are being applied to a wide range of problems.

The modeling techniques like Game theory, linear programming, dynamic programming, queuing theory, network techniques etc are seen implemented in various organizations for solving their problems.

Many of economic, managerial andsocialproblems are amenable to quantitative analysis and so management science has its applications in these fields of study.

Impact of management science

The impact of management science as a field ofstudy is quite remarkable. Many corporations, consulting companies' and universities put on nationally based seminarson specific management science modeling procedures. Many graduate programs of study offer degree in management science. Management science departments are well established in relatively large corporations and governmental agencies. Management consulting groups provide management science services to small and medium size organizations. Several personnel placement companies specialize in searching for and placing individuals with management science expertise.

Since 1972, the Institute of Management Science has recognized excellence in the application of management science in the form of the Edelman Award for management science achievement.

Given below are some of the areas where management science has its impact

- Air water pollution control
- City planning
- Personnel management
- Population planning
- Law enforcement
- Political campaigning
- Health science management
- Hospital administration
- Dietary planning
- Inventory management
- Maintenance of equipment
- Transportation
- Production scheduling
- Capital budgeting
- Assignment of job
- project management
- resource allocation
- replacements

- Queuing analysis

Managementscience – new challenges

Development of decision support and expert systems is a natural evolution of management science towards increasing intimacy with the decision maker.

The decision support systems concept represents a new and increasingly important vehicle for assisting managers to make decisions and holds the potential for improving the quality of decision making.

It is an interactive computerized system capable of providing direct, personal support for the individual decision maker. The final component of a decision support systemis a hardware.

An expert systemis a software packagethat attempts toemulate expert human performance in solving problems that require significant levels of human expertise. Anexample of anarea of applicability is that of medical diagnosis.

At present simpleexpert systemsare designed and they are relatively straight forward todevelop. More sophisticated intelligence expert systems,capable of displaying commonsense knowledge and capable ofdealing with conflicting informationrequire more powerful Artificial intelligence methods that cancurrently exist. Therefore the future of expert systems depends on more powerful artificial Intelligence methods.

Limitations of management science

Previous sections have brought out the positive side of management science. However, there is also the need to point out the negative side. Certain common errors and pitfalls can and have ruined the otherwise good work. Some of these pitfalls are quite obvious while others are go subtle and hidden that extreme care is required to locate their presence.

Management science has certain limitations. However,these limitations are mostly related to the problems of model building and the time and money factors involved in its application rather than its practical utility.

Computations

Management science tries to find out optimal solution taking into account all the factors. In the modern society, these factors are enormous and expressing the and establishing relationships among these. They require complicated calculations which can only be handled by machines.

Qualitative aspects

Management science provides solution only when all elements related to a problem can be quantified. All relevant variables do not lend themselves to quantification. Factors which cannot be quantified , find n lace in management science models. The science do not

take into account qualitative factors oremotionalfactorswhichmay be quite important incertain situations.

Knowledge gap

Management science, being a specialist's job, requires cooperation between mathematicians and statisticians, who might not be aware of the business problems. Similarly,a manager , fails to understand the complex working of managementscience. Thus there is a knowledge gap betweenthe two. Management itself may offer a lot of resistance due to conventional thinking.

Costs

When the basic data are subjectedto frequent changes, incorporating them into management science models is a costly affair. Moreover,a fairly good solution at present may be more desirable than a perfect management solution, available after some time.

Implementation

Implementation of decision is a delicate task. It must take into account the complexities of human relations and behaviors. Sometimes, resistance is offered only due to psychological factors.

Technique selection

Management science techniques are very useful but they cannot be used indiscriminately. Choice oftechnique depends upon the nature of problem, operating conditions, assumptions,objectives,etc. Thus identification and use of anappropriate technique is essential.

Not substitute

Managementscience only provides tools and cannot be a substitute of management. It only examines theresults of alternative courses of action andfinaldecisionis made by management withinits authority and judgment.

Sub-optimization

Sub-optimization is deciding in respect of a relatively narrow aspect of the whole business situations or optimization of a subsection of the whole. Functional departments sometimes, without taking care of wider implications, sub-optimize their functions. This may cause loss in that part of the organization which is left out of the exercise and as such should be avoided.

Review questions andexercises

How has management become a vital science
Define the term management
What is management science
Explains functions of management
Describe the role of managementscience

How is management science and decision making related
What is managementscience process?
\what are the stepsin solving a managemenTransportation problem
What are the merits of applying management science
Write notes onimpact of management science
What are thechallenges facing management science
Explain implementation of management science
Explainlimitations of management science
What is judgment phase?
State the new challenges in management science
What is sub optimization?
Explain implementation of management science
Explainlimitations of management science
Describe role of management science in decision making
Relate management science with industry and commerce
What is meant by objectivity?
Explain scientific method in management science
What are the new challenges facing management science
Explain limitatios of management science

UNIT II MODELLING

Management is a science based on observations and experimentatins.A solution may be extracted from a model by considering experiments and mathematical analysis. Construction ofmodels is the method of solution in management science.

AK Erlang in 1909, published his most important work containing the development of formulae on waiting line model, based on laws of statistics. In the area of inventory control, Ford W Harris deduced a model in 1905 on Economic Order Quantity. Walter Schewart used a graphic model in the form of control charts , for qualitycontrol of manufactured products. Thus models constitute an integral part of management science

Model

A model is an idealized representation ofa real life situation. It represents the aspects of reality, to be observed and experimented easily. The globe, map, control chart, PERT network, break even equation, balance sheet etc are models, because each represents one or few aspects of the real life situation. A map for example, represents physical boundaries of countries, along with a few other aspects such as depth of oceans, mountains etc.The objective of a model is to provide a means for analyzing the behavior of he system for the purpose of experimenting and improving its performances.

Type of models

On the basis of structure, models can be classified as iconic, analogue or symbolic

Iconicmodels

Iconic models represent the system as it is, but in different size. Thus iconic models are obtained by enlarging or reducing the size of the system. In other words, they are images.

Some common examples are photographs, drawings, model airplanes, ships, engines, globes, maps etc. A toy airplane is aniconic model of a real one. Iconic model of the sun and its planets are scaled down while the model of theatom is scaled up so as to make it visible tothenaked eye. Iconic models have got some advantages. They are specific and concrete. They are easy to construct. They can be studied more easily than thesystem itself.

They have certain disadvantages also. They are difficult to manipulate for experimentalpurposes. They cannot be used to study thechanges in operation of a system. It is not easy tomake any modification or improvement inthese models. Also adjustment withchanging situations cannot bedone inthesemodels.

Analogue models

In analogue models one set of properties is used to represent another set of properties. After theproblem is solved, the solution is reinterpreted in terms of theoriginal system. For example, contour lines on a map are analogues of elevation as they represent the rise and fall of heights. Analogue models are easier to manipulate than iconic models, but they are less specific andless concrete.

Symbolic models

In symbolic models, letters, numbers, and other types ofmathematicalsymbols are used to represent variable and the relationship between them. Thus symbolic models lare some kind of mathematical equations or inequalities reflecting the structure of thesystem

they represent. Inventory models, queuing models etc are the examples of symbolic models. Symbolic models are most abstract and most general . They are usually easiest to manipulate experimentally. They usually yield more accurate results, under manipulation

Characteristics of a good model

Models are integral part of management science. A good model should possess following charcteritics.

1. The number of variables or ctitical factors should be as few as possible.
2. The number of assumptions should be less
3. It could accommodate any changes or modifications.
4. It should be easy and straightforward to construct.
5. It could be applied repeatedly and in wide variety of situations.

Constructing a model

A model is built for solving a decision making problem. It passes through following steps

1, formulation of the problem

The decision problem should be properly formulated. For this, thecritical factorsdeterminingthesolution should be identified and expressed in understandable terms.

2. Criticality of factors

Once, a complete list of critical factors is prepared, the next step is to find whether or not to take each of these components into account. This is determined by finding the effect of various alternative courses of action on each of these factors. Generally one or more factors are independent of the changes made among the various alternative courses of action. Such factors may be temporarily dropped from consideration.

3. Relating factors

The next step is to identify the raltions between factors and expressing them in understandable style. It may be convenient to group certain factors of the system. For example, purchase price, freight charges and receiving cost of a raw material can be combined and related together and called raw matrial acquisition cost.

4. Substituting symbols

Once factors are identified and related, symbols are assigned to each of these factos. For lexampole, in a situation " a newspaper boy wants to decide the number of newpapers he shold order to maximize his expected profit', the variables may be number of new papers ordered per day, profit earned on each news paper, loss on each newpaper retuned, demand for news aper per day, probability of demand, net profit per day etc.... these variables may be assigned symbols like N, A , B ,D, P(D), P , respectively.

5. Expression

When the variables are identified and symbols are ssigned to them, the model can be expressed in an appropriate format. The model should be expressed, featuring the relations between the variables.

Methods of deriving solution

Solution to a management science problem may be extracted either by conducting experiments or by mathematical analysis. Some cases may require a combination of both. Following are such methods of deriving solution

Problem solutions using models consists of finding thevalues of thecontrolled variables that optimize themeasure of performance or of estimating them approximately. Models are generally solved byfollowing methods.

Analytic methods

In these methods all thetools of classical mathematics such as differential calculus and finite difference are available for thesolution of a mode.Various inventory modes are solved by theuse of these analytic methods.

Iterative method

Whenever the classical methods fail, we use iterative procedure. The classical methods may fail because of the complexity theconstraints or of thenumber of variables. In this procedure we start with a trial solution and a set of rules for improving it. The trail solution is improved by the given rules and is then replaced by this improved solution. This process of improvement is repeated until either no further improvement possible or the cost further calculation cannot be justified.

Simulation technique

The basis of simulation technique is random sampling of a variable's possible values. For this technique, some random numbers are required whichmay be converted into random variates whose behavior is known from past experience. Dargaer and Koc define Monte Carlo methods 'a combination of probability methods and sampling techniques providing solutions to complicated partial or integral differential equation" in short, Monte Carlo technique is concerned with experiments onrandom numbers and it provides solutions to complicated managemenTransportation problems.

Management science models

Management science models are widely applied in business and industry situations. The impact of management science as a field of study is quite remarkable. Given below are some of themodels which are been implemented in various organizations. They are also called management science tools.

Allocation models

Allocation models involve theallocation of resources to activities insuch a manner that some measure of effectiveness is optimized. Allocation problems can besolved by Linear and Non Linear Programming techniques. Linear Programming technique is used infinding a solution for optimizing a given objective such as maximizing profit or minimizing cost under certain constraints. It is a technique used to allocate scarce resource in an optimum manner in problems of scheduling product mix and so on. This technique includes an objective function, choice among several alternative limits or constraints using standard symbols and variables to belinear. Assignment models and transportation models are special cases of linear programming.

Sequencing model

These are concerned with placing items in a certain sequence or order for service. This is applied in large scale plants where there are large number of employees, tools and equipments to be used often. The technique offer least time consuming and least cost solution for servicing.

Queuing models

These are models that involve waiting for services. Inbusiness world, several types of interruptions occur. Facilities may break down and therefore repairs may be required. Power failures occur. Workers or the needed materials do not show up where and when expected. Allocation of facilities considering such interruptions is queuing models. The waiting line problems can be solved by Waiting Line theory or Queuing Theory. Waiting Line theory aims in minimizing thecosts of both servicing andwaiting.

Inventory models

These are models with regard to holding or storing resources. The decisions required generally entail the determination of how much of resources are to be acquired or when to acquire them. Inventory control claims at optimum inventory levels. Inventory planning is meant for optimum decisions about how much to buy and when to buy. Inventory theory technique is used for solving inventory problems. Thetechnique helps to minimize costs associated with holding inventories, procurement of inventories and theshortage of inventories.

Competitive models

These are models which arise when two or more people are competing for a certain resource. Game theory models are used to determine theoptimum strategy in a competitive situations.

Network models

Network models involve the determination of anoptimum sequence of performing certain operations concerning some jobs in order to minimize over alltime or cost. PETRT, CPM and other network techniques such as Gantt Chart come under network model.

Simulation models

Simulation is a technique of testing a model whichresembles a real life situation. This technique is used to imitate an operation prior to actual performance. There are two methods of simulation. Monte Carlo method and System simulation method.

Search Models

This model concerns itself with search problems. A searchproblem is characterized by theneed for designing a procedure to collect information on thebasis of which one or more decisions are made. Examples for search problem are advertising agencies search for customers, and personal departments' search for good executives.

Replacement models

These are models concerned withsituation that arise when some items such as machines, electric light bulbs need replacement because the same may deteriorate withtime or may break down or may become out of date due to new developments. This model is concerned withtheprediction of replacement costs and determination of themost economic replacement policy.

Scope of management science

Management science is applied in different areas of human activity, suchas project planning, capital budgeting, production planning, inventory analysis, accounting, market planning, quality control, plant location, personnel management etc. thevarious studies conducted, show that management science activities are being conducted by a large percentage of corporate organizations and are being applied lto a wide range of problems.

The modeling technique like Game theory, Linear programming, Dynamic programming, Queuing theory, network techniques etc are seen implemented in various organizations for solving heir problems.

Many of economic, managerial and social problems are amenable to quantitative analysis and so management science has its application inthese fields of study.

Many corporations , consulting companies anduniversalities put on nationally based seminars on specific management science modeling procedures. Many graduate programs of study offer degree in management science. Management science departments are qell established in elatively large corporations and governmental agencies. Management consulting groupsprovide management science services to small and medium size organizations. Several personnel placement companies specialize in searching for and oplacing individuals with management science expertise.

Since 1972, the Institute of Management Sciences has recognized excellence in the application of management science in theform of the Edelman Award for Management science achievement.

A number of innovative areas have emerged for theapplication of management science. They include city planning, air water pollution control, environmental planning, scenario development, population planning, politicl campaign strategies, health management, hospital administration, diagnosis, disease control., military operations, terrorist fighting, aerospace management, space vehicles launching, portfolio management, insurance andrisk management, transportationscheduling, production planning, capitalbudgeting, assignment facilities etc.

Principles of modeling

Following principles must be kept in mind while formulating models

Simplicity – mathematicians are of the habit of making complex models here as one should go in for simple model if it is sufficient. It means models must be kept simple and understandable.

Clarity – if we do not understand theproblemproperly, we can not apply theappropriate technique of management science. For example in case of allocations of scarce resources, the technique of LPP may be applied.

Validity – model must be validated before implementation, otherwise it can be implemented in phases for validating. Example, simplex method.

Decision – models are to support decision makers. Or models are to aid thedecision maker but not to replace them. Decision is tobe taken by the management itself.

Flexibility – model should be flexible enough to incorporate changes. It should give range where one solution is valid as in case of sensitivity analysis.

ICT – use of computers and information technology should be applied wherever it is possible steps should be clearlystated toenable the management science expert to develop computer software for future implementation of techniques.

Review Questions and Exercises

1. what is a model
2. Explain importance of modeling
3. How is modeling related experimentation
4. Control charts are popular models. Explain
5. Give a classification of models
6. How would you construct a model
7. What are iconic models
8. What are analogue models
9. What is the specialty of symbolic models.
10. Explain characteristics of models
11. State methods of deriving solutions from a model.
12. State steps inmodel building
13. Discus role of model building in decision making
14. What are general problem solution methods
15. Explain practical applications of models
16. Two companies A and B are competing for the same product. Their differenct strategies are fiven in the following matrix.

		Company A		
		A1	A2	A3
Company B	B1	2	-2	3
	B2	3	5	-1

Determine the type of model to describe this problem.

UNIT III MANAGERIAL DECISION THEORY

Problem solving and decision making have always been realized as complexprocessesofmanagement in which there has always remained a need to follow systematic procedure to dealwith thesituations. Management science is an approach of manager to solving managerial problems and decision making, by identifying , modeling, and stating the problems and testing and implementing solutions.

The success or failure that an individual ororganization experiences, depends to a large extent,on the ability of making appropriate decisions Making ofdecision requires a an enumeration of feasible and viable alternatives, projected consequences associated with such alternatives, and a measure of effectiveness by which preferred alternatives is identified. Managerial decision theory emerged in such an environment, to provide a method of decision making where data concerning occurrence of difference outcomes may be evaluated.

Decision and decision making

A decision is a process of choosing analternative course of action,when a number of alternatives exist. Decision making is an everyday process in life. It is a major part of a managers'' role too. The decision taken by a manager has far reaching effect on the business. Right decisions will have a salutary effect and wrong ones may prove to be disastrous.

Decisions may be tactical or strategic. Tactical decisions are those which affect the business in the short run. Strategic decisions are those which have far reaching effect during the course of business.

These days, in every organization whether large or small , the top management has to take some decision knowing that certain events beyond his control may occur, to make him regret the decision. He is uncertain as to whether or not these unfortunate events will happen . In such situations, the best possible decision can be made by the use of statistical methods. The methods try to minimize the degree to which the person is likely to regret the decision.

Decision making constitutes one of the highest forms of human activity. Statistics provides with tools for making wise decision in the face of uncertainty. This has led to the development ofstatistical decision theory.

Broadly defining, statistical decision theory is a term used to apply to those methods for solving decision problems in which uncertainly play a crucial role.

Various stages in the decision making process are perceiving the need for decision making, determining the objectives, collection of relevant information, evaluating alternative courses ofaction and choosing the best alternative.

Basic Concepts

Irrespective of type of decisions, there are certain essentialcharacteristicswhich are commontoall, aslisted below:

Decision maker

Decision maker is charged with responsibility of making the decision. That is, he has to select one from a set of possible courses of action.

Courses of action

Also called acts, they are the alternative courses ofaction or strategies that are available to the decision maker. The decision involves a selection among two ormore alternative courses of action. The problem is to choose the best of these alternative courses of action. To achieve an objective, If thedecision maker has alternative choices - A_1 . A_2 A_3 A4 . . . .then, all these acts form the action space.

Event

Events are the occurrences which affect the achievement of objectives. They are also called states ofnature The events constitute a mutually exclusive and exhaustive set of outcomes, which describe the possible behaviors of the environment in which the decision is made. The decision maker has no control over theevent which will take place and can only attach a subjective probability of occurrence of each.

Outcome

To every act in combination withevery stateofnature, there is an outcomeor consequence. The outcome is also knownas conditional value. That is,when the decision maker selects a particular state of nature, the result obtained is called the outcome. Outcomes may be evaluated in terms of profit or cost or opportunity loss or utility.

Pay off

The pay off can be interpreted as theoutcome inquantitative form whenthe decision maker adopts a particular strategy under a particular state of nature. it is themonetary gain or loss of eachsuch outcome. Pay offs can also be based on cost or time.

Opportunity loss

An opportunity loss is the loss incurred because of failure to take the best possible decision. Opportunity losses are calculated separately for each state ofnature that might occur.Given he occurrence of a specific state of nature, we can determine the best possible act. For a given state of natue, th opportunity loss of anact is the diferencebetweenth pay off of that act and thepay off for best act that could have been selected. Opportunity loss is also called regret.

Pay of Table

Pay off table is in a matrix form. It lists theacts andevents. It considerstheeconomics of the problemby calculating conditional pay offvalue for each act adevent combination.similarly regret table is a matrix showing opportunity loss for eachact under a state ofnature. Pay off tabl e consists of rows and columns. Acts are shown in rows and states of nature in columns.

Example

Pay off Table

States of nature

Acts	S1	S2	S3
A1	30	45	23
A2	25	34	53

A3	23	32	24
A4	15	24	43

In the pay off table every figure represents outcome or pay offs to the decision maker when he selects a particular act under a particular state of nature. For example, when A selects A2 , under the state of nature, S2, his gain is 34. Similarly when A selects A2 under thestate of nature S2, opportunityloss to A is 11 and so on....

Opportunity Loss Table

A Pay Off Table represents income or gains if a particular Act is chosen under a particular states of nature. The pay offs will differ according to various states of nature. Agood decision is that which will give maximum pay off under a state of nature. Then, there is a hypothetical loss, if we donot take the best act. Such loss is called opportunity loss or regret. Opportunity loss is pay of or profit lost due to not taking the best strategy, under a state of nature.

In order to calculate, opportunity loss or regret, the best pay off under each states of nature must be identified. Then, actual pay off must be deducted and shown in each cell.
Opportunity Loss = Max payoff under state of nature – actual payoff.

In the above example, maximum pay off under S1 = 30. In the cell, A1 S1, the opportunity Loss is 30 – 30 = 0, in cell A2 S1, opportunity loss is 30 – 25 = 5, and so on.

Opportunity Loss Table

States of nature

Acts	S1	S2	S3
A1	30	0	30
A2	5	11	0
A3	7	13	29
A4	15	21	10

Decision making situations

In any decision problem,the decision maker is concerned with choosing from among the available alternative courses of action,the one that yields the best result If the consequences of each choice are known with certainty, the decision maker can easily make decisions. But in most of real life problems,the decision maker has to deal with situations where uncertainty of the outcome prevail.

Decision making problems can be discussed under following heads, on the basis of their environments- decision making under certainty, decision making under uncertainty, decision making under risk, and decision making under competition.

Decision making under certainty

In this case, the decision maker knows with certainty,the consequences of every alternative ordecision choice. Thedecision makerpresumesthat only onestateof nature is relevant forhis purpose. He identifies this state of nature, takesit for granted and presumes complete knowledge as to its occurrence. Suppose A wants to invest in one of the financial institutions and each of them pays different ratesofreturn. He can select that institution which pays highest return because only one is best act.

Decision makingunder uncertainty

When the decision maker faces multiple states of nature, but he has no means to arrive at probability values to the likelihood or chance of occurrence of these states of nature, the problem is decision making under uncertainty. Such situations arise when a new product is introduced in the market o ra new plant is set up. In business,there are many problems of this nature. There the choice of decision largely de, ends on how the decision makerviews the situation.

Following choices are available before the decision maker in situations of uncertainty - Maximax, Minimax, Maximin, Laplace and Hurwicz Alpha criteria.

Maximax Decision Criterion

The term Maximax is the abbreviation of thephrase maximum of themaxima. It is also called thecriterion of optimism. An adventurous and aggressive de :ision maker chooses that act that would result in the maximum payoff possible . Suppose fo eachact there are three possible payoffs, corresponding to three states of nature as given inthefollowing decision matrix

Payoff Table

	States of nature		
Acts	**S1**	**S2**	**S3**
A1	220	160	140
A2	180	190	170
A3	100	180	200

The maximum of these three maximums is 220 which relates to A1. Consequently, according to the maximax criteria, the decision is to choose A1.

Minimax Decision Criterion

Minimax is just opposite to maximax. Application of the minimax criterion requires a table of losses instead of gains The losses are the costs to be incurred or the damages tobe suffered for each of the alternative act andstates ofnature. The minimax rule minmizes the maximum possible loss for a course of action. The term minimax is an abbreviation of the phrase minimum of maxima loss. Under each of the various acts,there is a maximum loss and the act that is associated with the minimum of the various maximum losses is the act to be undertaken according to the minimax criterion. Suppose the loss table is

Opportu ity loss table

	States of nature		
Acts	S1	S2	S3
A1	0	3	18
A2	4	0	14

A3	10	6	0

Maximum losses incurred by the various decisions . And the minimum amongthese three maximums is 10 which if offered by A3. According to Minimax criteria, the decision maker should take A3.

Maximin decision criterion (criterion of Pessimism)

The maximin criterionofdecisionmakingstandsforchoice betweenalternative courses of action assuming pessimistic view . Taking each act in turn, we note the worst possible results in terms of pay off andselecttheactwhich maximizes theminimum pay off. Supposethe pay off table is

Payoff table

States of nature

A1	-80	-30	30	75
A2	-60	-10	15	80
A3	-20	-2	7	25

Minima under each decision A1 = -80, A2 = -60, A3 = -20. According to Maximin criterion, A3 is to be chosen, which gives maximum pay off among minima. This way of decision making is alsocalled Waldian criterion.

Laplace criterion

As the decision maker has no information about the probabilityof occurrence of various events,the decision maker makes a simple assumption that each probability is equally likely. Theexpected Pay off is worked out on the basis of these probabilities. Then act having maximum expected pay off is selected.

	States of nature		
Acts	S1	S2	S3
A1	20	25	30
A2	12	15	20
A3	25	30	22

We associate equal probability for each event – 1/3 to each state of nature. So, as per Laplace criterion, expected pay off are

A1 = 20 x 1/3 + 25 x 1/3 + 30 x 1/3 = 25

A2 = 12 x 1/3 + 15 x 1/3 + 20 x 1/3 = 15.67

A3 = 25 x 1/3 + 30 x 1/3 + 22 x 1/3 = 25.67

Since A3 has maximum expected pay off, as per Lalace criterion, A3 is the Act to be selected.

Hurwicz Alpha criterion

This method is a combination ofmaximincriterionandminimax criterion. Inthis method, thedecisionmaker's degree of optimismsis representedby alpha -the coefficient ofoptimism. Alpha varies between 0 and 1.When alpha is = 0, there is total pessimism and when alpha is =1, there is total optimism. As per the criterion, Hurwicz value is calculated for each Act, considering maximum pay off and minimum pay off as per an Act. Hurwicz value is the total of products of maximum payoff and alpha, and minimum pay off and 1 – alpha.

Hurwicz value = Max pay off x alpha + mini pay off x 1- alpha for an Act.consider folowing pay off table. Hurwics alpha value given is = .6

Payoff table

States of nature

Acts	S1	S2	S3
A1	20	25	30
A2	12	15	20
A3	25	30	22

Hurwicz value for A1 = 30 x .6 +20 x .4 = 26

Hurwicz value for A2 = 20 x .6 + 12 x .4 = 16.8

Hurwicz value for A3 = 30 x .6 + 22 x .4 = 26.8

Since Hurwicz value is maximum for A3, it is the optimal Act. It is to be chosen.

Ex 3.1 A food products company is planning the introduction of a new productwithnewpackingtoreplacetheexistingproduct ata much higher priceA1, or a moderate price A2, or low price with a new package A3. The three possible states of nature of events are highincreaseinsales S1, no change insales S2,and decrease insales S3. Then, marketing department ofthecompany workedout thepay offs under each of these estates of nature and strategies. This is representedinthe followingtable.

Pay offs

States of Nature

Strategies	S1	S2	S3
A1	700	300	150
A2	500	450	0
A3	300	300	300

Which strategy should the manager concerned choose on the basis of

1. Maximax
2. Maximin
3. Minimax
4. Laplace criterion.

1. Maximax criterion

Max pay off as per each Act A1 = 700, A2 = 500, A3 = 300

Maximum of these maxima = 700. The optimal act is A1

Maximin criterion

Minimum pay off as per A1 = 150, A2 = 0, A3 = 300

Maximum of these minima = 300, which is provided by A3. So A3 is the optimal strategy.

Minimax Regret criterion

For this a Regret Table or Opportunity Loss Table is to be made as belo0w. opportunity loss is the difference between max pay off in a state of nature, and actual pay off, for each Act.

Opportunity Loss Table

Act	S1	S2	S3
A1	0	150	150
A2	200	0	300
A3	300	150	0

OL = Max pay of under an state of nature – Actual Pay off for an act.

Maximum opportunity loss for A1 = 150, A2 = 300, A3 = 300

Minimum of these maxima = 150. It is given by A1. Therefore A1 is the optimal Act.

Laplace criterion

As per Laplace criterion, stats of nature are equally likely. So the average pay may be found out. And maximum average pay off may be calculated for choosing optimal Act.

Average pay off A1 = 700 + 300 +150 ÷ 3 = 383.3

Average pay off A2 = 500 = 450 = 0 ÷ 3 = 316.67

Average pay off A3 = 300 + 300 + 300 ÷ 3 = 300

Since the average pay off is maximum for A1, it is the optimal Act, as per Laplace criterion.

Ex 3.2 the research departmentofconsumer products division has recommended tothemarketingdepartmentto launch a soap with three different perfumes. The marketing manager has todecide thetype of perfume tolaunch under the following estimated pay off for the various levels of sales.

Pay off Table

Perfume	Sales		
	S1	S2	S3
A1	250	15	10
A2	40	20	5
A3	60	25	3

Estimate whichshould be thedecision as per Maximax, Maximin, Laplace , and Hurwiz criteria. (givenα = .6)α

As per maxima – Maximum among maxima =250, 40, 60 = A1

As per Maximin – Max of Min pay off = 10, 5, 3 = A1

As per Laplace – Average pay off = 91.67 , 21.67, 29.33 = A1

As per Hurwicz α= .6 , 1- α = .4

Hurwics value for A1 = Max pay off x α + Mini pay off x 1- α

= 250 x .6 = 10 x .4 = 154

= 40 x .6 + 5 x .4 = 26

= 60 x .6 + 3 x .4 = 37.2

Maximum Hurwicz value is 154 which is for A1. Therefore it is optimal Act as per Hurwicz criterion.

Decision making under risk

In this situation, thedecisionmaker has to face several states of nature. But he has some knowledge or experience, which will enable him to assign probability to the

occurrence of each states of nature. The objective is to optimize the expected profit, or to minimize opportunity loss. For decision problems under risk, the most popular methods used are Expected Monetary Value criterion, and Expected Opportunity Loss criterion.

Expected Monterrey Value

When the probabilities can be assigned to the various states ofnature,it is possible tocalculate theexpectedoff for each course of action. These expected pay offsareknown as EMV.

The conditional value of each eventinthe pay off table is multiplied by its probability and the productis summed up. Theresulting number is the EMV for the act. The decision maker then selectsfromthe available alternatives,the actthat leads to the optimum expected outcome. thatthat is the

The criterion of selecting maximum expectedpay off under each act , sometimes is referred toas Baye's Decisionrule.

Expected Opportunity Loss

When the probabilities for various states ofnature areknown, it is possible tocalculatetheexpectedlosses foreach courseofaction. Expected opportunity loss is the difference between the maximum pay off under state of nature and the actual pay off obtained. Under this strategy, the course of action whichhas minimum expectedopportunity loss is chosen.

For calculating Expected Monetary Value, the probabilities of each states of nature should be given. Then EMV is the sum total of products of pay off and its concerned probability, as per an Act.

EMV = Total of pay off x probability

Ex . 3.4 You are given following pay off matrix

From the following pay off matrix, and details, calculate EMV and decidewhich of the Acts can be chosen.

Pay off Table

Perfume	Sales		
	S1	S2	S3
A1	25	400	650
A2	-10	440	740
A3	-125	400	750

Probabilities are .1, .7, .2 respectively.

EMV for A1 = 25 x .1 + 400 x .7 + 650 x ..2 =412.5

EMV for A2 = -10 x .1 + 440 x .7 + 740 x .2 =455

EMV for A2 = -125 x .1 + 400 x 0.7 + 750 x .2 =417.5

Since EMV is maximum for A2, choose the Act A2.

Ex 3.5 A management is faced with theproblem of choosing one of the products for manufacturing. The chance that market will be good , fair, or bad is .75, .15, and .10 respectively. Select the decision as per EMV criterion.

States of nature

Acts	Good	Fair	Bad
A	35000	15000	5000
B	50000	20000	-3000

EMV for A = 35000 x .75 + 15000 x .15 = 5000 x .10 = 29000

EMV for B = 50000 x .75 + 20000 x .15 + -3000 x .10 = 40800

As per Expected Monetary Value criterion, the optimal Act is B, because it gives maximum EMV.

Expected Opportunity Loss

In risk situations, managerial decisions can be taken on the basis of opportunity loss also. Here, probabilities of events or states of nature must be given.

When probabilities for various states of nature are known, it is possible to calculate Expected Opportunity Loss, for each course of action. Under this criterion, the strategy which has minimum expected opportunity loss would be chosen as optimal Act. Expected opportunity losses are calculated Act wise, ie, for each of the Act.

Opportunity loss is the loss due to not taking the best course of action or act. It is calculated as the difference between the best pay off under an Act and the actual pay off. In most cases, Opportunity Loss Table will have to be prepared from the given Pay off Table.

EOL = total of opportunity loss x probability for act.

Ex 3.6 A news paper boy buys paper at Rs 3 and sells at Rs 5 according to past experience demand per day has neve been less than 78 or greater than 80 papers. Prepare

1. pay off table
2. Opportunity loss table
3. Select EMV Decision, given , probabilities are .4, .3, and .3 respectively.
4. Select EOL decision

Ans 1. Preparation of pay off table.

A paper costs Rs 3 and can be sold at Rs 5, at a profit (pay off) of Rs 2.

An unsold paper brings a loss of Rs 3. Accordingly, the pay off table is prepared as below.

Pay Off Table

	Sales		
	S178	S279	S380
A178	156	156	156
A279	153	158	158
A380	150	155	160

2. Preparation of opportunity loss table.

Opportunity loss or regret is the loss due to not taking the best act, given a state of nature. It is obtained as the differecn e between the max pay off under a state of nature and the amout of actual pay off.

Opportunity Loss Table

	Sales		
	S178	S279	S380
A178	0	2	4
A279	3	0	2
A380	6	3	0

3. Decision as per EMV ariterion.

   EMV for A1 = 156 x .4 + 156 x .3 + 156 x .3 = 156

   EMV for A2 = 153 x .4 + 158 x .3 + 158 x .3 = 156

   EMV for A3 = 150 x .4 + 155 x .3 + 150 z .3 = 151.5

Maximum EMV is offered by two decisions – A1 and A2. So these two actions are optimal decisions.

4. Decision as per EOL criterion.

   EOL for A1 = 0 x .4 + 2 x .3 + 4 x .3 = 1.8

   EOL for A2 = 3 x .4 + 0 x .3 = 2 x .3= 1.8

   EOL for A3 = 6 x .3 + 3 x .3 + 0 x .3 = 2.7

   As per EOL criterion, that decision must be taken , which leads to minimum opportune ity loss. Minimum EOL is offered by two decisions – A1 and A2. There both of them are optimal decision, as per this criterion.

Expected pay off under perfect information

In management science, a decision maker could remove many of decisiD ddon and uncertainty problems, by obtaining complete and accurate information about future states of nature. Such information is referred to as perfect information. With perfect information, traders and business executives would know in advance how many events are like to happen, affecting decisions

A decision maker can calculate the cost of perfect or additional information. The perfect information concept leads to two possiblecalculations - Expected Profit with Perfect Information and Expected Value of Perfect Information.

Expected Profit With Perfect Information

Expected pay off under perfect information is the sum of products of maximum pay off under each state of nature and their respective probabilities. This is the maximum amount of expected pay off when perfect information regarding states of nature is available. This leads to the concept called expected Value of Perfect Information.

Expected Value of Perfect Information

A decision maker can expect average excpected pay off, even if no perfect information is available. Naturally, he will be eager to compare thecost of such perfect information.

Now Expected Value of Perfect Information is the difference between pay off under perfect information and pay off under normal information. This can be considered as thevalue of additional information in decision making process. In some case, it would be feasible to incur extra cost for acquiring additional and perfect information. Expected Value of

Perfect Information is the upper bound of the amount which the decision maker can spend for acquiring perfect information.

EVPI = EPPI - Max.EMV

Steps in calculating Expected Value 0f Perfect Information

- consider the Pay off Table
- Select highest pay off in each row.
- Calculate total expected pay off multiplying these pay offs with probabilities (EPPI)
- Calculate EMVs for each Act
- Identify maximum EMVamong several Acts
- Calculate difference between EPPI and Max EMV to get EVPI

Ex. 3.7 Compute EVPI from the Pay off table given below, known that probabilities of states of nature are .5, .4 and .1 respectively.

Pay Off Table

	Sales		
	S178	S279	S380
A178	30	20	40
A279	25	35	30
A380	22	20	35

EMV for A1 = 30 x .5 = 20 x .4 + 40 x .1 = 27

EMV for A2 = 25 x .5 + 35 x .4 + 30 x .1 = 29.5

EMV for A3 = 22 x .5 + 20 x .4 + 35 x .1 = 22.5

To ascertain EVPI, we want EPPI and Mamimum EMV. Then,

EVPI = EPPI - Max EMV.

EPPI = total of highest expected profits of every state of nature, ie, if complete information regarding state of nature is available.

= 30 x .5 + 35 x .4 + 40 x .1 + 33

Maximum EMV = 29.5

So, EVPI = 33 – 29.5 = 3.5

That is, the firm can invest upto Rs 3.5, in order to get complete information regarding uncertain state of nature.

Bayesian Rule in managerial Decision

Managerial decision maker assigns probabilities to various events whichis his subjective evaluation of likelihood of occurrence of various states on the basis of experience of past performance. When these priorprobabilities are used, the procedure is known as posterior analysis or Bayesian rule.

Bayesian rule of decision theory is an approach in which the decision maker selects a course of action on critical basis by using subjective evaluation of probability based on experience, past performance, judgment etc.

For making use of the Baye's principle in thestatistical decision problem, thedecision maker has to assign probabilities to eachstate of nature. These probabilities represent the strength of the decision makers' belief. Ie, a subjective evaluation regarding the likelihood of occurrence of various states of nature.

After determining probabilities, the Bays's principle must be used phase wise. The three phases are prior analysis, preposterous analysis and posterior analysis.

If prior analysis reveals a high EVPI, additional information are to be obtained. Prior probabilities may then, be revised on the basis of these additional information. By applying Baye's theorem of probability, the revised probabilities are computed. These probabilities are known as posterior probabilities. A further analysis of the problem, using these posterior probabilities give new expected pay offs. This revised analysis of the problem is known as posterior analysis.

Preposteriour analysis is done to assess the expected value of sample information as against the expected value of perfect information even before selecting a sample for additional information. This analysis involves the revision of probabilities using Bays's theorem. Posterior analysis involves arriving at a decision after revising probabilities.

Decision situations in certainty, uncertainty and risk have been discussed in this unit .Decision making under competition is a prominent decision making situation, which is dealt with in next unit

Review questions and exercises

1. What is a decision
2. State the importance of decision making in management science
3. What are tactical decisions
4. Explain strategic decisions
5. Explain statistical decision theory
6. State the stages in decision making process
7. What are the components of decision problem.what is Act
8. What ar states of natureexplain out comes
9. What are pay offs
10. Describe opportunity loss
11. What is regret
12. Prepare a hypothetical pay off table
13. How is Opportunity loss table constructed

14. Explain decision making situations
15. What is decision making under certainty
16. What is uncertainty in decision making
17. Explain risk situatin in decision making
18. How decision taken in competitive situatin.
19. A trader buys pen at Rs 4 and sells at Rs 6. From past experience, he knows that daily demand has between 20 and 24 pen. Construct a pay off tble.
20. From the below given pay off table, determine optimal acts, as per Maxmax, Maximin, Minimax, Laplace and Hurwics criterion. Hurwics alpha = .7.

Pay off tble

	S1	S2	S3	S4
A1	8	0	-10	6
A2	-4	12	18	-2
A3	14	6	0	8

21. Proctor and Gamble proposes to market three types of shampoos, with following marketing potential. What would be the decision as per –
    1. EMV criterion
    2. EOL criterion

Estimated sales

	S1	S2	S3
Egg shampoo	400	600	800
Clinic shampoo	150	350	400
Deluxe shampoo	100	130	180
Probabilities	.5	.3	.2

Find Expected Value of Perfect Information.

22. A company has an opportunity to computerize its records department. However, existing personnel have job secutiry under union agreement. The cost of three alternative programmes for the change over depend upon the attitude of the union and are estimated below.

Attitude	general retraining	selective retraining	Hire new
Against	940	920	900
Passive	810	800	820
Favour	700	710	860

Construct pay off table and opportunity loss table. Determine EMV and EOL decision. What is the value of perfect information.

UNIT IV DECISION MAKING UNDER COMPETITION - GAME THEORY

Many practical problems require decision making in a competitive situation where there are two or more opposing parties with conflicting interests. And where the action of one depends upon the action taken by the opponent, such a situation is termed as a competitive situation.

A great variety of competitive situations are seen in every daylife. Competitive situations occur frequently on economic and business activities. Management and labour relations, political battles and elections, etc are some of the examples of competitive situations. It is a type of decision theory which is concerned with the decision making insituations where two ormore rational opponents are involved under conditions of competition and conflicting interests.

Theory of games became popularwhenNewmanalong with Morgensten published the book titled 'Theory of Games and Economic Behavior' in 1944.

Game

Game is defined as an activity between two ormore persons, according to a set of rules at the end of which each person receives some benefit or satisfactionor suffers loss. In agame, there are two or more opposite parties withconflicting interests. They know the objectives and the rules of the game.An experienced player usually predicts with accuracy how his opponent will react if a particular strategy is adopted. When one player wins, his opponent losses

Characteristics of a competitive game

Acompetitive situation is called a game, if it has the following properties or characteristics.

1. There are finite number of competitors, called players
2. Each player has a list of finite number of possible courses of action.
3. A play is said to be played wheneach of the players chooses a single course of action from the list of courses of action available tohim.
4. Every play is associated with an outcome known as pay off.
5. The possible gain or loss of each player depends not only on the choice made by him but also the choice made by his opponent.

Assumptions of a game

1. The players act rationally and intelligently
2. Each player has a finite set of possiblecourses of action
3. The players attempt to maximize gains or minimize losses
4. All relevant information is known to eachplayer
5. The players make individual decisions
6. The players simultaneously select their respectivecourses ofaction
7. The pay off is fixed and determined in advance

Strategy

The strategy of a player is the predetermined rule by which a layer decides this course of action during the game.tht is, a strategy for a given player is a set of rules or programmes that specify which,of the available courses of action, he should select at each play.

There are two types of strategies, pure strategy and mixed strategy

Pure strategy

A pure strategy is adecision in advance of all players, always to choose a particular course of action. It is a predetermined course of action. The player knows it in advance.

Mixed strategy

A player is said to adopt mixed strategywhen he does not adopt a single strategy , all the time, but decides to choose a course of action for eachplay in accordance with some particular probability distribution. In a mixed strategy, we can not definitely say which course of action theplayer will choose. We can only guess on the basis of probability.

Player

Each participant of the game is called a player. In a competitive game,there will be two or more players, competingwith each other with conflicting interests.

Pay off

The outcome of a game in the form of gains or losses to the competing players for choosing different courses of action is known as pay offs.

Pay off matrix

In a game, the gains and the losses, resulting from different moves and counter moves, when represented in the form of a matrix, is known as a pay off matrix. Each element of the pay off matrix is the gain of the maximizing player when a particular course of action is chosen by him as against the course faction chosenby theopponent.

Ex. 4.1 Givenbelow is a pay off matrix.

		B	
		B1	B2
A	A1	2	-3
	A2	0	1

Here A is the maximizing player and B is the minimizing player. Each element in the matrix is the gain for A when he chooses a course of action against which B chooses another course ofaction. Forexample,when A chooses A2, and B chooses B1, the gain for A is known in the second row, first column. Hence it is 0.

Value of the game

The value of the game is the maximum guaranteed gain to the maximizing player A, if both the players use their best strategies. It is the expected pay off of a play when all the players of the game follow their optimal strategies.

Maximizingand minimizing players

If there are two player A and B, generally the pay off given in a pay off matrix indicate gains to A for each possible outcomes of the game,that is, each outcome of a game results into a gain for A. all such gains are shown in the payoff matrix. Usually each

row of the pay off matrix indicates gains to A for his particular strategy. A is called the maximizing player and B is called minimizing player. The pay off values given in each column of pay off matrix indicates the losses for B for his particular course of action. Therefore, if the element in the position A1B3 is a then A's gain is a and B's gain is 1 –a or B's loss is a, when A chooses the strategy A1 and B chooses strategy B3.

Maximin and Minimax

Each row in a pay off matrix represents pay offs in respect of every strategy of the maximizing player A . Similarly, each column represents pay offs in respect of every strategy of minimizing player B. Maximin is the maximum of minimum pay offs in each row . Minimax is the minimum of maximum pay offs in each column

For example

Ex 4.2

	B1	B2	B3
A1	5	3	2
A2	1	-2	0
A3	8	-1	1

Minimum in row A1 = 2

A2 = -2

A3 = -1

Maximum of these minima = Maximin =2

Maximum for column B1 = 8

" " " B2 = 3

" " " B3 = 2

Minimum of these maxima = Minimax = 2

Maximin principle

Here the minimizing player B lists his maximum losses from each strategyand selects hat strategy which corresponds to the least. This is minimax principle.

Saddle Point

A saddle point of a pay off matrix is that position in the pay off matrix where the maximin coincides with the minimax. Pay off at the saddle point is the value of thegame. In a game having a saddle point , optimum strategy of maximizing player is always to choose the row containing saddle point and for minimizing player, r tochoose the column containing saddle point. If there is more than one saddle point there will be more than one solution.

A game for which maximin for A = minimax for B, is called a game with saddle point. The element at the saddle point position is the value of thegame denoted by V

Example

Ex. 4.3

	B1	B2	B3	Row Min
A1	3	2	4	2
A2	-2	1	-3	-3
A3	0	-2	3	-2
Column Max	3	2	4	

Maximin = Max of Row min = 2

Minimax = Min of column max = 2

Maximin = Minimax = 2, which refers to A1 B2

Saddle Point = A1B2

Value of the game = v = 2

Different types of games

Games are categorized on the asis of

Number of players

Number of moves

Nature of pay off

Nature of rules

Zero sum game

In a game , if the algebraic sum of the outcomes or gains of all the players together is zero,the game is called zero sum game,otherwise it is called Non- zero sum game. That is, in zero sum game, the amounts won by all winners together is equal to the sum of the amounts lost by all together.

A game involving n players is called n persongame and agame withtwo players is called two persongame.

Two person zero sumgame

Two person zero sum game is the simplest of game models. There will be two persons in the conflict and the sum of the pay offs of both together is zero. That is, the gain of one is at theexpenseof theother. Such a game is also called rectangular game. The two person zero sum game may be pure strategy game or mixed strategy game.

Basic assumptions in two person zero sum game

1. There are two player
2. They have opposite and conflicting interests
3. The number of strategies available toeachplayer is finite
4. For each specific strategy, selected by a layer, there results a pay off
5. The amount won by one player is exactly equal to the amount lost by the other.

Limitations of Game theory

1. In fact, a player may have infinite number of strategies. but we assume that there are only finite number of strategies
2. It is assumed that each player has the knowledge of opponent's strategies. But it is not necessary in all cases.
3. The assumption that gain of one person is the loss of his opponent ,need not be true in all situations
4. Game theory usually ignores the presence of risk and uncertainty.

5. It is assumed that pay off is always known in advance. But sometimes it is impossible to know the pay off accurately.
6. It is assumed that the two persons involved in the game have equal intelligence. But it need not be.

Fair game

A game is said to be fair if the value of the game = 0. In a fair game, the win of maximizing player would be equal to the loss of the minimizing player.

Solution of pure strategy games

The maximizing player arrives at his optimal strategy on the basis of maximin criterion, while minimizing players' strategy is based on the minimax criterion. The game is solved by equating maximin value with minimax value. In this type of problems saddle point exists.

Ex 4. 4 From the following game matrix, find the saddle pint nd value of th game.

		B	
		B1	B2
A	A1	6	2
	A2	-1	-4

Row minima = 2, -4 = -4 Maximin = Maximum of minima = 2

Column maxima = 6, 2 = 6 minimax = Minimum of maxima = 2

So saddle point is A1B2 and value of the game is 2

Ex 4.5

For the following pay off matrix , determine the optimal strategies for both the firms M and N, and find the value of the game using maxinin - minimax principle

		M	
		M1	M2
N	N1	1	0
	N2	-4	3

Row minima = 0, -4 Max = 0

Column maxima = 1, 3 Mini = 1

Here, Maximin is not equal to Minimax. There is no saddle point.

Ex 4.6

Following is a pay off matrix. What is the value of game? Who will be the winner of the game? Why?

	Y	
X	1	-2
	2	-1

Row minima = -2 , -1 Max = -1

Column maxima = 2 , -1 Mini = -1

Maximin = Minimax

Therefore, saddle point exists X2Y2

Value of the game = -1

Since the value of the game is negative Y wins and X loses. Gain of X = -1

Ex. 4.7

Solve the game whose pay off matrix is given by

	B1	B2	B3	Row Min
A1	10	2	3	2
A2	6	5	7	5
A3	-7	4	7	-7
Column Max	10	5	7	

Maximin equals Minimax at A2B2

This game has a saddle point at A2 B2

The optimal strategy for player A is A2

The optimal strategy for player B is b2

The value of the game is 5.

Solution of mixed strategy problems

When there is no saddle point for a game problem, the minimax-maximin principle can not be applied to solve that problem. In those cases the concept of chance move is introduced. Here the choice among a number of strategies is not the decision of the player but by some chance mechanism. That is, predetermined probabilities are used for deciding the course of action. The strategies thus made are called mixed strategies. Solution to a mixed strategy problem can be arrived at by any of the following methods.

Probability method

This method is applied when there is no saddle point and the pay of matrix has two rows and two columns only. The players may adoptmixed strategies withcertain probabilities. Here the problem is to determine probabilities of different strategies of both players and the expected payoff of the game.

Example

	Player B	
Player A	B1	B2
A1	a	b
A2	c	d

Let p be the probabilityof A using strategy A1 and 1-p be the probabilityfor A using A2. Then we have the equation. Expectedgain of A if B chooses B1 = ap + c (1-p)

Expected gain of A if B chooses B2 = bp + d (1-p)

ap + c (1-p) = bp + d (1-p)

Solving theequation ,we get $p = \left(\frac{(d-c)}{(a+d)-(b+c)}\right.$

Similarly,let q and 1-q be respectively probabilities for B choosing strategies B1 and B2, then

aq + b(1-q) = cq + d(1-q)

Solving the equation q = = $\left(\frac{(d-b)}{(a+d)-(b+c)}\right.$

Expected value of the game = $\sum$ x prob = apq + bp (1-q) + c (1-P) q + d (1-P) (1-q)

Substituting the value of p and q and simplifying v = $\left(\frac{(ad-bc)}{(a+d)-(b+c)}\right.$

Therefore solution is Strategies of A are p, 1-p

Strategies for B are q , 1-q

Value of the game = v = p = $\left(\frac{(d-c)}{(a+d)-(b+c)}\right.$ $q = \left(\frac{(d-b)}{(a+d)-(b+c)}\right.$

$$v\left(\frac{(ad \quad bc)}{(a+d)-(b+c)}\right.$$

Example 4.2 Find p, q and v fromthefollowing problem

Here a = -2, b = -1, c = 2 and d = -3

P = $\left(\frac{(d-c)}{(a+d)-(b+c)}\right.$ $\left(\frac{(-3-2}{(-2-3)-\left(^{-1+2}=\right)}\right.=\frac{5}{6}$

$q = \frac{(d-b)}{(a+d)-(b+c)} = \frac{(-3--1)}{(-5-1)} = \frac{1}{3}$

$V = \frac{(ad-bc)}{(a+d)-(b+c)} = \frac{(-2\,x3-(-1\,x\,2)}{(-6)} = \frac{4}{3}$

Principle of dominance

The principle of dominance states that if the strategy ofla player dominates over another strategy in ll conditions, then the latter strategy can be ignored because it will not affect the solution in any way.A strategy dominates over the other only if it is preferable in all conditions.

Conditions

1. If all the elements in a row of apay off matrix areless than or equal to the corresponding elements ofanother row, thenthe latter dominates and soformer is ignored.,

Example

1 3 4 5
-2 1 4 0

Here every element of second row is less than or equal to the corresponding elements of 1 row.Therefore first row dominates and so second row can be ignored.

2. If all the elements in a column of a pay off matrix are greater than or equal to the corresponding elements of another column, then the former dominates and so latter is ignored.

2	6
-3	2
4	4
2	3

Here the elements of second column are greater than or equal to the corresponding elements of first column. So second column dominates and first column can be ignored.

3. If the linear combination of two or more rows or columns dominates a row or column,then the latter is gnored.

If all the elements of a row are less thanor equal toaverageof the corresponding elements of two other rows,then the former is ignored

For example,

2	0	2
3	-1	3
1	-2	1

Here the elements of third column are greater than or equal to the average of corresponding elements of two other columns, so third column dominates, and other columns can be ignored.

Ex 4.8 solve the game whose pay off matrix is given by

	B1	B2	B3	Row Min
A1	1	7	2	1
A2	6	2	7	2
A3	5	1	6	1
Column Max	6	7	7	

Applying principle of dominance , B3 is dominated by B1. So ignore B3. The reduced matrix is

	Player B	
Player A	B1	B2
A1	1	7
A2	6	2
A3	5	1

Now A3 is dominated by A2. So ignore A3. The resulting matrix is

	Player B	
Player A	B1	B2
A1	1	7
A2	6	2

Let p and 1-p be the probabilities for A choosing A1 and A2. Let q and 1-q, be the probabilities for B choosing B1 and B2. Then,

$p = \left(\frac{(d-c)}{(a+d)-(b+c)}\right) \quad p = \left(\frac{(2-6)}{(1+2)-(6+7)}\right) = 2/5 \quad 1-p = 3/5$

p A choosing A1 = 2/5

p A choosing A2 = 3/5

$$q = \frac{(d-b)}{(a+d)-(b+c)} \quad \frac{(2-7)}{(1+2)-(7+6)} = ½ \quad 1\text{-}p = ½$$

p B choosing B1 = ½

p B choosing B2 = ½

Expected value of the game = $\frac{(ad-bc)}{(a+d)-(b+c)} = 4$

A's mixed strategy = 2/5, 3/5

B's mixed strategy = 1/2, 1/2

Principle of dominance is applicable to pure strategy and mixed strategy problems.

Review Questions& Exercises

1. What is game theory
2. State the importance of game theory
3. What is the relation between competition and game
4. Who are players in a game . Define a competitive game
5. What is a pay off matrix
6. What are pure strategies?
7. Distinguish between pure and mixed strategies
8. Explain saddle point
9. What is two person zero sum game
10. What do you mean by Maximax –Minimax principle.
11. What are the assumptions of game theory
12. Explain dominance principle
13. State probability method of solving a game theory
14. Find saddle point of following game

15.

Find saddle point

		B	
		B1	B2
A	A1	1	0
	A2	-4	3

16 Solve the following game and find value of the game

	B1	B2	B3
A1	15	2	3
A2	6	5	7
A3	-7	4	0

17. Find strategies of the players and value of the game

		B	
A	6	8	6
	4	12	2

18. Two players A and B, without showing each other, put a coin on a table, with head or tail up. If the coins show the same side (both head and tail) the player A takes both the coins, otherwise B gets them. Construct the matrix of the game and solve it.

19. Given the pay off matrix for play A obtain the optimum strategies for both the players and determine the value of the game.

		Player B		
Player A		6	-3	7
		-3	0	4

20. Solve the game and find saddle point

	B1	B2	B3
A1	15	2	3
A2	6	5	7
A3	-7	4	0

UNIT 5 – LINEAR PROGRAMMING

Introduction

The most important function of management is decision making . A large number of decision problems faced by an executive involve allocations of resources to various activities with the objective of increasing profit or decreasing cost or both. When resources are unlimited, there is no difficulty, but such cases are very rare. Generally managements are confronted with the problem of limited resources. The manager has to take decision as to how best such resources be allocated among various activities to get desired results.

Programming problems in general deal with determining optimal allocation of limited resources to meet given objectives. The resources may be in the form of men, materials,machines etc. There are certain restrictions on thetotal amounts of eachproduct made. The objective is to optimize total profit or total cost, Using the limited resources, production can be planned ,subject to various restrictions. A linear programmingproblem includes a set of simultaneous linear equations for inequalities which represent the restrictions related to the limited resources and a linear function which expresses the objective function representing the total profit or cost.

The term linear means that all the relations in the problem are proportionate , and the term programming refers to theprocess of determining aparticular progamme or plan of action. Thus linear programming planning operations which have proportionate inter relations.

George B Dantzig is recognized as the pioneer of linear programming. In 1947 he published the first paper on simplex method. Since 1947, many other researchers joined him in developing and exploring new applications of linear programming. Now Linear programming problem is being applied to many areas of human activity. Its application has increased with thedevelopment of computer technology.

Definition

Linear programming may be defined as a method of determining an optimum programme of independent activities in view of available resources. The objective of Linear Programming Problem is to maximize profit or minimize cost, as the case may be subject to number of limitations known as constraints. For this, an objective function is constructed which represents total profit or total cost as the case may be. The constraints are expressed in the form of inequalities or equations. Both the objective function and constraint are linear relationship between the variables. The solution to a Linear Programming Problem shows how much should be produced (or sold or purchased) which will optimize the objective function and safety the constraints.

Uses of Linear Programming

Linear programming technique is used to achieve the best allocation of available resources. Available resources may be man-hours, money, machine-hours, raw materials etc. As most of these resources are available for a limited supply, the allocation of these should be done in a manner which will optimize the objective of maximizing profit or

minimizing cost. Linear programming is powerful quantitative technique which is useful to solve such problems. Consider following examples

1. A production manager wants to allocate the available machine time, labour and raw materials to the activities of producing different products. The manager would like to determine the number of unit of the products to be produced so as to maximize the profit.
2. A manufacturer wants to develop a production schedule and an inventory policy that will minimize total production and inventory cost.
3. A manager wants to allocate fixed advertising budget among alternative advertising Medias such as radio, television, newspapers and magazines. He wants to determine the media scheduled that maximizes the advertising effectiveness.

In the case of these similar problems, linear programming technique can be applied to arrive at best solution.

Applications of Linear Programming in Industry and Management

Linear programming is exclusively used to solve variety of industrial and managemenTransportation problems. Few example, where linear programming can be applied are given bellow.

1. ***Product Mix***: This problem essentially deals with determining the quantum of different product to be manufactured knowing the managerial contribution of each product and amount of available used by each product. The objective is to maximize the total contribution subject to constraints formed by available resources.
2. ***Product Smoothing***: This problem in which the manufacturer has to determine the best plan for producing a product with a fluctuating demand.
3. ***Media Selection***: The problem is to select advertising mix that will maximize the number of effective exposure subject to constraints like total advertising budget, usage of various media etc.
4. ***Travelling Salesman Problem***: The problem is to find the shortest route for a salesman starting from a given city, visiting of each specified cities and returning to the original point of departure.
5. ***Capital Investment***: The problem is to find the allocation to which maximize the total return when a fixed amount of capital is allocated to a number of activities.
6. ***Transportation Problem***: Using transportation technique of linear programming we can determine the distribution system that will minimize total shipping cost from several warehouses to various market locations.
7. ***AssignmenTransportation problems***: The problem of assigning the given number of personnel to different jobs can be tackled with the help of assignment model. The objective of may be to minimize the total time taken or total cost.
8. ***Blending Problem***:These problems are likely to arise when product can be made from variety of available raw materials of various composition and prices. The problem is to find the number of unit of each raw material to be blended to make one unit of product.
9. ***Communication Industry***: Linear programming methods are used in solving problems involving facility for transmission, switching, relaying etc.

10. ***Rail Road Industry***: Linear Programming technique can be used to minimize the total crew and engine expenses subject to restriction on hiring and paying the trainmen, scheduling of shipment, capacities of rail road's etc.
11. ***Staffing Problem***: Linear Programming technique can be used to minimize the total number of employees in restaurant, hospital, police station etc. meeting the staff need at all hours.

Requirements of Linear programming

Linear programming has been applied extensively in the past tomilitary,industrial, financial marketing and agriculture problems. Even thoughthese applications are diverse, all Linear programming problemshave four properties in common.

Objective

There must be an objective the firm wants to achieve. The major objectives of a manufacturer is to maximize profits. Whenever the term profit is used in this context of LP, it actually refers to contribution. In certain organizations the objective might be to minimize the cost. In many events, this objective must be clearly stated and mathematically defined.

Alternatives

There must be alternative courses of action, one of which will achieve the objective. For example, if a company produces three different products,management may use LP to decide how to allocate among them its limited resources. Should it devote al manufacturing capacity to make the first product, should it allocate theresources toproduce equal amounts of each product, should it locate theresources insame or other place? These questions have to be answered, through alternative options.

Limited resources

There are certain restrictions or limitations to our resources. For example, how many units of eachproduct to manufacture is restricted by available manpower and machinery. Financial portfolio is limited by the amount of money available to be spent or invested. We want, therefore, tomaximise or minimize a quantity subject , of limited resources or constraints.

Objectives

We must be able to express the firms' objective and its limitations as mathematical equations or inequalities. These equations must be linear. These linear mathematical relationships just mean that all the terms used in theobjective function and constants are of th first degree.

Assumptions

Following are the basic assumptions of linear programming. Technically these are additional requirements for an allocation problem.

Certainty

A very basic assumption is that the various parameters namely objective is known with certainty. Thus the profit or cost per unit of product , requirements of material and

labour per unit and availability of material andlabour, are to be given in the problem and theydo not change with thepassage of time

Proportionality

The assumption of linear programming model proportionality exists in the objective function and constraints inequalities. For example, if one unit product contributes Rs. 5 towards profit, then 10 units of product,, it would be Rs 50 and for 20 units it would be Rs 100 and so on. If theout put and sale is doubled, theprofit will also be doubled.

Additivity

The assumption of additivity underlying thelinear programming model is that in both, the objective function and constraints inequalities, the total of all the activities is given by the sum total of each activity conducted separately. Thus, the total profit intheobjective function is determined by thesum of the profits contributed by such of the products separately. Similarly, thetotalamount of theresources used is equal to thesum of theresources used by various activities.

Divisibility

The assumption of divisibility of the linear programming model is that the solution need not be inwhole number. Instead, they are divisible and may take any fractional value. If product cannot be produced in fraction, an integer may take any fractionalvalue. If product cannot be produced infraction, an integer programming problem exist and can be solved.

Finite choices

A linear programming also assumes that a finite number of choices are available to a decision maker to find out optimum solution. The solution, when there are infinite number of alternatives, activities and resources exist, theoptimum solution can be achieved.

Optimality

In the LP problem, in the maximum of profit or in the minimum oof cost, it always occurs at thecorner pints of the set of feasible solution. If the optimum solution is not sure to get, we may get different type of solution under different circumstances.

Merits of linear programming

Scientific approach

Linear programming problem helps in studying theinformation of anorganisation in sucha way that it depicts clear picture of theproblem. This scientific approach to theproblem is as valuable andnecessary as is thesolution.

Multiple solutions

Management problems are so complex that it is very difficult to arrive at the best alternative solution. With the use of LPP technique managers consider all possible solutions to the problem in the context of multiple solutions.

Cost benefit analysis

LPP helps the managers to plan and execute thepolicies of thetop management in such a way that costs or penalties involved are minimum. Management always put restrictions under which the manager must operate, and Lpp helps to make maximum use of limited resources.

Flexibility

After theplans are prepared , it can berevaluated for changing conditions. LP is one of the best techniques to be used under the changing circumstances to provide flexibility.

Optimum

The objective helps to allocate thescarce resources insuch a way that profit is maximum or cost is minimum and ensures optimum use of productive factors by indicating the best use of existing facilities.

Demerits of linear programming

Divisibility

We have assumed that LP model are based on divisibilityof resources. It means all solution variables should have any value, but in certainsituations, like in a product mix problem, It can be fractional units.

Linearity

In actual practice many objective functions and constraints can be expressed linearly. In case therelation is non linear, the solution obtained by LPP can not be taken as correct

Certainty

Coefficients of theobjective function and theconstraints equation must be known. These coefficients should not change during theperiod ofstudy. But practically in real life, situations, this is not possible.

Conflicting goals

If the management has conflicting, multiple goals, then linear programming problem will fail to give efficient solutions.

Unused capacities

In certainsituations, when by product is produced with thescarp material, then additional resources involve and if by product is not produced thenit will remain unutilized. Suchsituations may not be handled withlinear programming problems.

Complexity

If there are alarge number of variables andconstraints, then formulated mathematicalmodel become complex. Its solution involve largenumberof calculations and iterations whichcan be solved with thehelp of computer only.

Optimality

Maximum profit solution or minimum cost solution always occurs at a corner point of the set of feasible solutions. But sometimes, these points may not be obtained. That is why there are infeasible solutions.

Formulation of Linear programming problem

Construction of suitable model to explain the given situation is the starting point in LPP. The problem must be stated in an understandable manner, as per following steps.

1 variables

Any problem will have cetin critical factors, whichwill ultimately determine thesolution to theproblem.Such critical factors must be traced and identified as variables.

For example , ina product mix problem, the variables will be the number of eachtypes of products to be produced.

Objective

This is a keyrequirement that we should identify the objective or goal of a problem. Generally, theobjective may be maximization of profit or minimization of cost or loss. Mathematically theobjective is expressed in theform of an equation, relating thevariables and the profit or cost on each type of product.

Constraints

Next, the resources which are limited in nature must be ascertained. They are called constraints, which are stated as conditions in theproblem,. Constraints are those relating to availabilityofresources, or conditions on quantities to be purchased or made or sold etc. Constraints are expressed inthe form of inequalities or equations.

Non negativity

It is an expression that thevariables cannot assume negative values of negative quantities. Otherwise theobjective function or constraints may be nullified. Non negativity is expressed as equations.

Format

Finally, state the problem in a format as below

Find values of x and y, which

Maximizes $Z = 10x + 20y$

Subject to $2x + 3y \leq 30$

$4x + 3y \leq 36$

Where $x, y \geq 0$

Ex 5.1

The manager of anoil refinery must decide on the optimalmix of 2 possible blending processes of which theinputs and outputs per productin run are as follows

	crude A	Crude B	Diesel X	Diesel B
1	6	3	6	9
2	5	6	5	5

Maximum availability of crude A and B are 250 units and 200 units respectively. The market requirement shows that at least 150 units of Diesel X and 130 units of Diesel Y must be produced. The profits per production run from process 1 nd 2 are Rs 40 and Rs 50 respectively. Formulate theproblem formaximizing theprofit

Let x, y be th e number of production runs of the two processes respectively.

The objective is to maximize profit $Z = 40 \times x + 80 \times y = 40x + 80y$

The constraints of theproblems are :

$6x + 5y \leq 250$ crude material constraint

$3x + 6Y \leq 200$ crude materialconstraint

$6x + 5y \geq 150$ demand constraint

$9x + 5y \geq 130$demand constraint

Now theformulated problem is

Max	$Z = 40x + 80y$
Subject to	$6x + 5y \leq 250$
	$3x + 6Y \leq 200$
	$6x + 5y \geq 150$
	$9x + 5y \geq 130$
Where	$x, y \geq 0$

EX 5.2: A manufacturer of furniture makes two products, chair and tables. Processing of these products is done on two machines A & B. A chair requires 2 hours on machine A and 6 hours on machine B. A table requires 5 hours on machine A and no time on machine B. There are 16 hours of time per day available on machine A and 30 hours on machine B.Profit gained by manufacturer from a chair is RS 1 and from table is RS 5 respectively. Formulate the problem into L.P.P. in order to maximize the total profit.

Ans: Let x_1 be the number of chairs and x_2 be the number of tables produced.

Profit from chair = $1 \times x_1$

Profit from table = $1 \times x_2$

$\therefore$ Total Profit = $Z = x_1 + x_2$

Constraints

(i) Machine A

Time required to for chairs = $2 \times x_1 = 2x_1$

Time required for tables = $5 \times x_2 = 5x_2$

$\therefore$ Total time required = $2x_1 + 5x_2$

Available time = 16 hrs

$\therefore 2x_1 + 5x_2 \leq 16$

(ii) Machine B

Time required to for chairs = $6 \times x_1 = 6x_1$

Time required for tables = $0 \times x_2 = 0x_2$

$\therefore$ Total time required = $6x_1 + 0$

Available time = 30 hrs

$\therefore 6x_1 + 0 \leq 16$

Thus L.P.P. is find x_1 and x_2 find which

Maximize: $Z = x_1 + x_2$

Subject to $2x_1 + 5x_2 \leq 16$

$6x_1 \leq 30$

$x_1 = 0, x_2 = 0$

EX: 5. 3: A home resourceful decorator manufactures two types of lamps say A and B. Both lamp go through two technicians first a cutter and second a finisher. Lamp A requires 2 hours of cutter's time and 1 hour of finisher's time; Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 104 hours and finisher has 76 hours of available time per eachmonth. Profit on the Lamp A is RS. 6.00 and on one B Lamp is RS. 11.00. Formulate a mathematical model.

Ans: A decorator manufactures x1 and x2 Lamps pf type A and B respectively.

Total profits (in RS) = $Z = 6x_1 + 11x_2$

Constraints

(i) Total time of the cutter used in preparing x1 lamps of type A and x2 of type B is $2x_1 + x_2$. But available time only 104 hours.

$\therefore 2x_1 + x_2 \leq 104$

(ii) Similarly, the total time of the finisher used in preparing x_1 lamps of type A and type B is $x_1 + 2x_2$. But available time is 76 hours.

$\therefore x_1 + 2x_2 \leq 76$

Hence, the decorator's problem is to find x_1 and x_2 which

Maximize $Z = 6x_1 + 11x_2$

Subject to $2x_1 + x_2 \leq 104$

$x_1 + 2x_2 \leq 76$

$x_1 \geq 0, x_2 \geq 0$

EX 5. 4: A Company produces two types of cow boy hats. Each hat of the first type requires twice as much labour time as second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second types to 150 and 250 hats. Assuming that the profit per hat are RS 8 and for type I and RS 5 for type II, formulate the problem as a linear programming model in order to determine the number of hat to be produced of each type of so as to maximize the profit.

Ans: Let number of hats of Type A and Type B produced be x_1 and x_2 respectively.

Total profit = $(8 \times x_1) + (5 \times x_2) = 8x_1 + 5x_2$

Constraints

(i) Time required:

for hat of first type = $2 \times x_1 \times t$

for hat of second type = $x_2 \times t$

where 't' is the time for one hat of second type.

Total time required = $2x_1t + x_2t$

time available for that of 500 second per type = 500 t

$2x_1t + x_2t \leq 500$ t

ie $2x_1 + x_2 \leq 500$.

(ii) x_1 is to be limited to 150 i.e. $x_1 \leq 150$

(iii) x_2 to be limited to 250 i.e. $x_2 \leq 250$

$\therefore$ The linear programming problem is to find x_1 and x_2 which

Maximize: $Z = 8x_1 + 5x_2$

Subject to $2x_1 + x_2 \leq 500$

$x_1 \leq 150$

$x_2 \leq 250$

$x_1, x_2 \geq 0$

EX: 5.5: A firm can produce three types of cloths say A, B and C. Three kind of wool were required for it, say red wool; green wool and blue wool. One-unit length of type B cloth needs 2 yards of red wool and 3 yards of red wool, 2 yards green and 2-yard blue wool, and

one unit of type C need 5 yards of green wool and 4 yards of blue wool. The firm has only 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one-unit length of type A cloth RS 3.00, of type B cloth is RS 5.00 and of type C cloth is RS 4.00. Formulate mathematical model to the problem.

Ans:

Kinds of Wool	Type of Cloth			Stock of wool available with the firm (in yards)
	A	B	C	
Red	2	3	0	8
Green	0	2	5	10
Blue	3	2	4	15
Income from one unit	3	5	4	

Let the firm produce x_1, x_2, x_3 yards of three types of cloths A, B and C respectively.

Total profit in RS of the firm is given by

$$Z = 3x_1 + 5x_2 + 4x_3$$

Constraints:

Total quality of red wool required to prepare x_1, x_2, x_3 and C is $2x_1 + 3x_2 + 0x_3$. Since the stock of red wool available 8 yards only. $2x1 + 3x2 + 0x3 \leq 8$

Similarly,

the constraints connecting the quality of green wool is $0x_1 + 2x_2 + 5x_3 \leq 10$

and blue wool is $3x_1 + 2x_2 + 4x_3 \leq 15$.

Hence, problem of the firm formulated as linear programming problem is to find x_1, x_2, x_3 which

Maximize $Z = 3x_1 + 2x_2 + 4x_3$

Subject to $2x1 + 3x2 + 0x3 \leq 8$

$0x_1 + 2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x1 \geq 0, x2 \geq 0, x3 \geq 0$

EX: 5. 6: A marketing manager wishes to allocate his annual advertising budget of RS 20000 in two media vehicles A & B. The unit cost of a message in media A is RS 1000 and that of B is 1500. Media A is monthly magazine and not more than one insertion is desired in one issue. A least 5 message should appear in the media B. The expected effective audience for unit message in media A is RS 40000 and for media B is 55000. Develop mathematical model.

Ans: Let x_1 and x_2 be annual number of insertion for media Aand B respectively.

The effective audience = 40000 per issue A and 55000 per issue B.

Total audience (Z) = $40000x_1 + 55000x_2$

The constraints are: -

1. Total cost is limited to RS 20000. $1000x_1 + 15000x_2 \leq 20000$
2. Number of issues of "A" in a year is 12 and number of insertions should not exceed one per issue.

   So x_1 should exceed 12. i.e. $x_1 \leq 12$
3. The least of '5' in 'B' $x_2 \geq 5$

Therefore, the problem is to find x_1 and x_2 which

Maximize $Z = 40000x_1 + 55000x_2$

Subject to

$$1000x_1 + 15000x_2 \leq 20000$$
$$x_1 \leq 12$$
$$x_2 \geq 5$$
$$x1, x2 \geq 0$$

Review Questions and Exercises

1. What is optimal utilization of resources.
2. What does the term linear mean
3. What is meant by linear programming
4. What are the requirements of linear programming
5. State the assumptions in linear programming
6. What are alternative courses of action
7. What is meant by objective function
8. Explain proportio0nality
9. Explain additivity
10. What is divisibility
11. Explain optimality
12. What is non negativity
13. State the characteristics of LPP
14. Explain applications of LPP
15. State steps in formulation of LPP
16. Explain merits of Lp model
17. What are disadvantages of LPP
18. A trader wants to purchase a number of fans and sewing machines. He has only Rs 5760 toinvest and has space atmost for 20 items. A fan costs him Rs 360 and a sewing machine Rs 240. His expectation is that lhe can sell a fan at a profit of Rs 22 and a sewing machine at a profit of Rs 18. Assuming that he can sell all the items he can buy, how should he invest his money in order to maximize his profit.
19. A firm has two types of pens – A and B . Pen A is a superior quality and pen B is lower quality. Profits on pen a and Pen B are Rs 5 and Rs 3 respectively. Raw materials required for each pen A is twice as that of pen B. the supply of raw material is sufficient only for 1000 pen B per day. Pen A requires a special clip and only 400 such clips are available per day. For pen B, only 700 clips are available per day. Formulate the problem into a LPP.

20. A toy company manufacture two types of dolls a basic version doll A and a deluxe Version doll B. Each doll of type B takes twice as long as to produce as one type A and the company would have time to make maximum of 2,000 per day if it produced only the basic version. The supply of plastic is sufficient to produce 1,500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are 600 per day available. If the company makes profit of Rs.3.00 per doll and Rs.5.00 per doll respectively on doll A and B, how many of each should be produced per day in order to maximize profit?
21. A small scale manufacture has production facilities for producing two different products. Each of the products requires three different operations: grinding, assembly and testing. Product I requires 15, r20 and 10 minutes grind, assembly and testing respectively where-as Product II requires 7.5, 40 and 45minutes for grinding, assembly and testing .The production run calls for at least 7.5 hours of grinding time, at least 20 hours of assembly time and at least 15 hours of testing time. If Product I costs Rs.60 and Product II costs Rs.90 to manufacture, determine the number of each product the firm should produce in order to minimize the cost of operations.
22. A company produces two articles A and B. There are two different departments through which the articles are processed, viz assembly and finishing. The potential capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. Production of one unit of A requires four hours in assembly and 2 hours in finishing. Each of the unit B requires 2 hours in assembly and 4 hours in finishing. If profits is Rs..8 for each unit of A and Rs..6 for each unit of B find out the number of units of A and B to be produced each week to get maximum profit(solve graphically)

UNIT 6 GRAHIC SOLUTION TO LPP

A Linear Programming Problem is formulated in order to obtain a solution, ie, to ascertain the unknown variables – x and y. an optimal solution to an LPP is obtained by choosing from several values of decision variable. An optimal solution will be one set of values that satisfies thegiven set of constraint simultaneously and also provides maximum profit or minimum cost, as per given objective function.

LPPs can be solved by two methods - Graphic method or Simplex method.. Lpps involving two variables can be solved by graphical method. In th graphical method, the constraints are plotted as straight lines. The two variables are represented on x and y axis. A feasible area is identified and solution obtained by trial and error method.

Graphic method

Graphic method applies the method of two dimensional graph, consisting of x and y axis. Linear programming problems involving two variables can be solved by Graphical method .This method is simple and easy to apply. A layman can easily apply this method to solve a LPP.

But Linear programming problems involving more than two variables cannot be solved by this method. Each constraint is represented by a line. If there are many constraints, many lines are to be drawn. This will make the graph difficult to read.

Steps in Graphic method

The procedure for solving a LPP by graphic method is

1. Formulate the problem into a Linear Programming Problem.
2. Each inequality in the constraint may be written as equality.
3. Draw straight lines corresponding to the equations obtained in step 2.
   (So there will be as may straight lines as there are constraint inequations.)
4. Identify the feasible region. Feasible region is the area which satisfies all the constraints simultaneously.
5. The vertices of the feasible region are to be located and their co-ordinates are to be measured with the objective function.

How to draw constraint Lines

For each constraint inequation, there will be a straight line. A straight line connects two points. These points are obtained as below . For example, an inequation is $20x + 30y = 120$

First take $y = 0$, then $20x = 120$. And $x = 120/20 = 6$ = point one
Then take $x = 0$, $30y = 120$, and $y = 120/30 = 4$ = point two.

Now draw straight line connecting 6 and 4 on the two axes. Similarly all constraints inequations are plotted on the graph as separate straight lines.

Optimality of graphical solution

While obtaining optimal solution to LPP by the graphical method, following theorems are relevant

1. The collection of all constraint inequations constitute convex set whose extreme points correspond to basic feasible solution
2. There are finite number of basic feasible solutions within feasible solution base
3. In a convex set of feasible solutions form a polygon, at least one of the corner points gives optimal solution.
4. If the optimal solution occurs at more than one point, one of the solution can be accepted at optimal combination point.

Graphs of equations

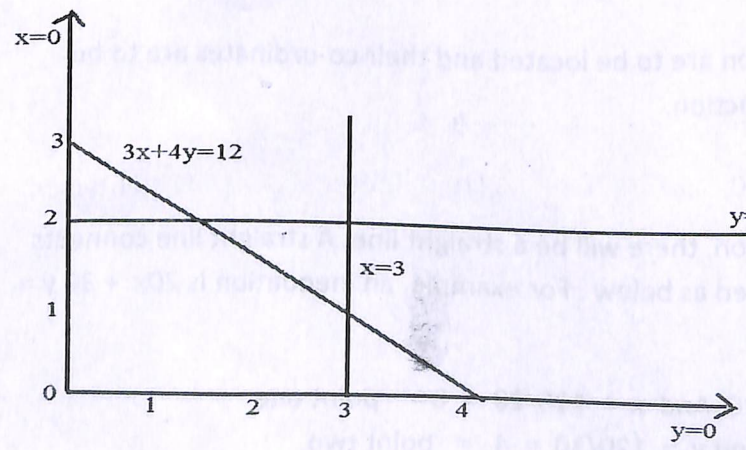
How to draw a line of equation is a critical aspect. Each line represents an equation. A line consists of two points, which are derived from equations. For example the equation $3x = 4y = 12$ can be drawn as a line on a graph.

First Put 0 for x, then $3x = 0$, $4y = 12$. So, $y = 3$

Then put 0 for y, $4y = 0$. Now, $3x = 12$. So $x = 4$

Thus we get the two points - $x = 4$ and $y = 3$. Plot them and join them, and we get the line.

Similarly, two other equations $x = 3$, and $y = 2$, can be drawn as straight lines. These three lines are depicted below.



Ex.6. 1: Indicate on a graph paper, the region satisfying the following restraints.

$X \geq 0, y \geq 0, 2x+y \leq 20, 2y + x=20$

Under the above conditions maximize the function x+3y

Ans:

I step: write all the constraints in the form of equation: then they are

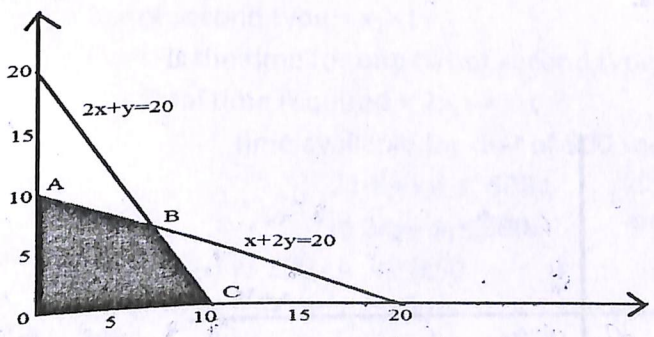
x=0; y=0

2x+y=20

x+2y=20

II step: Draw these lines:

1) For x=0 draw the y-axis
2) For y=0 draw the x-axis
3) For 2x+y=20, find two points on the line
   Put any value for x, say 0 then y=20
   therefore, one the point is (0, 20)
   Put any value for y, say 0 then 2x=20 or x=10
   Therefore, Another point is (10, 0)
   Plot points (0, 20) and (10, 0) and join them we get the straight line DC
4) For x+2Y=20 also find two points
   Put x=0, we get y=10 therefore, (0, 10) is a point
   Put y=0, we get x=20 therefore, (20, 0) is also a point
   Plot (0, 10) and (20, 0) and join them we get this line, AE.
   Draw all the four lines. They provide the boundaries of the feasible region.
   The feasible region is OABC (shaded)



Co-ordinates of O,A,B,C are (0,0),(0,10),(6.7,6.7),(10,0).[Coordinates of B is obtained by solving the equations 2x+y=20 and x+2y=20]

Substituting these values in the function x+3y as shown below:

Point	x	y	x+3y
O	0	0	0+0=0
A	0	10	0+30=0
B	6.7	6.7	6.7+20.1=26.8
C	10	0	10+0=10

Therefore, Maximum of x+3y is 30 which is at the point A

Therefore, The solution is x=0,y=10 and maximum value of x+37 is 30

Ex. 6. 2: solve following problem graphically

Max: $Z=60x_1+40x_2$

$2x_1+x_2 \leq 60$

$x_1 \leq 25$

$x_2 \leq 35$

$x_1 \geq 0, x_2 \geq 0$

Ans: Reading the constraints as equations.

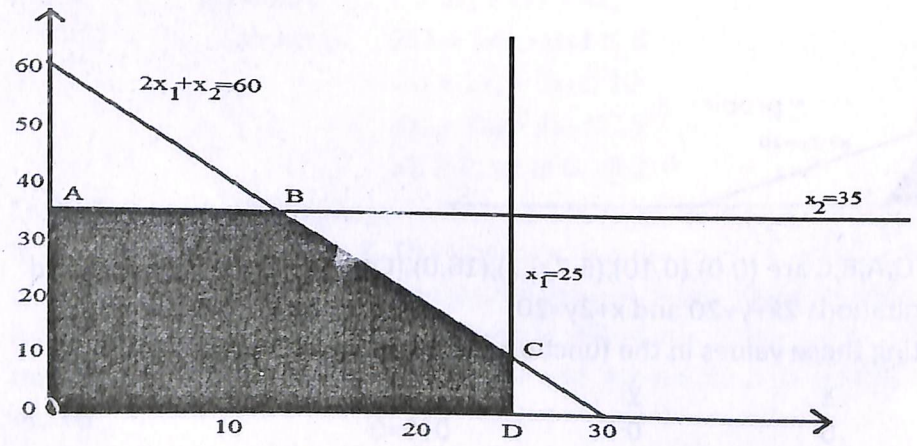
$2x_1+x_2 = 60$ .....(1)

$x_1 = 25$ .....(2)

$x_2 = 35$ .....(3)

$x_1 = 0$ and $x_2 = 0$ .....(4)

1) $2x_1+x_2 = 60$
   Put $x_1 = 0$, then $x_2 = 60$ So one point is (0, 60)
   Put $x_2 = 0$, then $2x_1 = 60$ therefore, $x_1=30$. So another point is (30, 0)
   Plot (0, 60) and (30, 0) and join them, we get the line $2x_1+x_2 = 60$
2) $x_1 = 25$ is a line parallel to x_2–axis
3) $x_2 = 35$ is a line parallel to x_1–axis
4) $x_1 = 0$ and $x_2 = 0$ are two axis.



The feasible region is OABCD(shaded).

Coordinates of A,B,C and D are respectively (0,35),(12.5,25),(25,10),(25,0). Coordinates of B and C are obtained by solving the equations of the lines passing through the points. Substituting the values of x_1 and x_2 in Z.

Point	x_1	x_2	$Z=60x_1+40x_2$
0	0	0	0
A	0	35	0+1400=1400
B	12.5	35	750+1400=2150
C	25	10	1500+400=1900
D	25	0	1500+0=1500

Z is highest for the point B.

Therefore, Solution is $x_1=12.5, x_2=35$ and $Z=2150$

Ex.6 . 3 A toy company manufacture two types of dolls a basic version doll A and a deluxe Version doll B. Each doll of type B takes twice as long as to produce as one type A and the companywould have time to make maximum of 2,000 per day if it produced only the basic version. The supply of plastic is sufficient to produce 1,500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are 600 per day available. If the company makes profit of Rs.3.00 per doll and Rs.5.00 per doll respectively on doll A and B, how many of each should be produced per day in order to maximize profit?

Ans: Let x_1 dolls of type A and x_2 dolls of type B be produced per day.

therefore, Total profit, $Z=3x_1+5x_2$ (Rs.)

Total time per day consumed to prepare x_1 and x_2 dolls of type A and B is $x_1(t) + x_2(2t)$ which should be less than 2000t where 't' is the time required for one doll of type.

Therefore, $x_1t+2x_2t \leq 2000t$ $\quad x_1+2x_2 \leq 2000$

Since plastic is available to produce 1500 doll only, $x_1+x_2 \leq 1500$

Fancy dress is available for 600 dolls only therefore, $x_2 \leq 600$

Hence the Linear programming problem is as follows.

Maximize $\quad Z=3x_1+5x_2$

Subject to $\quad x_1+2x_2 \leq 2000$

$x_1+x_2 \leq 1500$

$x_2 \leq 600$

$x_1 \geq 0, x_2 \geq 0$

First we consider the constraint as equation.

Therefore, $x_1+2x_2 = 2000$

$x_1+x_2 = 1500$

$x_2 = 600$

Put $x_1 = 0$, $x_2 = 0$,

Therefore, (0, 1000) and (1000, 0).

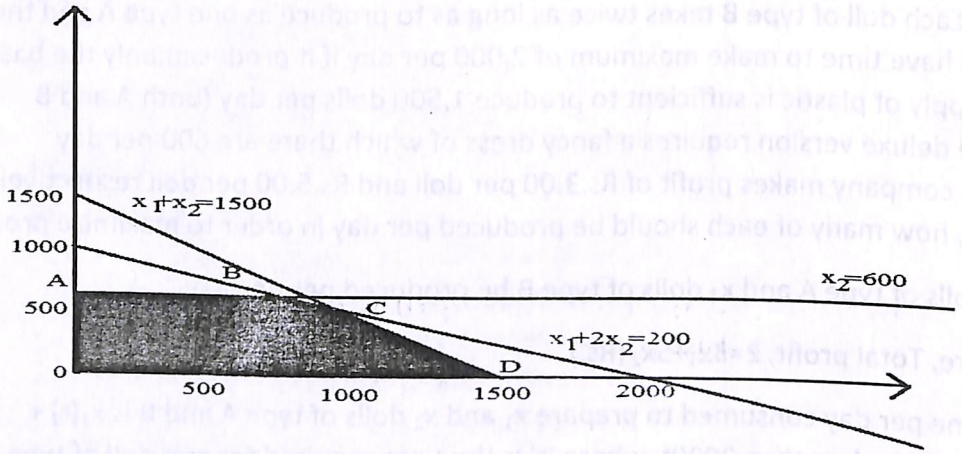
Putting $x_1 = 0$, $x_2 = 1500$ and

Therefore, (0, 1500) and (1500, 0) are the points.

$x_2 = 600$ is parallel to x_1 axis.

$x_1 = 0$ and $x_2 = 0$ are the two axes.

Draw all the lines



The feasible region is OABCD (shaded)

Points	x_1	x_2	$Z=3x_1+5x_2$
O	0	0	0
A	0	600	3000
B	800	600	5400
C	1000	500	5500
D	1500	0	4500

Z is maximum at the point C.

Therefore, The solution is $x_1=1000$, $x_2=500$, $z=5500$.

Therefore, The company should manufacture 1000 units of doll A and 500 units of doll B in order to have maximum profit of Rs.5500.

Ex 6..4:Solve the L.P.P

Max: $Z=2x_1+3x_2$

subject to

$x_1+x_2 \leq 30$

$x_2 \geq 3$

$0 \leq' x2 \leq 12$

$x_1 - x_2 \leq 0$

$0 \leq x_1 \leq 20$

Ans: The third constraints can be split into $x_2 \geq 0$, $x_2 \leq 12$.

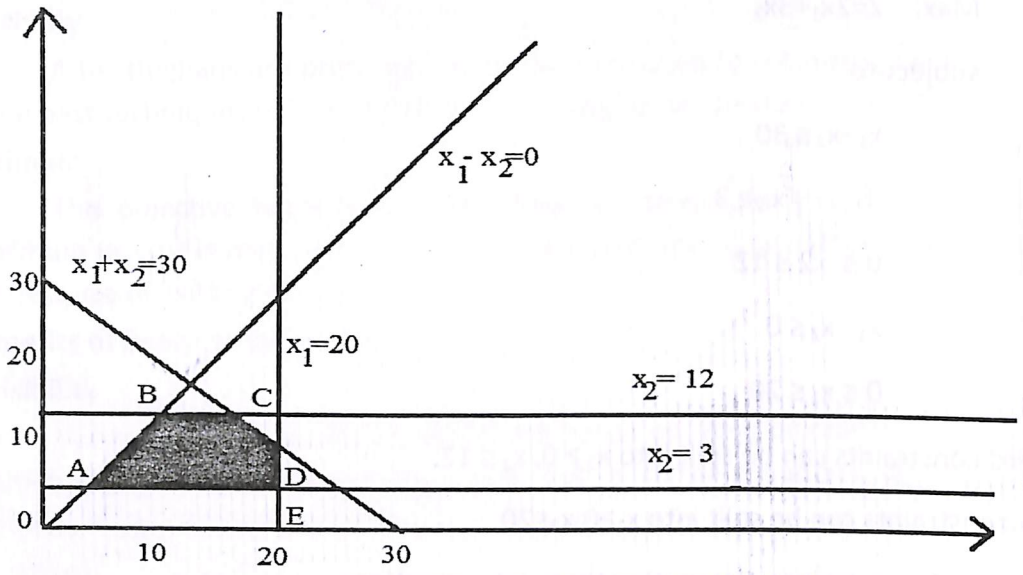
Fifth constraints can be split into $x_1 \geq 0, x_2 \leq 20$.

So we get 7 constraints. Converting them into equations,

$x_1 + x_2 = 30$
$x_2 = 3$
$x_2 = 12$
$x_1 - x_2 = 0$
$x_1 = 20$
$x_1 = 0$
$x_2 = 0$

For the line $x_1 + x_2 = 30$, when $x_1 = 0$, $x_2 = 30$ and when $x_2 = 0$, $x_1 = 30$.
Therefore,(0,30) and (30,0) are two points.

$x_2 = 3$ is parallel to x_1 - axis.
The line $x_2 = 12$ is parallel to x_1 axis.
For the line $x_1 - x_2 = 0$,put $x_1 = 0$ then $x_2 = 0$ therefore(0,0) is a point.
Then put $x_1 = 20$, then $x_2 = 20$ therefore, (20,20) is another point.
The line $x_1 = 20$ is parallel to x_2 axis (or y-z=axis)
$x_1 = 0$ and $x_2 = 0$ are x_2-axis and x_1-axis respectively.
Draw the lines



The feasible region s A B C D E (shaded)

Points	x_1	x_2	$Z=2x_2 + 3x_2$
A	3	3	15
B	12	12	60
C	18	12	72
D	20	10	70
E	20	3	49

The maximum Z is obtained at the point C.

Therefore, The solution is $x_1 = 18$ $x_2 = 12$,$Z=72$

Ex 6. 5: Solve graphically the following linear programming problem.

Minimize: $Z=3x_1 + 5x_2$

Subject to $-3x_1 + 4x_2 \leq 12$

$2x_1 - x_2 \geq -2$

$2x_1 + 3x_2 \geq 12$

$x_1 \leq 4, x_2 \geq 2$

$x_1, x_2 \geq 0$

Ans: Reading $2x_1 - x_2 \geq -2$ as $-2x_1 + x_2 \leq 2$

Reading al constraints as equations:

$-3x_1 + 4x_2 = 12$ .....(1)

$-2x_1 + x_2 = 2$ .....(2)

$2x_1 + 3x_2 = 12$.....(3)

$x_1 = 4$ .....(4)

$x_2 = 2$ .....(5)

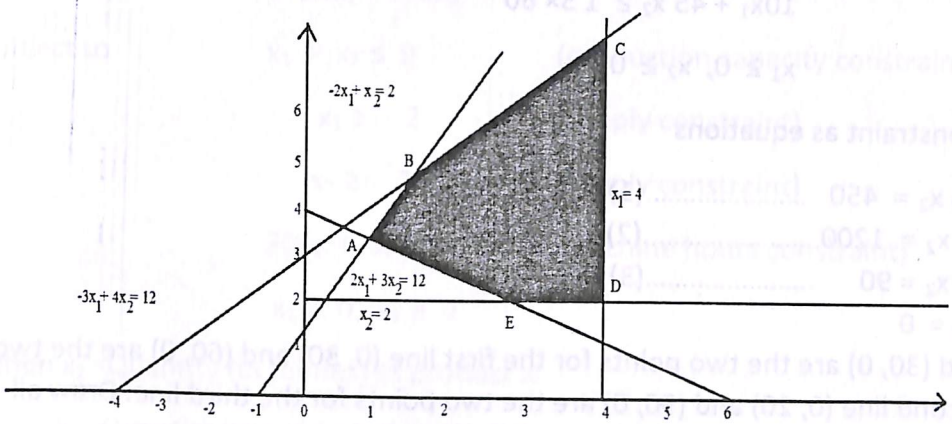
$x_1 = 0, x_2 = 0$ .....(6)

(0,3) and (-4,0) are two points on equation(1)

(0,2) and (-1,0) are two points on equation(2)

Similarly (0,4) and (6,0) are two points n equation (3)

Equations (4) and (5) are lines parallel to x_2 - axis and x_1 - axis respectively. $x_1 = 0$ and $x_2 = 0$ are the two axes.



A B C D E is the feasible region (which s shaded)

The co-ordinates of the vertices of this feasible region and value of Z for those co-ordinates are given below.

Points	x_1	x_2	$Z=(3x_1 + 5x_2)$
A	.75	3.5	19.75
B	.8	3.6	20.4
C	4	6	42
D	4	2	22
E	.3	2	19

Z is minimum at E Therefore, The solution to the problem is

$x_1 = 3, x_2 = 2$ and $Z = 19$

Ex. 6. 6: A small scale manufacture has production facilities for producing two different products. Each of the products requires three different operations: grinding, assembly and

testing. Product I requ-ires 15, r20 and 10 minutes grind, assembly and testing respectively where-as Product II requires 7.5, 40 and 45minutes for grinding, assembly and testing .The production run calls for at least 7.5 hours of grinding time, at least 20 hours of assembly time and at least 15 hours of testing time. If Product I costs Rs.60 and Product II costs Rs.90 to manufacture, determine the number of each product the firm should produce in order to minimize the cost of operations.

Ans: Let x_1 and x_2 the number of units of Product I and II produced.

The mathematical model is

Minimize $Z=60x_1+90x_2$

Subject to $15x_1 + 7.5\,x_2 \geq 7.5 \times 60$

$20x_1 + 40\,x_2 \geq 20 \times 60$

$10x_1 + 45\,x_2 \geq 15 \times 60$

$x_1 \geq 0,\ x_2 \geq 0$

Reading all the constraint as equations

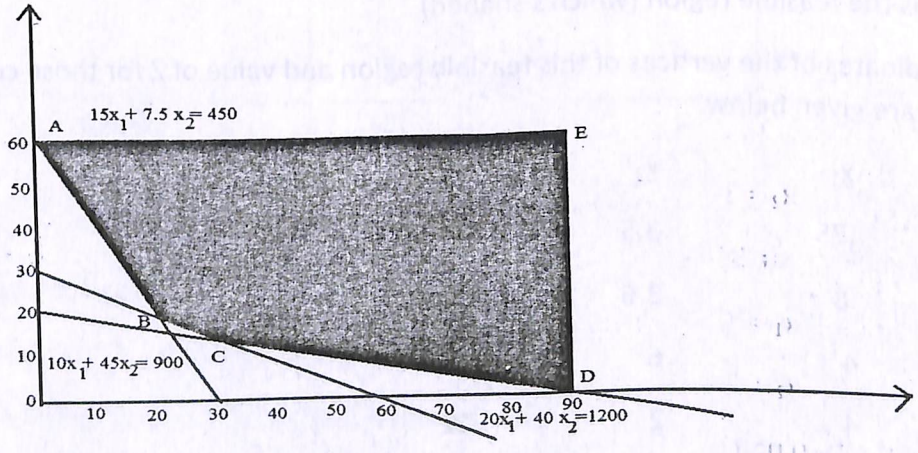
$15x_1 + 7.5\,x_2 = 450$ .....(1)

$20x_1 + 40\,x_2 = 1200$ .....(2)

$10x_1 + 45\,x_2 = 90$ .....(3)

$x_1 = 0,\ x_2 = 0$

(0, 60) and (30, 0) are the two points for the first line (0, 30) and (60, 0) are the two points for the second line (0, 20) and (90, 0) are the two points for the third line. Draw all the lines.



The feasible region is ABCDE

The co-ordinates of the four vertices of the feasible region are given below.

Points	x_1	x_2	$Z=(60x_1 + 90x_2)$
A	0	60	5400
B	20	20	3000
C	36	12	3240

D	90	0	5400
E	-	-	-

Z is minimum at B.

Therefore, 20 units of products I and 20 units of product II may be manufactured so that the cost is minimum of Rs.3000

Ex. 6. 7: A firm makes two products X and Y, and has total production capacity of 9 tonnes per day, X and Y requiring the same production capacity. The firm has permanent contract to supply at least 2 tonnes of X and at least 3 tonnes of Y per day to another company. Each tonnes of X requires 20 machine hours production time and each tone of Y requires 50 machine hours production time, the daily maximum possible number of machine hours is 360. All the first output can be sold and the profit made is Rs.80 per ton of x and Rs.120 per ton of y. It is required to determine schedule for maximum profit and calculate this profit.

Ans: The given information can be presented in appropriate mathematical form as follows.

Maximize $Z=80x_1 + 120x_2$

Subject to

$x_1 + x_2 \leq 9$ (production capacity constraint)

$x_1 \geq 2$ (supply constraint)

$x_2 \geq 3$ (supply constraint)

$20x_1 + 50x_2 \leq 360$ (machine hours constraint)

$x_1 \geq 0,\ x_2 \geq 0$

When x_1 =Quantity (in tonnes) of product X

x_2 =Quantity (in tonnes) of product Y

Reading the inequality as equations

$x_1 + x_2 = 9$ ..... (1)

$x_1 = 2$ ..... (2)

$x_2 = 3$ ..... (3)

$20x_1 + 50x_2 = 360$ ..... (4)

$x_1 = 0$ ..... .(4)

$x_2 = 0$ ..... .(6)

(0, 9) and (9, 0) are the two points on the equation (1)

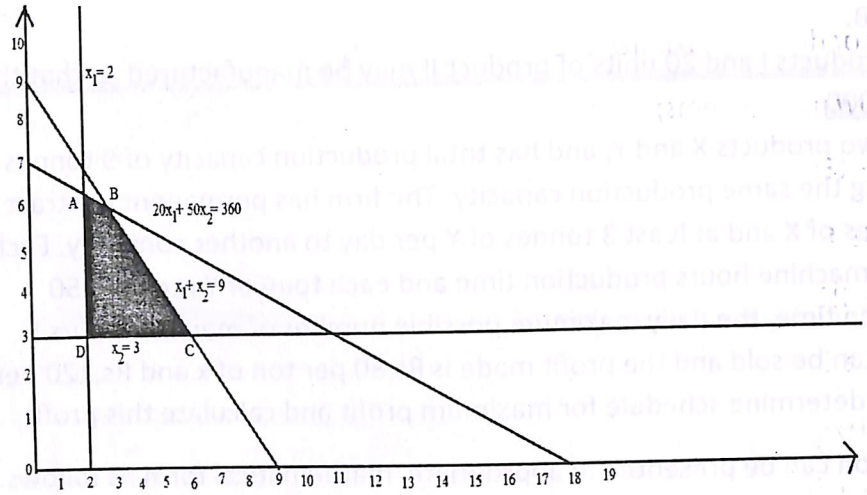
Equation (2) is parallel to x_2 axis.

Equation (3) is parallel to x_1 axis.

(0, 7.2) and (18, 0) are the two points on the equation (4)

$x_1 = 0$ and $x_2 = 0$ are the two axes .Draw all these 6 lines.

Feasible region is ABCD.



Coordinating of the four vertices of the feasible region are given below.

Points	x_1	x_2	$Z=(80x_1 + 120x_2)$
A	2	6.4	928
B	3	6	960
C	6	3	840
D	2	3	520

Maximum value of z = 960 and the solution point is B.

Therefore, $x_1 = 3$, $x_2 = 6$ with max Z = 960.

Hence the company should produce 3 tonnes f product X and 6 tonnes of product Y in order to get a maximum profit of Rs.960.

Review Questions and exercises

1. What do you understand by 'Graphic method' of solving a LP problem?
2. What are methods of solutions of solving LPP
3. How is to draw line of equation
4. Explain the various steps involved n solving LPP by graphic method.
5. What is a feasible region ? What will be the shape of a feasible region ?
6. What are the merits of graphical method of solving a LPP
7. What are the limitations of graphical method of solving t LPP.
8. What are constraints?
9. Draw lines for following equations

$12x + 12y \leq 840$

$3x + 6y \leq 300$

$8x + 4y \leq 480$

Under the above conditions maximize the function $5x + 7y$

10. Solve graphically following problems;

Maximize $Z=22x_1 + 18x_2$

Subject to $3x_1 + 2x_2 \leq 48$

$x_1 + x_2 \leq 20$

$x_1 \geq 0, x_2 \geq 0$

11.. Maximize $Z=3x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 450$

$2x_1 + x_2 \leq 600$

$x_1, x_2 \geq 0$

12. Max: $Z = 5x_1 + 8x_2$

S.t $3x_1 + 2x_2 \leq 36$

$x_1 + 2x_2 \leq 20$

$3x_1 + 4x_2 \leq 42$

$x_1, x_2 \geq 0$

13. Maximize: $Z=5x_1 + 3x_2$

S.t $2x_1 + x_2 \leq 1000$

$x_1 \leq 400$

$x_2 \leq 700$

$x_1, x_2 \geq 0$

14. Max $Z=40x+ 80y$

S.t $2x + 3y \leq 48$

$0 \leq x \leq 5$

$0 \leq y \leq 10$

15. Max: $Z=40000x_1 + 55000x_2$

S.t $10x_1 + 15x_2 \leq 200$

$0 \leq x_1 \leq 12$

$0 \le x_2 \le 5$

16. Max: $Z=8000x_1 + 70000x_2$

S.t $3x_1 + x_2 \le 66$

$x_1 + x_2 \le 45$

$x_1 \le 20$

$x_2 \le 40$

$x_1 \ge 0, x_2 \ge 0$

17. Max: $Z=3x_1 + 5x_2$

S.t $x_1 + x_2 \le 2000$

$x_1 + x_2 \le 1500$

$x_1 \ge 600$

$x_1 \ge 0$

$x_2 \ge 0$

18. Max: $Z=3x_1 + 2x_2$

S.t $-2x_1 + x_2 \le 1$

$x_1 \le 2$

$x_1 + x_2 \le 3$

$x_1 \ge 0$

$x_2 \ge 0$

19. Max: $Z=300x_1 + 400x_2$

S.t $5x_1 + 4x_2 \le 400$

$3x_1 + 5x_2 \le 150$

$5x_1 + 4x_2 \ge 100$

$8x_1 + 4x_2 \ge 80$

$x_1, x_2 \ge 0$

20. Max: $Z=2x_1 + 3x_2$

S.t $x_1 + x_2 \le 1$

$3x_1 + x_2 \le 4$

$x_1 \ge 0, x_2 \ge 0$

21. Max: $Z=6x_1 - 2x_2$

S.t $2x_1 - x_2 \leq 2$

$x_1 \leq 3$

$x_1 \geq 0, x_2 \geq 0$

22. Min: $Z= -x_1 + 2x_2$

S.t $-x_1 + 3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

23. Min: $Z=600x_1 + 400x_2$

S.t $3x_1 + 3x_2 \geq 40$

$3x_1 + x_2 \geq 40$

$2x_1 + 5x_2 \geq 4$

$x_1, x_2 \geq 0$

24. A company produces two articles A and B. There are two different departments through which the articles are processed, viz assembly and finishing. The potential capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. Production of one unit of A requires four hours in assembly and 2 hours in finishing. Each of the unit B requires 2 hours in assembly and 4 hours in finishing. If profits is Rs..8 for each unit of A and Rs..6 for each unit of B find out the number of units of A and B to be produced each week to get maximum profit(solve graphically)

UNIT 7 SIMPLEX METHOD OF LPP

Graphic method of LPP is limited to two variables. We have to look to other procedure which offers an efficient means of solving more complex LPP. Although the graphical method of solving LPP is an invaluable aid to understand the basic structure, the method is of limited application in industrial problems as the number of variables occurring there, is substantially large. So another method known as simplex method is suitable for solving LPP with a large number of variables. The method though an iterative process, progressively approaches and ultimately reaches to themaximum or minimum value of the objective function. The method also helps the decision maker to identify the unmatching constraints, unbounded solution, multiple solution and infeasible solution .

Simplex method was originally developed by G.B Dantzig, an American mathematician. It has the advantage of being universal, ie, any linear model for which the solution exists can be solved by it. In principle, it consists of starting with a certain solution of which all that we know is that, it is feasible,, ie, it satisfies non negativity conditions. We improve this solution at consecutive stages, until after a certain finite number of stages we arrive at optimal solution.

For arriving at the solution of LPP by this method the constraint and the objective function are presented in table known as simplex table. Then following a set procedure and rules, the optimal solution is obtained making step by step improvement.

Thus simplex method is an iterative (step by step) procedure in which in systematic step from an initial Basic Feasible solution to another Basic Feasible solution and finally, in a finite number of steps to an optimal Basic Feasible solution, in such a way that value of the objective function at each step is better (or at least not worst) than that at preceding steps. In other words simplex algorithm consists of the following main steps.

(1) Find a trial Basic Feasible Solution of Linear Programming Problem.
(2) Test whether it is an optimal solution or not.
(3) If not optimal, improve the first trial Basic Feasible Solution by a set of rules.
(4) Repeat step(2) and step (3) till optimal solution is obtained.

How to construct simplex table?

Simplex table consists of rows and columns. If there are 'm' original variables and 'n' introduced variables, then there will be 3+m+n columns in the Simplex table [n' introduced variables are slack, surplus or artificial variables].

First column (B) contains the basic variables. Second column (c) shows the coefficient of basic variables in the objective function. Third column (x_B) gives the value of the basic variables. Each of the next 'm + n' columns contain coefficient of the variables in the constraints, when they are converted into equations.

In a simplex table there is a vector associated with every variable. The vector associated with the basic variables is unit vectors.

Basic Concepts

Simplex method makes use of certain mathematical terms and basic concepts, as described below.

Feasible Solution

A feasible solution in a Linear Programming Problem is a set of values of the variables which satisfy all the constraints and non-negative restriction of the problem.

Optimal Solution

A feasible solution to a Linear Programming Problem is said to be optimum if it's optimizes the objective function, Z, of the problem. It should either maximize profit or minimize loss.

Basic Feasible Solution

A feasible solution is a Linear Programming Solution in which the vectors associated to non-zero variables are linearly independent is called a basic feasible solution.

Slack Variables

If a constraint has a sign $\leq$(less than or equal to) then in order to make it an equality (=) we have to add some variable to left hand side. The variable which are added to left hand side of the constraints to convert them into equality is called slack variables. The value of this variable usually can be interpreted as an amount of unused resources S1 , S2 are usually taken as slack variables..

For example, consider the constraint $2x_1 + x_2 \leq 800$

In order to convert the constraints into equation, we add s1 into left hand side, then we have $2x_1 + x_2 + s_1 = 800$. Then, s_1 is the slack variable.

Surplus Variable

If a constraint has sign $\geq$then in order to make equality we have subtract some variables from left hand it's side. The variables are subtracted from left hand side of the constraint to convert them into equalities are called surplus variable. The value if this variable can be interpreted as the amount of over and above of the required minimum level.

For example, consider constraint $2x_1 + 4x_2 \geq 12$

In order to convert this equation, we subtract s_2 from left hand side of the inequality. Then $2x_1 + 4x_2 = 12$. Then s_2 is the surplus variable.

Simplex Table

Simplex table consists of rows and columns. If there is 'm' original values and 'n' introduced values, then there will be 3 + m + n columns in the simplex table. [Introduced variables are slack, surplus or artificial variables].

First column (B) contains the basic variables. Second column (C) shows the coefficient of the basic variable in the objective function. Third column (X_B) gives values of

basic variables. Each next 'm + n' columns contain coefficient of variables in the constraints, when they are converted into equations.

Basis (B)

The variables whose values are not restricted to zero in the current basic solution, are listed in one of the simplex table known as Basis (B).

Basic Variables

The variables are in the basis are called basic variables, and other known as non-basic variables.

Vector

Any column or row of simplex tables is called a vector.So we have x1 - vector, x2 - vector etc.

In a simplex table, there is a vector associated with ever, variable. The vector associated with the basic element are unit vectors.

Unit Vector

A vector with one element 1 and all other elements are zero, is unit a vector.

Eg: $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$ are unit vectors

Net Evaluation (Δ_j)

(Δ j) is the net profit or loss if one unit of the variable in the respective column is introduced. That is Δ_j shows that is profit (or loss) if one unit of $x_j$is introduced. The row containing $\Delta_j$values is called net evaluation row or index row.

$\Delta_j = z_j - c_j$

Where c_j is the coefficient of x_j variables in the in the objective function and z_j is the sum of the products of coefficients of basic variable in the objective function in the vector x_j.

Minimum Ratio

Minimum ration is the lowest non negative ratio in the replacing ratio column.

The replacing ratio column (θ) contains values obtained by dividing each element in $x_B$column (the column showing the values of the basic variable) by the corresponding elements in the incoming vector.

Key Column (Incoming Vector)

The column which has highest negative $\Delta_j$is maximization problem or the highest positive$\Delta_j$in the minimization problem, is called incoming vector.

Key Row (Outgoing Vector)

The row which relates the minimum ratio, is Outgoing vector

Key Element

Key element is that element of the simplex table which lies both in the key column and key row.

Iteration

Iteration means step by step process in simplex method to move from one basic feasible solution to another.

Steps in Simplex Method

1. Formulate problem into Linear Programming Problem.
2. Convert the constraints into equations by introducing slack variables, or surplus variables
3. Construct starting simplex table
4. Conduct test of optimality by evaluating profit contribution - $\Delta_j = Z_j - C_j$.
5. If **(Δ_j) is negative, the solution is not optimal**
6. Find Incoming and outgoing vectors
7. The incoming vector is with highest negative Δ_j
8. The outgoing vector is with lowest minimum ratio.
9. Identify intersection point as key element and note key column and key row
10. Change the values in the rows

11. In the key row, get 1 at the position of key element. Divide all the elements of the row, by the key element.

12. To get new elements of the other row, add to old row element, - key column element x revised key row element. = (old row element + (- key column element x revised key row element)

13. Obtain next simplex table with the changes.

14. Ascertain Improved feasible solutions by reading B column and x_B column together..

15.Test the above improved Basic Feasible Solution for optimality

16. If solution is not optimal, then repeat Step 5 and 6 until solution is finally obtained.

›*NOTE*: A minimization problem can be converted into maximization problem by changing the sign of the coefficients in objective function.

∴To Min: $Z = 4x_1 - 2x_2 + x_3$, we can Max $Z' = -4x_1 + 2x_2 - x_3$, subject to the same constraints.

Net Evaluation ($_j$)

$_j$ is the net profit or loss of one unit of the variable in the respective column introduced. That is $_j$ shows what is the profit (or loss) if one unit of x_j is introduced. The row containing $_j$ values is called net evaluation row or index row.

$_j = z_j - c_j$

where c_j is the coefficient of x_j variables in the objective function and z_j is the sum of product coefficient function of basic variable in the objective function and the vector x_j

Ex. 7.1 Max $Z = 6x + 8y$

Subject to $30x + 20y \leq 300$

$5x + 10y \leq 110$

By adding slack variable S1 and S2

$30x + 20y + S1 = 300$

$5x + 10y + S2 = 110$

B	C_B	X_B	x	y	S_1	S_2	Minimum Ratio
s_1	0	300	30	20	1	0	300/20 = 15
s_2	0	110	5	10	0	1	110/10 = 11
		Z_j	0	0	0	0	
		C_j	6	8	0	0	
$_j = Z_j - C_j$			-6	-8	0	0	

Since Zj - Cj values are negatives, Solution is not optimal.

Lowest Minimum Ratio = 11, SO out going vector S2

Maximum Zj -Cj value = -8. So incoming vector = y.

Key element is 10.

To convert key lement 1, divide all key row elements by 10 - 11, 5/10 1, 0, 1/10.

To get new elements of the other row - old row element + -key column elemnt x new row elemnt.= 300 = -20 x 11 = 80....20,0, 1, -2

IMPROVED SIMPLEX TABLE

B	C_B	X_B	x	y	S_1	S_2	Minimum Ratio
s_1	0	80	20		1	-2	80/20 = 4
y	8	11	5/10		0	1/10	11/5/10 = 5.5

				0 1			
		Z_j C_j	4 6	8 8	0 0	8/10 0	
	$_j = Z_j - C_j$		-2	0	0	8/10	

Since one value -2 is negative, solution is not still optimal. So prepare next improved simplex table. Replace S1 b y x

B	C_B	X_B	x	Y	S_1	S_2	Minimum Ratio
x Y	6 8	4 9	1 0	0 1	1/20 -1/40	-1/10 -3/20	
		Z_j C_j	6 6	8 8	0 4/40	0 12/40	
	$_j = Z_j - C_j$		0	0	0	8/10	

Since no Zj - Cj values are negative, this solution is optimal.

X = 4 and y = 9. Maximum profit will 6x4 + 8x9 = = 96

Ex. 7.2 Solve:

Maximize $Z = 5x + 3y$

Subject to $x + y \leq 2$

$5x + 2y \leq 10$

$3x + 8y \leq 12$

$x, y \geq 0$

Ans: Introducing slack variable and converting the constraints into equation; we have

$x + y + s_1 = 2$

$5x + 2y + s_2 = 10$

$3x + 8y + s_3 = 12$

These are entered in columns and rows in the initial simplex table given below.

B	C_B	X_B	X_1	X_2	S_1	S_2	S_3	Repl. Ratio $\Theta = x_B \div x_1$
s_1	0	2	1	1	1	0	0	2/1= 2
s_2	0	10	5	2	0	1	0	10/5= 2
s_3	0	12	3	8	0	0	1	12/3= 4
		Z_j	0	0	0	0	0	
		C_j	5	3	0	0	0	
$_j = Z_j - C_j$			-5	-3	0	0	0	

Solution is not optimal as two values of $_j$ are negative.

Max negative is -5, for X, and min ratio is 2, for s_1.

Therefore, x is the incoming vector and S_1 is the outgoing vector.

There are two minimum ratios. We select that at the top

Key column is Xand Ker row is s_1 and the key element is 1.since the key element is 1, there is no need of make it 1. So the values remain the same in the next table.

Add , -5 multiples of the elements I row to II row elements and -3 multiples of the elements of I row to III row elements in order. The resulting elements are taken in second simplex table.

[**Note:** -5 and -3 are elements in the key columns when their signs are changed]

II Simplex Table

B	C_B	X_B	X_1	X_2	S_1	S_2	S_3
x_1	5	2	1	1	1	0	0
S_2	0	0	0	-3	-5	1	0
S_3	0	6	0	5	-3	0	1
		Z_j	5	5	5	0	0
		C_j	5	3	0	0	0
		$_j$	0	2	5	0	0

Z_j values shown above , are (5 x 1 + 0 x 0 + 0 x 0) , (5 x 1 + 0 x -3 + 0 x 5) , (5 x 1 + 0 x -5 + 0 x -3) , (5 x 0 = 0 x 1 + 0 x 0) , (5 x 0 + 0 x 0 + 0 x 1) = 5 , 5 , 5 , 0 , 0]

Since no $_j$values in the II simple table is negative, the solution is optimum.

X1 = 2 , S2 = 0, S3 = 6

Other two variables , s_1 and x_1 which are not in B column are zero.

Therefore, $\underline{x_1 = 2}$, $\underline{x_2 = 0}$

and Z = (5 x 2) + (3 x 0) = $\underline{10}$

Ex .7. 3

A company manufactures two products P_1 and P_2. The company has two types of machines A and B. Product P_1 takes 2 hours on machine A and 4 hours on machine B, whereas product P_2 takes 5 hours on machine A and 2 hours on machine B. The profit realized on the sale of one unit of product P_1 is Rs. 4. If machine A and B can operate 24 and 16 hours per day respectively , determine the weekly output for each product in order to maximize the profit. [Assume a day week]

Ans: Let x_1 and x_2 be the units of product P_1 and P_2 manufactured per week. Then the LPP is

Max: $Z = 3x_1 + 4x_2$

S.t $2x_1 + 5x_2 \leq (24 \times 5)$

$4x_1 + 2x_2 \leq (16 \times 5)$

$x_1, x_2 \geq 0$

Introducing the slack variables s_1 and s_2 the constraints are

$2x_1 + 5x_2 + s_1 = 120$

$4x_1 + 2x_2 + s_2 = 80$

These are entered in the columns of the starting simplex table given below.

Starting Simplex Table is

B	C_B	X_B	x	y	S_1	S_2	Minimum Ratio
S1	0	120	2	5	1	0	120/5 = 24
S2	0	80	4	2	0	1	80/2 = 40
		Z_j	0	0	0	0	
		C_j	3	4	0	0	
$_j = Z_j - C_j$			-3	-4	0	0	

Highest negative $_j$ is -4 which relates to X_2.

Therefore, X_2 is the incoming vector.

Minimum ratio (Θ) = 24 which relates to s_1

Therefore, S_1 is the outgoing vector. Key element = 5.

Divide the I row (key row) by 5.

We have (24 2/5 1 1/5 0)

Add -2 multiple of these elements with the corresponding elements of II row.

(-2 x 24) + 80, (-2 x 2/5) + 4, (-2 x 1/5) + 0, (-2 x 0) + 1 = (32, 16/5, 0, 2

B	C_B	X_B	X_1	X_2	S_1	S_2	Repe Ratio(O)=$x_B \div x_1$
X_2	4	24	2/5	1	1/5	0	$24 \div 2/5 = 60$
s_2	0	32	16/5	0	-2/5	1	$32 \div 4/5 = 10$ ←
		Zj	8/5	4	-4/5	0	
		Cj	3	4	0	0	
		j	-7/5	0	-4/5	0	

Highest negative $_j$ is for X_1 Therefore, X_1 is the incoming vector.

Minimum ratio is for s_2. S_2 is the outgoing vector.

Key element = 16/5

To convert the elements of X_1 row,

Divide II row elements by 16/5 (from 3rd column to 5th column)

Therefore, New second row is (10, 1 , 0 , -1/8 , 5/16)

Adding -2/5 multiples of these with the I row elements

We get (20, 0, 1, 1/4, -1/8)

III Simplex Table

B	C_B	X_B	X_1	X_2	S_1	S_2
x_2	4	20	0	1	1/4	-1/8
x_1	3	10	1	0	-1/8	5/16
		Z_j	3	4	-5/8	7/16
		C_j	3	4	0	0
		j	0	0	5/8	7/16

No $_j$ is negative. Therefore, Solution is optimum.

Therefore, Optimum solution is $x_1 = 10$, $x_2 = 20$ and Z = (3 x 10) + (4 x 20) = 110

The weekly output for product P_1= 10 units and for product P_2 = 20 such that the maximum profit = 110 Rs

Ex 7.4

Bajaj Motors wants to produce two new models - Gus an d Tus., with capacity of 48 hours per week. Production of an Gus requires 2 hours and production of an AM –FM radio will require 3 hours. Each AM radio will contribute Rs.40 to profit while an AM –FM radio

will contribute Rs.80 to profits. The marketing department after extensive research has determined that a maximum of 15 AM radios and 10 AM –FM radios can be sold each week (1) Formulate a linear programming model to determine the optimal production mix of AM –FM radios that will maximize profits (2) solve the problem by simplex method.

Ans: let x_1 be number of units of Gus and Tus.

Then the LPP is

MAX: $Z = 40x_1 + 80x_2$

S.t $2x_1 + 3x_2 \leq 48$

$x_1 \leq 15$

$x_2 \leq 10$

$x1 \geq 0,\ x2 \geq 0$

Introducing the slack variables, constraints are

$2x_1 + 3x_2 + s_1 = 48$

$x_1 + s_2 = 15$

$x_2 + s_3 = 10$

Enter these in columns of the simplex table given below.

Objective function is

$Z = 40x_1 + 80x_2 - 0s_1 + 0s_2 + 0s_3 = (40\ 80\ 0\ 0\ 0)\ (x_1\ x_2\ s_1\ s_2\ s_3)$

$C_j = 40, 80, 0, 0, 0$

Starting Simplex Table

B	C_B	X_B	x	y	S_1	S_2	S3	Minimum Ratio
S1	0	48	2	3	1	0	0	48/3 = 16
S2	0	15	1	0	0	1	1	15/0 = 00
S3	0	10	0	1	0	0		10/12= 10

	Z_j	0	0	0	0	
	C_j	40	80	0	0	
$_j = Zj - Cj$		-40	-80	0	0	

Highest negative $_j$ is -80. So incoming vector is X_2.

Minimum ratio is 10. So the out going venctor is S3.

Key element = 1. Since it is 1, the key row S3 elements ned not be changed. S2 row also need not be changed, because it already contains o. only S1 row should be changed. For this, take -3 multiple of third row elements and add to the first row elements.

-3 x 10 + 48 = 18 -3 x 0 +2 = 2 -3 x 1 +3 = 0 -3 x 0 +1 + 1 -3 x 0 + 0 = 0

-3 x 1 +0 = -3

B	C_B	X_B	x	y	S_1	S_2	S3	Minimum Ratio
S1	0	18	2	0	1	0	-318/2	= 9
S2	0	15	1	0	0	1	1	15/ 1 =15
S3	80	10	0	1	0	0		10/0 = 0
		Z_j	0	80	0	0	80	
		C_j	40	80	0	0	0	
	$_j = Zj - Cj$		-40	0	0	0	80	

X1 is the incoming vector and S1 is theoutoing vector.. Key element is 2. So, divide 1 row by 2 and add -1 multiples of new 1 row elemtsn to the 2 row.

III SIMPLEX TABLE

B	C_B	X_B	x	y	S_1	S_2	S3	Minimum Ratio
X1	40	9	1	0	1/2	0	-3/2	18/2 = 9
S2	0	6	0	0	-1/2	1	3/2	15/ 1 =15
X2	80	10	0	1	0	0	1	10/0 = 0
		Z_j	40	80	20	0	20	
		C_j	40	80	0	0	0	
$_j = Z_j - C_j$			0	0	20	0	20	

No $_j$ is negative. Therefore, Solution is optimum

X1 = 9, X2 = 10 and Z ==360 +800 = 1160

20. Show that the following L.P.P. has alternative (multiple) solution.

Maximize $Z=4x_1 + 4x_2$

S. t $x_1 + x_2 \leq 6$, $x_1 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$

21. Show that the solution following L.P.P in unbounded

Maximize $Z=2x_1 + 3x_2$

S. t $x_1 - x_2 \leq 2$, $x_1 + x_2 \geq 4$, $x_1 \geq 0$, $x_2 \geq 0$

22. Show that the following L.P.P has no feasible solution

Maximize $Z=4x_1 + 3x_2$

S. t $x_1 - x_2 \leq -1$, $-x_1 + x_2 \leq 0$, $x_1 \geq 0$, $x_2 \geq 0$

23. A company produces two articles A and B. There are two different departments through which the articles are processed, viz assembly and finishing. The potential capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. Production of one unit of A requires four hours in assembly and 2 hours in finishing. Each of the unit B requires 2 hours in assembly and 4 hours in finishing. If profits is Rs..8 for each unit of A and Rs..6 for each unit of B find out the number of units of A and B to be produced each week to get maximum profit(solve graphically)

REVIEW QUESTIONS AMD EXERCISES

1. what are the methods of solving linear programming
2. What is simplex solution
3. Explain iteration
4. What is initial feasible solution
   what is optimal solution
5. Explain slack variables
6. What are surplus variables
7. How is simplex constructed
8. What is a vector
9. What unit vectors.\
10. Explain net evaluation
11. What is Minimum Ratio
12. What is key element
13. What are key column and Key row
14. Explain steps in simplex method
15. What are the advantages of simplex method
16. Solve by simplex method
    Maximize $Z=22x_1 + 18x_2$
    Subject to $3x_1 + 2x_2 \leq 48$

    $x_1 + x_2 \leq 20$

    $x_1 \geq 0, x_2 \geq 0$

17. Find values of X1 and X2, using simplex solution method of lpp:

    Maximize $Z=3x_1 + 4x_2$

    Subject to $x_1 + x_2 \leq 450$

$2x_1 + x_2 \leq 600$

$x_1, x_2 \geq 0$

18. Apply simplex m kethod to solve Max:

$Z = 5x_1 + 8x_2$

S.t $3x_1 + 2x_2 \leq 36$

$x_1 + 2x_2 \leq 20$

$3x_1 + 4x_2 \leq 42$

$x_1, x_2 \geq 0$

19. Calculate values of X1 and X2

Maximize: $Z=5x_1 + 3x_2$

S.t $2x_1 + x_2 \leq 1000$

$x_1 \leq 400$

$x_2 \leq 700$

$x_1, x_2 \geq 0$

20. Solcve using simplex

$Z=40000x_1 + 55000x_2$

S.t $10x_1 + 15x_2 \leq 200$

$0 \leq x_1 \leq 12$

$0 \leq x_2 \leq 5$

21.Solve as per Simplex.

Max: $Z=8000x_1 + 70000x_2$

S.t $3x_1 + x_2 \leq 66$

$x_1 + x_2 \leq 45$

$x_1 \leq 20$

$x_2 \leq 40$

$x_1 \geq 0, x_2 \geq 0$

22 Apply simplex

$Z=3x_1 + 5x_2$

S.t $x_1 + x_2 \leq 2000$

$x_1 + x_2 \leq 1500$

$x_1 \geq 600$

$x_1 \geq 0 \quad x_2 \geq 025.$

23. Solve using simplex method

Max: $Z=3x_1 + 2x_2$

S.t $-2x_1 + x_2 \leq 1$

$x_1 \leq 2$ where $x1, x2 \geq 0$

$x_1 + x_2 \leq 3$

UNIT 8 LINEAR PROGRAMMING - SPECIAL CASES

In most cases, the objective of a linear programming problem will be to maximise profit. Generally linear programming problems and models are designed with the objective of maximizing resources and profit. However, LPPS can be redesigned to accommodate special cases. Such special cases include minimization of cost or losses, infeasibility, unbounded solution,and multiple optimal solutions.

Minimization problems

When the objective is to maximize profit or resource, the objective function is expressed as an equation consisting of contribution from each product in quantitative terms. When the objective is to minimize cost or loss, the objective function is expressed as equation, considering the variable cost per unit, relating to each product or combination of products.

In order to solve minimization problems, first it is converted into a maximization problem by changing the signs of coefficient in theobjective function.

Steps in minimization problem

1. Consider given objective function and constraints
2. Convert signs of the objective function from positive signs to negative sings.
3. Convert constraint inequalities into equation andadd necessary slack variables.
4. Prepare initial simplex t5able
5. Find outgoing variable and incoming variable.
6. Convert all values including key element.
7. Conduct optimality test – see for any negative Zj - Cj
8. Prepare improved simplextable, if current solution is not optimal.
9. Continue this process until optimal solution is obtained.

Ex 8. 1.Solve Minimise $Z = x - 3y + 2z$

Subject to $3x - y + 3z \leq 7$

$-2x + 4y \leq 12$

$-4x + 3y + 8z \leq 10$

Converting signs, in theobjective function, and adding slack variables -

$Z = -x + 3y - 2z$

Subject to $3x - y + 3z \leq 7$

$-2x + 4y \leq 12$

$-4x + 3y + 8z \leq 10$

Initial simplex table

B	C_B	X_B	x	y	z	S_1	S_2	S_3	Minim Ratio
S_1	0	7	3	-1	3	1	0	0	7/-1 = -7
S_2	0	12	-2	4	0	0	1	0	12/4 = 3
S_3	0	10	-4	3	8	0	0	1	$10/3 = 3_{1/3}$
		Z_j	0	0	0	0	0	0	
		C_j	-1	3	-2	0	0	0	
$_j = Zj - Cj$			1	-3	2	0	0	0	

As one zj - cj value is negative, the solution is not optimal. Therefore we have to identify outgoing vector - S2, and incoming venctor = y, change the values and prepare II simplex table.

II Simplex Table

B	C_B	X_B	x	y	z	S_1	S_2	S_3	Minim Ratio
S_1	0	10	5/2	0	3	1	1/4	0	10/5/2 = 4
y	3	3	-2/4	1	0	0	1/4	0	3/ -1/2 = -6
S_3	0	1	-5/2	0	8	0	-3/4	1	1/-5/2= -2/3
		Z_j	-3/2	3	0	0	3/4	0	
		C_j	-1	0	-2	0	0	0	
$_j = Zj - Cj$			-1/2	0	2	0	3/4	0	

Since one $_j$ value is in negative, the solution is not optimal and outgoing vector isS1 and incoming vector is x. Changing the values, the third Simplex Table is

B	C_B	X_B	x	y	z	S_1	S_2	S_3	Minim Ratio
x	-1	4	1	0	6/5	2/5	1/10	0	
y	3	5	0	1	3/5	1/5	3/10	0	
S_3	0	11	0	0	11	1	-1/2	1	
		Z_j	-1	3	3/5	1/5	8/10	0	
		C_j	-1	3	-2	20	0	0	
	$_j = Z_j - C_j$		0	0	13/5	1/5	8/10	0	

All Zj - Cj values are positive or zero. There is no negative value. Therefore the obtained solution is optimal.

Infeasibility

In some cases linear programming problem has no feasible solution. It means there is in feasible solution. And there are no points that simultaneously satisfy all constraints in the problem, common region do not develop in the first quadrant. Objective function does not pass through any point of all theconstraints. It is pertinent to note that infeasibility arises only if all the constraints are in the first quadrant. It is similar to unboundedness.

When no solution satisfying all constraints is obtained for a Linear Programming Problem there exists no feasible solution.

For such problems, when graphic method is applied we get no feasible region. If we apply simplex method, at least one artificial variable remains in the basis so that Z value can be expressed only in terms of M.

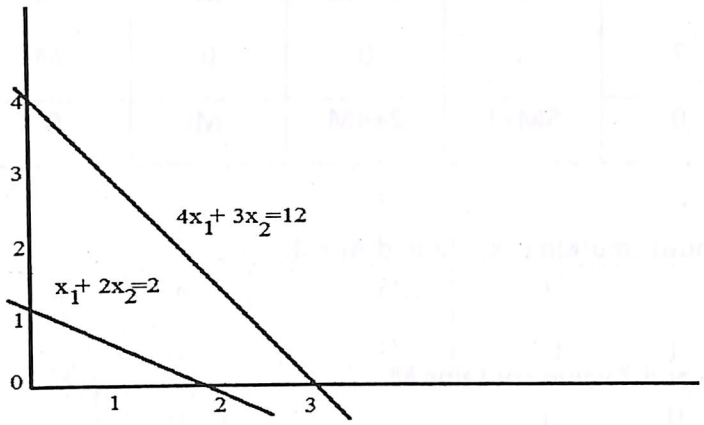
Ex 8.2 : Solve: Max $Z=2x_1 + 3x_2$

s.t $x_1 + 2x_2 \leq 2$

$4x_1 + 3x_2 \geq 12$

$x_1, x_2 \geq 0$

Applying graphic method



Feasible region satisfying both the conditions does not exist.

Therefore, Solution is not feasible.

Let us apply simplex technique and find the solution.

Constraints

$x_1 + 2x_2 + S_1 = 2$

$4x_1 + 3x_2 - S_2 + A_1 = 12$

Objective function $2x_1 + 3x_2 + 0s_1 + 0s_2 - MA_1$

Simplex Table I

B	C_B	X_B	X_1	X_2	S_1	S_2	A_1	$=X_{B\div X1}$
S_1	0	2	1	2	1	0	0	2
A_1	-M	12	4	3	0	-1	1	3
		Z_j	-4M	-3M	0	M	-M	
		C_j	2	3	0	0	-M	
		j	-4M-2	-3M-3	0	M	0	

Second Simplex Table

B	C_B	X_B	X_1	X_2	S_1	S_2	A_1
x_1	2	2	1	2	1	0	0
A_1	-M	4	0	-5	-4	-1	1

Z_j	2	4+5M	2+4M	M	-M
C_j	2	3	0	0	-M
j	0	5M+1	2+4M	M	0

The solution is optimum and the optimum solution is $x_1 = 2$ and $A_1 = 4$.

Therefore, Z=(2 x 2) (-M x 4) = 4 - 4M

Basic contains artificial variable and Z value contains M

Therefore, Solution is infeasible as Z contains M

(2) **Unbounded solutions**

When a Linear Programming problem does not have finitely valued solutions, the solution is said to be unbounded. If, in a problem, the solution of a variable can be made infinitely large without violating constraints, the solution obtained is unbounded. For such problems feasible region is unbounded.

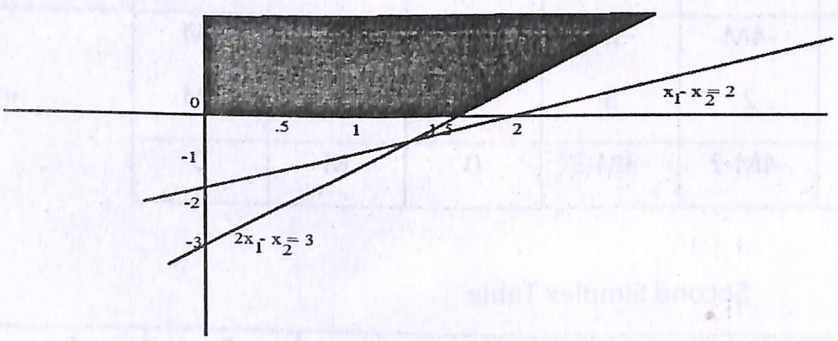
Ex.2: Solve:

Max $Z=2x_1 + x_2$

s.t $x_1 - x_2 \leq 2$

$2x_1 - x_2 \leq 3$

$x_1 , x_2 \geq 0$



Feasible region is bounded. Therefore, Solution is unbounded.

Applying Simplex method

Ans:

Constraints are

$x_1 - x_2 + s_1 = 2$

$2x_1 - x_2 + s_2 = 3$

Objective function $Z = 2x_1 + x_2 + 0s_1 + 0s_2$

Simplex Table I

B	C_B	X_B	X_1	X_2	S_1	S_2	
s_1	0	2	1	-1	1	0	2
s_2	0	3		-1	0	1	1.5
		Z_j	0	0	0	0	
		C_j	2	1	0	0	
		$z_j - c_j$	-2	-1	0	0	

Simplex Table II

B	C_B	X_B	X_1	X_2	S_1	S_2	
s_1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	-1
x_1	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	-3
		Z_j	2	-1	0	1	
		C_j	2	1	0	0	
		j	0	-2	0	1	

Now both the values f are negative.

So, non-negative minimum ratio is not obtained.

Therefore, There is no outgoing vector.

Therefore, The solution is unbounded.

(3) Alternative Optimum Solution (Multiple solution)

In the final simplex table we find that net evaluation values ($_j$ values) of all basic variables are zero and of non basic variables not zero. If a $_j$ value of a non basic variable is also zero, then more than one solution is possible for the problem. That is the problem has alternative optimum solution.

If we apply graphic method, the feasible region will show two solution points.

For example, consider the following problem.

Ex.3: **Maximize $Z = 4x_1 + 4x_2$**

Subject to $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 \leq 6$

$x_1, x_2 \geq 0$

Ans: Applying simplex method.

Starting Simplex Table

B	C_B	X_B	X_1	X_2	S_1	S_2	S_3	$= C_B \div X_1$
s_1	0	16	1	2	1	0	0	10
s_2	0	6	1	1	0	1	0	6
s_3	0	4	[1]	0	0	0	1	4
		Z_j	0	0	0	0	0	
		C_j	4	4	0	0	0	
		j	-4	-4	0	0	0	

Second Simplex table

B	C_B	X_B	X_1	X_2	S_1	S_2	S_3	$= C_B \div X_2$
s_1	0	6	0	2	1	0	-1	3
s_2	0	2	0	[1]	0	1	-1	2
x_1	4	4	1	0	0	0	1	∞

Z_j	4	0	0	0	4
C_j	4	4	0	0	0
j	0	-4	0	0	4

III Simplex Table

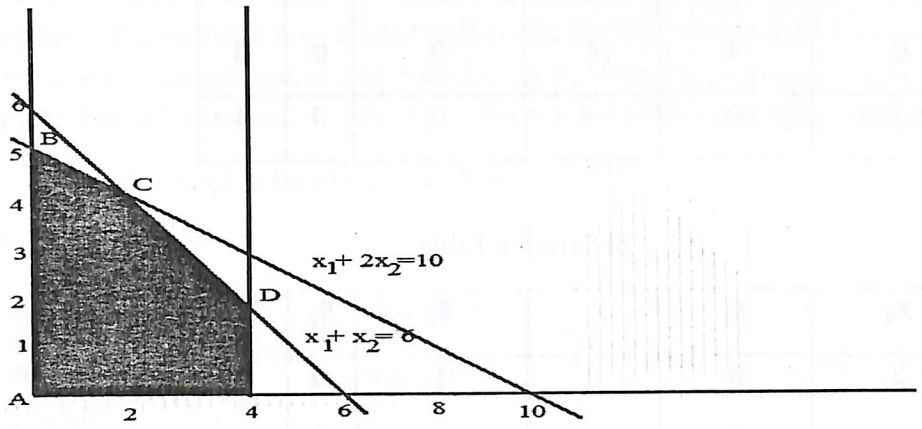
B	C_B	X_B	X_1	X_2	S_1	S_2	S_3
s_1	0	2	0	0	1	-2	1
x_2	4	2	0	1	0	1	-1
x_1	4	4	1	0	0	0	1
		Z_j	4	4	0	4	0
		C_j	4	4	0	0	0
		j	0	0	0	4	0

Therefore, The solution is optimum. Optimum solution is $x_1 = 4$, $x_2 = 2$

$_j$ values of $S_3 = 0$.But S_3 is a non basic variable. Therefore, this LPP has more than one solution.

Let us solve the problem graphically

Points	x_1	x_2	Z
A	0	0	0
B	0	5	20
C	2	4	24
D	4	2	24
E	4	0	16



There are two solutions

$x_1 = 2$, $x_2 = 4$, $z=24$

$x_1 = 4$, $x_2 = 2$, $z=24$

(4) **Tie in incoming vectors (more than one key columns)**

If two $_j$ values are largest (negative in the case of maximization problems and positive in case of minimization problems),then the question arises which is to be selected, for determining the incoming vector. In such situation, we may select arbitrarily one of them, preferably left most.

In the case of tie between a decision variable and a slack (or surplus) variable, the decision variable should be selected.

(5) **Tie in key rows (more than one minimum ratio)**

When there are more than one minimum ratio we can solve the problem in the following manner.

Each element in the tied rows should be divided by positive coefficients of the key column in that row. Moving left to right, column by column (first unit matrices), the row which first contains the smallest algebraic ratio has the outgoing slack variable. Before this, artificial variables , if any, should be removed. Seethe problem done in page 39 as Ex.2

	S_1	S_2
Row1	$^1/_1 = 0$	$^0/_1 = 0$
Row 2	$^0/_5 = 0$	$^1/_5 = .2$

Here smallest is 0. It occurs in Row 1. Therefore, S_1 is the outgoing vector.

(6) **Degeneracy and Cycling**

A basic feasible solution of a linear programming problem is said to be degenerate if at least one of the basic variables is zero.

One of the basic theo rems of Linear Programming is that the number of non zero valued variables in a LPP should be equal to the number of constraints. Therefore, if one of the basic variable is zero, the number of non-zero valued variables become one less than the number of constraints [non-basic variables are always zero]. This situation is called degeneracy because from this simplex table, we cannot continue to reach next simplex table as the variable to be replaced is already zero.

The main drawback of degeneracy is the increase in the computation which reduces the efficiency of the simplex method.

Degeneracy occurs in two stages (1) The degeneracy appears in a LPP at the very first iteration (2) Degeneracy occurs in the subsequent iteration.

(1) **Degeneracy in first iteration**

Suppose the right hand side of one of the constraints is zero, then the corresponding X_B value will be zero so that one of the basic variables is zero. Therefore the solution is degenerate See the following example.

Ex.3: Max $Z=2x_1 +3x_2 + 10x_3$

s.t $x_1 + 2x_3 \leq 0$

$x_2 + x_3 \leq 1$

$x_1, x_2, x_3 \geq 0$

Ans: The constraints are

$x_1 + 2x_3 + s_1 = 0$

$x_2 + x_3 + s_2 = 1$

$z=2x_1 + 3x_2 + 10x_3 + 0s_1 + 0s_2$

Starting Simplex Table

B	C_B	X_B	X_1	X_2	X_3	S1	S_2	Θ
s_1	0	0	1	0	2	1	0	0
s_2	0	1	0	1	1	0	1	1

Here the basic variables are s_1 and s_2. The solution is $s_1 = 1$ and $s_2 = 1$.

Since one of the basic variables is zero the solution is degenerate.

NOTE:

In some cases there will be degeneracy in the first iteration but in the subsequent stage the degeneracy disappears.

Review Questions EXCERCISES

1. what is objective function in Linear Programming
2. explain minimization problem
3. what is infeasibility of solution

4. state the steps in minimizing problem
5. how can minimization be solved
6. what Alternative optimal solution
7. explain the situation of multiple options
8. what is unbounded solution n
9. Explain Tie in vectors
10. What is tie in Key rows
11. What is degeneracy
12. What is cycling inlinear programming
13. Explain 'Degeneracy' and its implication in Linear Programming. How is degeneracy resolved ?
14. .When is the solution to a LPP infeasible ?
15. Solve the following LPP

Max. $Z=x_1 + x_2$

S.t $x_1 - x_2 \geq 0$

$3x_1 - x_2 \leq -3$

$x_1 , x_2 \geq 0$

16. Solve Max. $Z=3x_1 + 4x_2$

S.t $x_1 - x_2 \leq 1$

$-x_1 + x_2 \leq 2$

$x_1 , x_2 \geq 0$

17. Solve Max. $Z= 3x_1 + 2x_2$

S.t $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1 , x_2 \geq 0$

18. Solve using simplex method

Max. $X= 5x_1 + 8x_2$

S.t $3x_1 + 2x_2 \geq 3$

$x_1 + 4 x_2 \geq 4$

$x_1 + x_2 \geq 5$

$x_1 \geq 0, x_2 \geq 0$

19. Show that the following has multiple optimum solution.

Max. $X= 4x_1 + 4x_2$

S.t $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 \leq 4$

$x_1 , x_2 \geq 0$

20. . Show that the following L.P.P. has alternative (multiple) solution.

Maximize $Z=4x_1 + 4x_2$

S. t $x_1 + x_2 \leq 6 , x_1 \leq 4 , x_1 \geq 0 , x_2 \geq 0$

21. Show that the solution following L.P.P in unbounded

Maximize $Z=2x_1 + 3x_2$

S. t $x_1 - x_2 \leq 2 , x_1 + x_2 \geq 4 , x_1 \geq 0 , x_2 \geq 0$

22. Show that the following L.P.P has no feasible solution

Maximize $Z=4x_1 + 3x_2$

S. t $x_1 - x_2 \leq -1 , -x_1 + x_2 \leq 0 , x_1 \geq 0 , x_2 \geq 0$20. Show that the following L.P.P. has alternative (multiple) solution.

Maximize $Z=4x_1 + 4x_2$

S. t $x_1 + x_2 \leq 6 , x_1 \leq 4 , x_1 \geq 0 , x_2 \geq 0$

21. Show that the solution following L.P.P in unbounded

Maximize $Z=2x_1 + 3x_2$

S. t $x_1 - x_2 \leq 2 , x_1 + x_2 \geq 4 , x_1 \geq 0 , x_2 \geq 0$

22. Show that the following L.P.P has no feasible solution

Maximize $Z=4x_1 + 3x_2$

S. t $x_1 - x_2 \leq -1 , -x_1 + x_2 \leq 0 , x_1 \geq 0 , x_2 \geq 0$

23. A company produces two articles A and B. There are two different departments through which the articles are processed, viz assembly and finishing. The potential capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. Production of one unit of A requires four hours in assembly and 2 hours in finishing. Each of the unit B requires 2 hours in assembly and 4 hours in finishing. If profits is Rs..8 for each unit of A and Rs..6 for each unit of B find out the number of units of A and B to be produced each week to get maximum profit(solve graphically)

UNIT 9 TRANSPORTATION MODEL - BASIC CONCEPTS

Transportation is concerned with the movement of pruducts, from a source, viz, plant, factory or workshop etc, to a destination, such as ware house, retail store, etc. Transportation may be done through road, rail, air, water, pipeline or cable routes using trains, trucks, planes, boats, ships and telecommunication equipments, as per the situations emerging.

In the context of the present day competitive market situations, the decision taken by the organisatin to find better ways for transporting their products from a number of sources to a number of destinations has become extremely crucial. In this connection, it is pertinent, to find answers for following questions:

1. How to transport the products from sources to destinations in a cost effective manner?
2.When to transport such products from sources to destinations ?

Cost effective transportation is challenging task to organizations. Transportation model provides a powerful framework to meet this challenge and ensure the efficient movement and timely availability of products. In this context, the objective for any organisation is to minimize transportation costs while meeting the demand for the products.

Transportation costs generally depend upon the distance between the source and he destination, the mode of transportation chosen and the size and quantity of products to be shipped. In many cases, there are several sources and destinations for the same product, which brings significant level of complexity of the problem of minimizing transportation costs.

The decisions regarding transportation of products are dependent on several factors. For example, the accessibilityof suitable mode of transportation is affected by the decisions regarding approporiate location of the business. The mode of transportation chosen affect the decisions regarding the form of packing used for the producrts and the size or frequency of the shipments made.However, transportation costs may be reduced by sending larger quantity in a single shipment. The inter-relationship of these decisions implies that the successful planning and scheduling can help to decrease transportation costs. In other words, the transportation problem is with the objective of optimizing transportation resources.

The earliest development of this problem was done by Hitchcock in 1941, and thereafter, Charnes and Copper proposed an alternative technique for solving the same. In this area, the Charnes and Copper method was considered as the stepping stone method.

The transportation problems are special type of linear progrmming problems, of which, the objective is to transport various quantities of a single homogenous commodity , to different destinations, in such a way that total transportation cost is minimum . Transportations problems give direct relevance to decisions in the area of distribution, policy making, channel selection etc, where the objective is to minimize cost. Here the availability as well as requirements of various centres are finite and constitute limited resources. It is assumed that cost of shipping is linear.

Transportation problems are particular class of allocation problems also. The objective in the decision problems is to transport various amounts of a single homogenous commodity, that are stored at several origins, to a number ofdestinations. The

transportation cost is effected insuch a way that the destination's demands are satisfied within the capacityof distribution origins, and that the total transportation cost is minimum For example, a manufacturing concern has three plants located in different cities in India. There are 4 retail shops in different cities of the country which can absorb al theproducts stored. Then the transportation problem is to determine the transportation schedule that minimizes the total cost of transporting manufactured products from various plans to various retail shops.

The name transportation problem is derived from the term transport to which it was first applied. But the transportation technique is applicable to to other problems also, for example – machine allocation, product mix etc. Transportation technique can be applied not only to the cost minimizing problems, but also to time minimizing problems, distance minimizing problem, profit maximizing problems etc.

Basic assumptions in transportation model

1. Total quantity available for distribution is equal to total requirements in different destinations together.
2. Unit transportation cost from one origin to a destination is certain
3. Unit cost is independent of the quantity transported
4. Objective is to minimize total transportation cost.

Uses of transportations technique

1. It is helpful in minimizing trasporttion costs from factories to warehouses or from ware houses to markets
2. It assists in determining lowest cost loction for new factories
3. It can identify minimum cost production schedule.
4. It can determine ideal locations for factories or ware houses, so that transportation cost is minimum,

Badic concepts

Several concepts ae involved in operating transportation model

Feasible solution

A feasible solution to a transportation problem is a set of non negative individual alloctions which satisfy the row and column sum restrictions. So, for feasibility, the sum of allocation in the rows must be equal to the availability in that row. Similarly sum of the allocation in the column must be equal to the demand in that column.

Basic feasible solution

A basic feasible solution to a m x n transportation problem is said to be a basic feasible solution , if the total number of alocations is exactly equal to m + n - 1.

Optimal solution

A fessible solution, basic or no,t is said to be optimal if it minimises the total transportation cost.

Non degenerate basic feasible solution

A feasible solution of m x n transportation problem is said to be non degenerate basic feasible solution if the number of allocations is equal to m + n - 1, and the allocations are in independent positions.

Loops in transportation table

Allocatins are said to in indepndent positions, if it is impossible to increase or decrease any allocation without either changing the positoin of the allocation or violating the rum requiremtnts. Therefore when the allocations are in indepndent positions, it is

impossible to tranfer from any allocation back to itself through a series of horizontal or vertical jumps .

Steps for solving transportation problem

1. Set up transportation table with m rows representing the origins and n columns representing the destinatins
2. Develop initial feasible solutions to the problem.
3. Test whether the soluton is optimal or not
4. If the solutins in not loptimal , modify the allocaltions
5. Repeat steps 4 and 5 until an optimal solution is obtained

If there are m rows and n columsn, ther will be mn cells or spaces. Each cell is known by two numbers , one representing the row and the other representing the column. For example, cell 2,3 means the cell falling in the second row and third column.

Initial basic feasible solution

Initial feasible solutions are those which satisfy the rim requirements. That is, the allocations made in every row taken together is equal to the availability shown in that row. Similarly for each column, the total alloction should be equal to the requirement in that column.

The initial soluton can be obtained either by inspection or by some rules. The commonly used methods for finding initial solution are North West Corner Rule, Lowest Cost entry Method or Vogels Approximisatin Method.

Optimal solution of Transportation Problem

There are two steps to find the optimal soluton of the transportation problem

1. Find an initial basic feasible solution.
2. Obtain an optimal solution by makig successive improvement to initial basic feasible solution,until no further decerease in the transportation cost is possible

Methods for initial basic feasible solutions

Tranpsorttion table presents information relating to supply at origins and demand at diestination centres. Ehen the information is presented in the form a transportation table, attmept can be made to obtain a feasible solution. A feasible solution or basic feasible solution is a set of non-negative allocations which satisfy row and column sum restrictions. For feasibility, sum of allocations in the rows must be equal to availabity in that row. Similarly, sum of allocations in the columns must be equal to demand in that column

Following are important methods of developing an initial feasible solution.

1. North West Corner Method (NWCM)
2. Lowest Cost Entry Method (LCEM)
3. Vogel's Approximatin Method (VAM)

North West Corner Method

This is the most systematic and easiest method for obatinaing initial fassible solution for a transportation problem. This method starts by allocating from the left top cell onwards. This will give solution, which may or may not be optimal.

Steps in NWC method

1. Provide an emply m x n marix, with necessary columns and rows
2. Allocate to the left – top cell (1,1) maximum possible amount which is minimum of row total and column total. So either a row or a column total gets exhausted.

3. Cross off that row or column as the case may be.
4. Consider reduced matrix. In that matrix, allocate to the cell (1,1) maximum possible amount (which is minimum of present row total or column total.
5. Repeat the above steps until all available quantities are exhausted.

EX 10.1

EX. 10.1:Find initial feasible solution to the transportation problem given below, by North West Corner rule.

Origins	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

Allocate to cell (1, 1) minimum of 5 and 7, i.e. 5.Thus O_1 row total is exhausted, since the supply of O_1 is completely met. So, cross of the row O_1

	D_1	D_2	D_3	
	5			
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
	7	9	18	

Consider reduced matrix after deleting O_1 row. Now allocate the cell (1, 1) min of 8 and 2 i.e. 2. Thus column D_1 is exhausted and it is crossed off.

	D_1	D_2	D_3	
	2			
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
	2	9	18	

	D_2	D_3	100
O_2	3	1	6
O_3	4	7	7
O_4	6	2	14
	9	18	

Consider reduced matrix.Allocate to cell (1, 1) min of 6 and 9, i.e. 6. Thus O_2 row is crossed off.

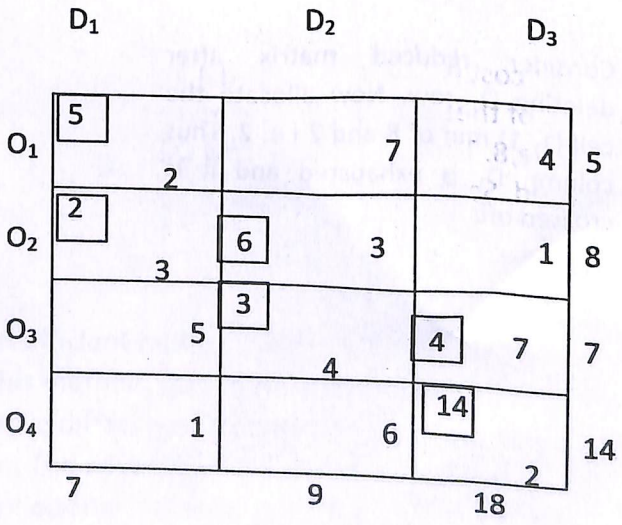
	D_2	D_3	
O_3	4	7	7
O_4	6	2	14
	3	18	

Allocate to cell (1, 1) min of 7 and 3, i.e. 3. Thus D_2 column is crossed off.

	D_3	
O_3	7	4
O_4	2	14
	18	

Finally allocate 4 to cell 1,1 and 14 to 2,1.

Thus, various allocations made to the cells are shown below and the solution is-



Thus the various allocations made to the cells are shown here and the solution is -

Total transportation cost

$= (5 \times 2) + (2 \times 3) +$

$(6 \times 3) + (3 \times 4) + (4 \times 7)$

$+ (14 \times 2)$ = RS 102

Lowest Cost Entry Method

Choose the cell having the lowest cost in the matrix. Allocate there as much as possible which is the minimum of the row total and column total. Thus either a row total or a column total is exhausted. Cross off corresponding row or column. From the reduced matrix, locate the cell having the lowest cost. Allocate to that cell maximum possible quantity, thus leading to further reduced matrix. Continue this process until all the available quantities are exhausted.

Ex.

Find initial feasible solution to the following transportation problem by lowest cost entry method

Origins	D_1	D_2	D_3	Supply
O_1	2	3	7	5
O_2	3	3	1	8
O_3	5	4	7	7
5_4	1	6	2	14
Demand	7	9	18	

The lowest cost is 1 in cells 2,3 and 4,1. select one of thes say 2,3.allocate min of 8 and 18, ie,8. Thus the total of row O2 is exhausted. So cross off the row F2

7 9 18

	W_1	W_2	W_3	
F_1	2	7	4	5
F_2	3	3	[8] 1	8
F_3	5	4	7	7
F_4	1	6	2	14

	W_1	W_2	W_3	
F_1	2	7	4	5

F_3	5	4	7	7
	7	6	2	14
F_4	1			

The lowest cost in thereduced matrix is in the cell 3,1. Allocate min of 7 0 14, ie, 7. Thus the total of column W1 is exhausted. Cross off the

	W_2	W_3	
		4	
F_1	7		5
	4	7	7
F_3			
	6	7	7
F_4		2	

Allocate to the cell 3,2 min of 7 or 10, Ie, 7 and cross off row F4

Allocate to thecell 2,1 units 7

	7	4
	7	
	4	7

Finally allocate 3 to the cfell 1,2 and 7 to The Cell 2,1 and 2 to the cell 1,1.

	2		3		
	7		4		

The solution is
2 x 7 + 3 x 4 +8 x 1 +7 x 4 +
7 x 1 = 7 x 2 = 83

O_1	5		2	7	3 4	5
O_2	3		3		8 1	8
O_3		5	7	4	7	7
O_4	7	1		6	7 2	14
	7		9		18	

Vogel's approximation Method

Vogels Approximation method is an efficient method for finding initial basic feasible solution of a transportation problem . In this method, each allocation of initial basic feasible solution is made on the basis of non negative difference between the smallest and second smallest cost in each row and each column. The advantage of this method is that it gives an initial basic feasible solution which ie either optimal solution or nearer to the optimal solution.

This method is preferred over the other two methods because the initial basic feasible solution obtained with VAM is either optimal or very close to the optimal solution. Therefore, the amount of time required to calculate the optimum solution is reduced In vogel's approximation method , the basis of allocation is unit cost penalty, ie, that column or row which has the highest unit cost penalty diffrerence,between the lowest and the next higher cost is selectd first, for allocation and the subsequent allocations in cells are also done keeping in view the highest unit cost penalty.

Steps in VAM

1. Construct the cost, requirement and availability matrix, ie, cost matrix with column and row information
2. Compute penalty for each row and column in the transportation table. The penalty for a given row and column is merely the difference between the smallest cost and the next smallest cost element in that particular row or column.
3. Identify the row and column with largest penalty. In this identified row or column, choose the cell which has the smallest cost and allocate the maximum possible quantity to this cell. Delete the row or column, in which capacity requirement is exhausted
4. Repeat step 1 to 3 for the reduced table until the entire capacities are used to fill the requirement at different warehouses.
5. From step 4 we will get initial basic feasible solution. Now for IFS find the initial transportation cost by multilying the cell allocation by unit cost.

Though this method takes more tme as compared to other two methods, it gives better solution, and saves more time in reaching the optimal solution.

Find initial solution for the transportation problem by Vogel's method

Origins	W_1	W_2	W_3	Supply
F_1	2	3	7	5
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
Demand	7	9	18	

Ans.

Write penalties in brackets for all columns and rows

Penalties for rows $4-2=2$ $3-1=2$ $5-4=1$ $2-1=1$

Penalties for columns $2-1=1$ $4-3=1$ $2-1=1$ these are shown in brackets

	W_1	W_2	W_3	104
	5			
	2	7	4	5(2)
	3	3	1	8(2)
	5	4	7	7)1)
	1	6	2	14(1)
	7	9	18	
	(1)	(1)	(1)	

Maximum penalty 2 is associatged with row F1 and row F2. Select any one of these say row F1. Allocte min of 5 and 7 = 5 to the cell 1,1 which has the lowest cost in the row F1. Cross off row F1

	W_1	W_2	W_3	
			8	
F2	3	3	1	8(2)
F_3	5	4	7	7(1)
				14(1)
F_4	1	6	2	
	2	7		
	2	9	18	
	(2)	(1)	(1)	

Maximum penalty 2 is associated with row F_2 and column W1. Select any one of these say row F2. Allocate min of 8 and 18 = 8 to the cell 1,3 which has tghelowest cost in the row F_2. Cross off row F_2.

	W_1	W_2	W_3	
F_3	5	4	7	7(1)
			10	14(1)
F_4	1	6	2	
	2	9	10	
	(4)	(2)	(5)	

Maximum penalty 5 in column W3. Allocate10 to the cell 2,3. Cross off column W3.

	5	4
	2	
	1	6
	2	9
	(4)	(2)

Allocate 2 to the cell 2,1 and cross column W1.

7	4	
2		
	6	

Allocate 2 to 2,1 and 7 to 1,1

9

Thus the solution is

$5 \times 2 + 8 \times 1 + 7 \times 4 + 2 \times 1 + 2 \times 6 + 10 \times 2 = 80$

O_1	5 2	7	4	5
O_2	3	1	8 1	8
O_3	5	7 4	7	7
O_4	2 1	2 6	10 2	14
	7	9	18	

By applying North West Corner method or Lowest Cost Entry method or Vogel's Approximatin method, an initial feasible solution can be obtained. It may or may not be an optimal solution. Therefore , the next step is to examine whether the solution can be improved and made optimal. For this, we conduct test of optimality. This is discussed in the next unit.

Review Questions and Exercises

1. What is menat by transportation?
2. What are different modes of transportation?
3. Why transportation is considered crucial?
4. Explain Tranporttation Model
5. State factors in determining tranportation cost
6. What is the objective in transportation
7. What are origins
8. Explain destinations
9. How can transportation cost be refuced

10. Explain nature of transportation problem
11. What are basic assumptions in transportation model
12. State uses of transportation technique
13. What is feasible solution
14. What is Initial feasible solution
15. Explain optimal solution
16. Explain loop in transportation problem
17. State steps in solving transportation model
18. Explain methods for Initial Feasible Solution
19. State steps in NWC method
20. What is the procedure in LCE method
21. Explain steps in VAM
22. Solve following transportation problem by NWC method

Origins	W_1	W_2	W_3	Supply
F_1	12	13	17	25
F_2	13	13	11	32
F_3	5	4	7	12
F_4	11	16	12	24
Demand	27	29	28	

23. Find lowest cost of transportation for the following problem by Lowest Cost Entry Method

Plants	D_1	D_2	D_3	Markets
O_1	6	8	9	6
O_2	5	6	5	10
O_3	4	5	7	9
5_4	5	6	6	12
Demand	8	7	12	

24. Solve following transportation problem using VAM

Origins	D_1	D_2	D_3	Supply
O_1	7	6	8	7

O_2	6	6	7	9
O_3	5	5	7	9
5_4	5	6	5	12
Demand	8	8	16	

UNIT 10 TRANSPORTATION – OPTIMAL SOLUTIONS

After obtaining an initial basic fesible solution of a transportation problem, our objective is to find the optimal solution of the problem. For this purpose , we have to improve the solution for obtaining the optimal solution. In this situation, we shall have to study the effect of allocating a unit in an unoccupied cell after making adjustment in the solution to satisfy the rim requirements. This net change in the total cost resulting from the unit allocation in a cell is called net evaluation of that cell. If the net evaluation be positive for some cells, the new solution increases the total cost and if net evaluation be negative, then the new solution reduces the total cost. This implies that the total cost can be further reduced by allocating some quantity in the cells whose net evaluations are negative.

Therefore, if the net evaluations for all the unocuupiedd cells be greater than or equal to zero, then there is no scope for decreasing the total cost further and hence the current solution will be optimal. This method is known as stepping stone method.

In this method, the net evaluations for unoccupied cells are computed by loop formation in each iteration which is a highly difficult job. To overcome this tedious job, a relatively easier method, like MODI or Modeified Distribution method or u-v method is used for net evaluatin for each cell in a similar way . Thus, there are two prominent methods available for testing optimality – the stepping stone method and modified distribution method.

Stepping stone method

The stepping stone method is an iteractive technique, from moving an initial feasible solution to an optimal feasible solution. In order to apply the stepping stone method, to transportation problem, one rule about the number of shipping routes being used must first be observed. In this rule, the number of occupied routes must always be equal to one less than the sum of the number of rows plus the number of columns.

For testing the solution for possible improvement, its approach is to evaluate the cost effectiveness of shipping goods via transportation routes not currectly in the solution.

Steps in stepping stone method

1. Prepare a Transportation Table with a given unit cost of transportation along with the rim reuirements
2. Determine an initial basic feasible solution using any method preferabley VAM.
3. Evaluate all unoccupied cell for the effect of transferring one unit from an occupied cell to the unoccupied cell. This transfer is made by forming a closed path (loop) that retains the supply and demand condition of the problem.
4. Compute net change in cost along the closed path, and calculate for all unoccupied cells.
5. Check the sign of each of the net change in the unit transportation costs. If all net changes are plus signs, or zeor, then we have obtained an optimal solution, other wise go to step 6.

.

6. Select the unoccupied cell with most negative net change among all unoccupied cells. If two minus values are equal, select that one which will result in moving as many units as possible into the selected unoccupied cell with the minimum cost.
7. Assign as many units as possible to unoccupied cell satisfying rim conditions. The mximum number of units to be assigned are qequal to the smaller circled number ignoring sign among the occupied cells with minus value in the closed path.
8. Go to step 3 and repeat the procedure until all unoccupied cells are evaluated and the net change is positive or zero vlues.

Since stepping stone method involves tedious calculations, we resort to Modified Distributon method

Modified Distribution Method (MODI)

The Modified Distribution Method allows us to compute improvement indices quickly for each unoccupied cell without drawing all of the closed paths or loops. Because of this it can often provide considerable time savings over the stepping stone method for solving transportation problems.

MODI provide a new means of finding the unused route with the lrgest negative imporovment index. Once the largest index is identified, we are required to trace only one closed path, just as with the stepping stone approach, this path helps to determine the maximum number of units that$_t$ can be shipped via the best unused route.

Steps in MODI method

1. When the initial basic feasible solution is obtained, some cells are occupied, ie, allocation made and others unoccupied. Number of occupied cells is m+n -1. Let C_{ij} be the cost of cell $_{ij}$.
2. Determine m+n numbers called $U_i + V_j = C_{ij}$ corresponding to each occupied cell. For example, if one of the occupied cells is 2,3 then, the equation is $U_2 + V_3 = C_{23}$, whefre C_{23} is the cost in the cell 2,3.
3. For solving the equation, we take one of U_i or V_j values as zero, since number of unknown is one more than thenumber of equations.

4. Calculate cell evaluations known as d_{ij} values for unoccupied cells, by the formula $d_{ij} = C_{ij} - (U_i + V_j)$
5. If all d_{ij} are positive, the solution is optimal and unique. If at least one of them is zero and others positive , the solution is optimal. But altenative solution exists. If atleast one d_{ij} value is negative, the solution is not loptimal
6. If the solution is not optimal, make reallocations. Give maximum allocation to the cell for which d_{ij} is negative making one of the occupied cells empty.

Solve following transportation problem

Origins	W_1	W_2	w_3	Supply
F_1	2	7	4	5
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
Demand	7	9	18	

Ans: We have solved this problem by Vogel's Method. Initial solution obtained is

	I	II	III	Supply
1	5 (2)	(7)	(4)	5
2	(3)	(3)	8 (1)	8
3	(5)	7 (4)	(7)	7
4	2 (1)	2 (6)	10 (2)	14
Demand	7	9	18	

To test the optimality: Occupied cells are (1,1), (2,3), (3,2), (4,1), (4,2), (4,3). The costs in these cells are 2, 1, 4, 1, 6, 2. Therefore the equations are :

$U_1 + V_1 = 2$ $\quad U_4 + V_1 = 1$

$U_2 + V_3 = 1$ $\quad U_4 + V_2 = 6$

$U_3 + V_2 = 4$ $\quad U_4 + V_3 = 2$

Take any one of U and V values as zero, say U_4 (which occurs more number of times) be 'zero'.

When $U_4 = 0$, $V_1 = 1$, $V_2 = 6$, $V_3 = 2$, $U_1 = 1$, $U_2 = -1$, $U_3 = -2$.

Shown below are three matrices, (C_{ij}), $(U_i + V_j)$, (d_{ij}). The (C_{ij}) matrix is that containing costs of all unoccupied cells. $(U_i + V_j)$ matrix is that containing $U_i + V_j$ values for unoccupied cells. (d_{ij}) matrix is that containing the difference, (C_{ij}) - $(U_i + V_j)$ values. The occupied cells are kept blank.

(C_{ij})

X	7	4
3	3	X
5	X	7
X	X	X

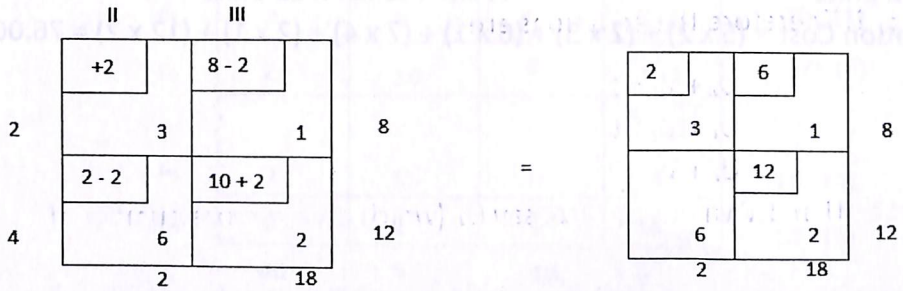
$(U_i + V_j)$

			U_i
X	1 + 6 = 7	1 + 2 = 3	1
-1 + 1 = 0	-1 + 6 = 5	X	-1
-2 + 1 = -1	X	-2 + 2 = 0	-2
X	X	X	0
$V_j \rightarrow$	1	6	2

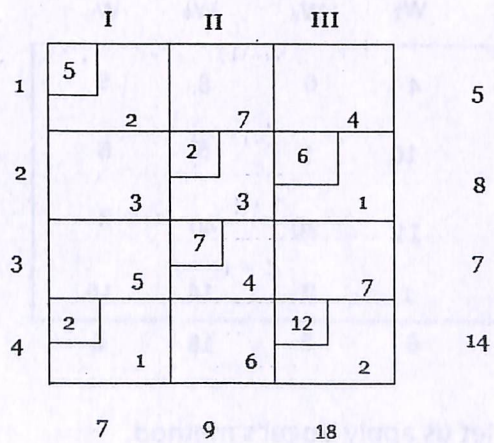
$d_{ij} = C_{ij} - (U_i + V_j)$

X	7 - 7 = 0	4 - 3 = 1
3 - 0 = 3	3 - 5 = -2	X
5 + 1 = 6	X	7 - 0 = 7
X	X	X

Since one of d_{ij} values is negative the solution is not optimal.
Make reallocations. Give maximum possible allocation to the cell where negative d_{ij} value occurs, ie., the cell (2, 2).
Consider four cells shown below. Transfer 2 from (4, 11) to (2, 11). Correspondingly subtract 2 from (2, 111) and add 2 to (4, 111).



The revised allocation gives the following matrix.



To test the optimality again; the equations of U_i and V_j values of occupied cells are

$U_1 + V_1 = 2$
$U_2 + V_2 = 3$
$U_2 + V_3 = 1$
$U_3 + V_2 = 4$
$U_4 + V_1 = 1$
$U_4 + V_3 = 2$

Put $U_4 = 0$ so that $V_3 = 2$, $V_1 = 1$
$U_1 = 1$, $U_2 = -1$, $V_2 = 4$, $U_3 = 0$

The matrix for C_{ij},, $U_i + V_j$ values of unoccupied cells, and d_{ij} matrx are

(C_{ij})

×	7	4
3	×	×
5	×	7
×	6	×

$(U_i + V_j)$

×	4 + 1 = 5	2 + 1 = 3
1 - 1 = 0	×	×
0 + 1 = 1	×	0 + 2 = 2
×	0 + 4 = 4	×
$V_j \longrightarrow$	1	4	2

1
-1
0
0

(d_{ij})

×	2	1
3	×	×
4	×	5
×	2	×

No d_{ij} value is negative. Therefore, the solution is optimum. Therefore, the optimal solution is,

From 1 to I : 5 units
From 2 to II : 2 units
From 2 to III : 6 units
From 3 to II : 7 units
From 4 to I : 2 units
From 4 to III : 12 units

The total transportation Cost = (5 x 2) + (2 x 3) + (6 x 1) + (7 x 4) + (2 x 1) + (12 x 2) = 76.00

Ex. 5: Solve the following Transportation Problem

		To					
		W_1	W_2	W_3	W_4	W_5	Available
From	F_1	3	4	6	8	9	20
	F_2	2	10	1	5	8	30
	F_3	7	11	20	40	3	15
	F_4	2	1	9	14	16	13
Required		40	6	8	18	6	

Ans: To find the initial basic solution, let us apply Vogel's method.

Table I

	W_1	W_2	W_3	W_4	W_5	
F_1	3	4	6	8	9	20 (1)
			8			
F_2	2	10	1	5	8	30 (1)
F_3	7	11	20	40	3	15 (4)
F_4	2	1	9	14	16	13 (1)
	40	6	8	18	6	
	(0)	(3)	(5)	(3)	(5)	

Table II

	W_1	W_2	W_4	W_5	
F_1	3	4	8	9	20 (1)
F_2	2	10	5	8	22 (3)
				6	
F_3	7	11	40	3	15 (4)
F_4	2	1	14	16	13 (1)
	40	6	18	6	
	(0)	(3)	(3)	(5)	

Table III

	W_1	W_2	W_4	
F_1	3	4	8	20(1)
F_2	2	10	5	22(3)
F_3	9 / 7	11	40	9(4) ←
F_4	2	1	14	13(1)
	40 (0)	6 (3)	18 (3)	

Table IV

	W_1	W_2	W_4	
F_1	3	4	8	20(1)
F_2	2	10	5	22(3)
F_4	2	6 / 1	14	13(1) ←
	31 (0)	6 (3)	18 (3)	

Table V

	W_1	W_4	
F_1	3	8	20(5)
F_2	2	5	22(3)
F_4	7 / 2	14	7(12) ←
	31 (0)	18 (3)	

Table VI

	W_1	W_4	
F_1	20 / 3	8	20(5) ←
F_2	2	5	22(3)
	24 (1)	18 (3)	

Table VII

W_1	W_4	
4 / 2	18 / 5	22
4	18	

The initial BFS is

	W1	W2	W3	W4	W5	
F1	20 / 3	4	6	8	9	20
F2	4 / 2	10	8 / 1	18 / 5	8	30
F3	9 / 7	11	20	40	6 / 3	15
F4	7 / 2	6 / 1	9	14	16	13
	40	6	8	18	6	

To test the optimality

For occupied cells the equation containing U_i, V_j

$U_1 + V_1 = 3$ Put $U_2 = 0$,

$U_2 + V_1 = 2$ then , $V_1 = 2$

$U_2 + V_3 = 1$ $V_3 = 1$

$U_2 + V_4 = 5$ $V_4 = 5$

$U_3 + V_1 = 7$ $U_3 = 5$

$U_3 + V_5 = 3$ $V_5 = -2$

$U_4 + V_1 = 2$ $U_4 = 0$

$U_4 + V_2 = 1$ $V_2 = 1$

$U_1 = 1$,

The matrix for C_{ij},, $U_i + V_j$ values of unoccupied cells, and d_{ij} matrix are

X	4	6	8	9
X	10	x	X	8
X	11	20	40	X
X	X	9	14	16

X	2	2	6	-1
X	1	x	X	-2
X	6	6	10	X
X	X	1	5	-2

X	2	4	2	10
X	9	x	X	10
X	5	14	30	X
X	X	8	9	18

No D_{ij} value is negative. Therefore, solution is optimal

F1 to W1 : 20
F2 to W3 : 8
F3 to W1 : 9
F4 to W1 : 7
F2 to W1 : 4
F2 to W4 : 18
F3 to W5 : 6
F4 to W2 : 6

Total transportation cost= 20 X 3 + 4 X 2 + 8 X 1 + 18 X 5 + 9 X 7 + 6 X 3 + 7 X 2 + 6 X 1 = 267

Review Questions and Exercises

1. What is initial basic feasible solution
2. State different methods of basic feasible solutions
3. Explain optimal solution
4. What is meant by optimality test
5. What are the popular lmethods of optimal solutions
6. What is the speciality of stepping stone method
7. Describe MODI method

8. What are the drawbacks of MODI method
9. What are the constraints of stepping stone method
10. What is U_i V_j equation
11. Explain degeneracy in transportation
12. What is an unbalanced transportation problem
13. How will you solve a maximization transportation problem
14. Solve following transportation problem whose cost matrix availability at each plant and reqirment at each warehouse are given as gollows:

	Warehouse				
Plant	W1	W2	W3	W4Aavailability	
P1	190	300	500	100	70
P2	700	300	400	600	90
P3	400	100	600	200	180
Requirement	50	80	70	140	

15. Following table shows all the necessary information on the available supply to each warehouse, the requirement of each market and the unit transportation cost from each ware house to each market. Find a basic feasible solution and then an optimal solution.

	Market				
warehouse	1	2	3	4Supply	
A	5	2	4	3	22
B	4	8	1	6	15
C	4	6	7	5	8
Requirement	7	12	17	9	

16. A manufacturer wants to ship 8 loads of his product as shown below. The matrix gives the mileage from origin O to destination D'

	A	B	C	Available
X	50	30	220	1
Y	90	45	170	3
Z	250	200	50	4
Req	4	2	2	

Shipping costs are Rs 10 per load mile. What shipping should be used.

UNIT 11 ASSIGNMENT MODEL

Assignment model deals in allocting the various resources or items to various activities on one to one basis to each way that the time or cost involved is minimized and sale or profit is maximized. Such types of problem can also be solved with the help of simplex method or by transportation method ,but simpler and more efficient methods for getting the solution is available through assignment models.

Several problems of management have a structure identical with the assignment problem. A departmental head may have six people available for assignment and six jobs to assign. He may like to know which job should be assigned to which person, so that all these jobs can be completed in the shortest possible time. Likewise, a truck company, may have an empty truck in each of cities 1,2,3,4, 5 and 6 and 6 routes - A.B, C, D, E, F, and G. He would like to ascertain the assignment of trucks to various cities so as to minimize the total distance coved. Similarly in a marketing set up , , one could assign a particular salesman a particular territory with a view to maximize oval sales, by making an estimate of sales performance for different territories

Assignment problem is a special case of the transportation problem in which the objective is to assign a number of origins (or persons) to the equal number of destinations (or tasks) at a minimum cost. For example, a departmental head may have four persons available for assignment and four jobs to fill. Then his interest is to find best assignment which will be in the best interest of the department.

It may be noted that with n facilities and n jobs, there are n possible assignments. Now in the assumption that each one of the persons can perform each one job,one at a time, then the problem is to find an assignment that is which job should be assigned to which person , so that total cost of performing all the jobs is minimised. For this purpose the assignment problem is mathematically modeled.

Assumptions in assignment

1. There are finite number of persons and jobs
2. Number of persons must be equal to number of jobs.
3. All the persons are capable of taking up all the jobs, with different time or cost.
4. Number of columns are always equal to number of rows.
5. There is exactly one occupied cell in ach row and each column of the table.

Solution methods of assignment problem

An assignment problem is similar to transportation problem. It can be solved by the following four methods.

Enumeration method

In this method, a list of all possible assignments among the given resources and activities is prepared. Then an assignment involving the minimum cost, time or distance or maximum profit is selected. If two or more assignments have the same minimum cost, time or distance or maximum profit, then the problem has multiple optimal solutions.In that case, one of the solution is taken arbitrarily, as optimal solution.

In general, if an assignment problem involves n worker or jobs, then there are n factorial numbers of possible assignments. For example, for n=6 wor ers or jobs problem, we have to evaluate a total of 6 factorial or 720 assignments. However, when n is large,, the method is not suitable for manual calculations. This method is suitable only for small number of jobs and persons.

Simplex method

Since an assignment problem can be formulatd as a 0 or 1 integar linear programming proble, it can be solved by simplex method. From the mathematical formulation, it is seen that an assignmen problem has n x n decision variables and n = n equations. In particular, for a problem, involving 5 workers or jobs, there will be decision variables and 10 equalities. So it is difficult to solve manually.

Transportation method

Since an assignment problem is a special case of transportation problem, it can alco be solved transportation methods. However, every basic feasible solutions of a general assignment problem having a square matrix of or der n should have n = n – 1 assignments. But due to special structure of this problem, any solution cannot have more than n assignments. Thus the assignment problem is degenerate. In order to emove debgeneracy, n – 1 nummber of dummy allocations will be required in order to proc ed with transportation model. Thus the problem of degeneracy at each solutic n makes the transportation method computationally inefficient for solving an assig ıment problem.

Hungarian method

An efficient method for solving assignment problem was developed by Hungarian meathematician D. Konig. Therefore, this method is known as Hungarian method.

Difference between Transportation problem and Assignment problem

Both Transportation problem and Assignment problems are special type of linear programming problems. They deal in allocating various resources to various activities, so as to minimize time or cost. However there are following differences between them.

(1) Transportation problem is one of the sub classes of linear programming problems in which the objective is to transport various quantum of a commodity that are initially stored at various origins to different destinations in such a way that the transportation cost is minimum. The assignment problem is a special case of Transportation problem in which the objective is to assign a number of origins to the equal number of destinations at a minimum cost.

(2) In Transportation problems number of rows and number of columns need not be equal. In Assignmen problems the number of persons and number of tasks are equal so that number of rows and number of columns are equal.

(3) Transportation problems are said to be unbalanced if the total demand and total supply are not equal while Assignmen problems are unbalanced when the number of rows are not equal to number of columns.

(4) In Transportation problems a positive quantity is allocated from a source (origin) to a destination. In Assignmen problems a source (job) is assigned to a destination (a man)

Hungarian Method – steps

1 . Subtract the smallest element of each row, in the cost matrix, from every element of that row

2. Subtract the smallest element of each column, in the cost matrix, from every element of that column
3. Staring with row 1of matrix obtained, examine all row having exactly one zero element. Enclose this zero within □ showing that assignment is made there.
4. Cross out all other zeros in the column (in which we mark □)to show that they cannot be used to make other assignments. Proceed in this way until the last row is examined.

4. Examine all columns with one unmarked zero. Mark □ at this zero and cross all the zeros of the row in which □ is marked. Proceed in this way until the last column is examined

5. Continue these operations successively until we reach any of the following two situations.

(i) all the zeros are enclosed by □ or crossed, or (ii) the remaining unmarked zeros lie at least two rows or columns

In case of (i), we have a maximal assignment and in case (ii) still we have some zeros to be treated for which we use the trial and error method. After the above operations, there arise two situations.

(i) it has an assignment in every row and every column so that we got the solution then the assignment is complete.

(ii) it does not contain assignment in all rows and all columns. In the second situations the following procedure may be followed.

1. Draw the minimum number of horizontal and vertical lines necessary to cover all zeros at least once. For this following method is adopted.

(i) Mark (√) all rows for which assignment have not been made.

(ii) Mark (√)for columns which have zeros in marked rows.

(iii) Mark (√)for rows (not already marked) which have assignment in marked columns.

(iv) Repeat step (ii) and (iii) until the chain of marking ends.

(v) Draw lines through unmarked rows and through unmarked columns to cover all the zeros.

This procedure will yield the minimum number of lines that will pass through all zeros.

5. Select the smallest of the element that is not covered by lines. Subtract it from all the elements that do not have line through them, add it to every element that lies at the intersection of two lines and leave the remaining elements of the matrix unchanged.
6. Now re-apply the step 3 to 5 to the modified matrix.

Ex. 11 . 1: Find optimum solution to the following assignmente problem , showing the cost for assigning workers to jobs.

		Job		
		x	y	z
	A	18	17	16
Workers	B	15	13	14
	C	19	20	21

Ans:

Subtracting the smallest element of every row from all the elements of that row.

	X	y	z
A	2	1	0
B	2	0	1
C	0	1	2

Subtracting the smallest element of every column from all the elements of that column.

	X	y	z
A	2	1	0
B	2	0	1
C	0	1	2

Mark ▭ tozero in every row, starting from the first row. By now all the rows and all the columns have zero assignment.

	X	y	z
A	2	1	[0]
B	2	[0]	1
C	[0]	1	2

Therefore Solution is A to z, B to y and C to x

Total cost for assignment = 16+13+19= <u>48 Rs.</u>

Note: for zero assignment, first consider rows with only one zero, starting with the first row. Then consider all columns with only one zero, starting with the first column. When a zero of a row is assigned, all zeros of the column in which it lies, must be crossed out. Similarly when a zero of a column is assigned, all, zeros of the row, in which it lies, must be crossed out. After column assignment, again rows, then columns and so on

Ex. 11.2: Solve the following minimal assignment problem

		Man			
		1	2	3	4
	I	12	30	21	15
Job	II	18	33	9	31
	III	44	25	21	21
	IV	14	30	28	14

Ans: Subtracting the smallest element of each row from every element of that row

		Man			
		1	2	3	4
	I	0	18	9	3
Job	II	9	24	0	22
	III	23	4	0	0
	IV	0	16	14	0

Ssubstracting smallest elemnt of each column

	1	2	3	4
I	0	14	9	3
II	9	20	0	22
III	23	0	3	0
IV	0	12	14	0

Starting with row 1 we make zero assignment , containing only one zero and cross out the zero in the column in which it liesThen starting with column 1, we mark zero to the zero in the column containing only one unmarked or uncrossed zero

		1	2	3	4
	I	0	14	9	3
Job	II	9	20	0	22
	III	23	0	3	0
	IV	0	12	14	0

Since every row and every column have one assignment, we have the complete optimal zero assignment. So the solution is

Job:	I	II	III	IV
Man:	1	3	2	4

Therefore Assign first job to first man. Second job to third man, third job to second man, fourth job to fourth man. Total cost = 12+9+25+14 = 60

Ex. 11.3: Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows

		Job				
		L	M	N	O	P
Persons	A	8	4	2	6	1
	B	0	9	5	5	1
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine optimum assignment schedule.

Ans:

Subtracting the least element of every row from all the elements of that row

	L	M	N	O	P
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	4
D	4	3	1	0	3
E	4	0	3	4	0

Subtracting the least element of every column from all the elements of that column

	L	M	N	O	P
A	7	3	0	5	0
B	0	9	4	5	4
C	1	6	6	0	4
D	4	3	0	0	3
E	4	0	2	4	0

Assigning zeros in the one zero rows f irst. Then assigning zeros in the columns have only one unmarked or uncrossed zero.

	L	M	N	O	P
A	7	3	~~0~~	5	[0]
B	[0]	9	4	5	4
C	1	6	6	[0]	4
D	4	3	[0]	~~0~~	3
E	4	[0]	2	4	~~0~~

Now all rows and all columns get assignment. The solution is A to P, B to L, C to O, D to N, and E to M. Total cost = 1+0+2+1+5=9 Rs.

After considering all one zero rows and one– zero columns, suppose, still the assignment is not complete, select row with two zeros and assign arbitrarily any one zero, crossing out other zeros of that row and the column. After exhausting two-zero rows and columns, select three-zero rows and columns and so on. The method is called Trial and Error. In such cases there will be more than one optimum solution.

Ex. 4: Given below is the time (days) required when a particular programme is assigned to a particular programmer

	Programmers			
Programmes	A	B	C	D
1	12	10	8	9
2	8	9	11	7
3	11	14	12	10
4	9	9	8	9

Assign the programmers to the programmes in such a way that the total computing time is least.

Ans

	A	B	C	D
1	3	1	0	1
2	0	1	4	0
3	0	3	2	0
4	0	0	0	1

Subtract the least element of each row from all the elements of that row. Then subtract the least element of each column from all the elements of that column.

Make zero assignments in one zero rows and then one zero columns, now find that assignment is not complete but two rows and two columns have two zeros

Select arbitrarily first zero of the second row of Table II and complete the assignment.

	A	B	C	D
1	3	1	[0]	1
2	[0]	1	4	0
3	0	3	2	[0]
4	0	[0]	0	1

We can have an alternative solution if we had selected last zero of the second row

The two solutions are

(a) 1 to C, 2 to A, 3 to D, 4 to B = 8+8+10+9 = 35
(b) 1 to C, 2 to D, 3 to A, 4 to B = 8+7+11+9 = 35

In both the cases total time = 35 Days

Ex. 5: Solve the following assignmen problem showing costs for assigning 3 men to 3 jobs.

		Men		
		A	B	C
	I	2	6	2
Jobs	II	1	4	1
	III	5	3	8

Subtracting the smallest element of every row from all the elements of that row.

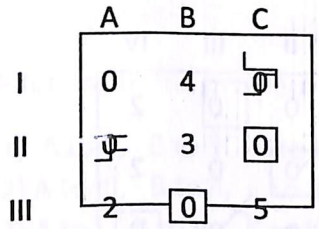
	A	B	C
I	0	4	0
II	0	3	0
III	2	0	5

Subtracting the smallest element of every column from all the elements of that column.

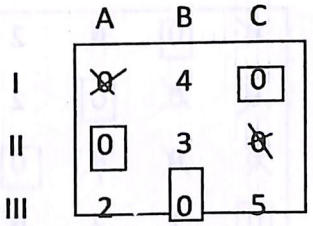
	A	B	C
I	0	4	0
II	0	3	0
III	2	0	5

Now make zero assignment. We find that third row has only one zero. It is first assigned. Then we have no one-zero row or column. So we select zero rows and then two zero columns starting from the first row. Arbitrarily assign one of these zeros in I rows and get one solution. Then consider the other zero and get the next solution.

Solution I



Solution II



Solutions: 1) I to A, II to C, III to B
2) I to C, II to A, III to B
Total cost for both the cases = 6 Rs.

Ex. 6: Solve the minimal assignmen problem whose effectiveness matrix is

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

Ans: Subtracting the smallest element of each row from every element of that row, Then, subtracting the smallest element of each column from the elements of that column.

	I	II	III	IV
A	0	0	0	2
B	0	0	0	2
C	0	0	0	0
D	0	1	3	0

None of the rows or columns contains exactly one zero. Therefore the trial and error method is followed. Now we start searching for two zeros. Starting with the first row coming to last row we find two zeros. We make the assignment the first zero and cross out all the other zeros of the first column and last row. Now starting with I column we find column IV which contain one zero and so make assignment and cross out all other zero's of this row. Now again starting with first row to find row containing only one zero and columns for one zero. But no assignment is possible. Again we start with first row searching for two zero and find the first row containing two unmarked zeros and cross out the other zeros of the column and row in which assignment Is made.

Proceeding in this manner, we find that many solutions are possible as shown below

Solution I

	I	II	III	IV
A	~~0~~	[0]	~~0~~	2
B	~~0~~	~~0~~	[0]	2
C	~~0~~	~~0~~	~~0~~	[0]
D	[0]	1	3	~~0~~

Solution I

	I	II	III	IV
A	~~0~~	0	[0]	2
B	~~0~~	~~0~~	0	2
C	~~0~~	0	~~0~~	[0]
D	[0]	~~1~~	~~3~~	0

Solution III

	I	II	III	IV
A	[0]	~~0~~	~~0~~	2
B	~~0~~	[0]	~~0~~	2
C	~~0~~	~~0~~	[0]	~~0~~
D	~~0~~	1	3	[0]

Solution IV

	I	II	III	IV
A	[0]	~~0~~	~~0~~	2
B	~~0~~	~~0~~	[0]	2
C	~~0~~	[0]	~~0~~	~~0~~
D	~~0~~	1	3	[0]

Solution V

	I	II	III	IV
A	~~0~~	[0]	~~0~~	2
B	[0]	~~0~~	~~0~~	2
C	~~0~~	~~0~~	[0]	~~0~~
D	~~0~~	1	3	[0]

Solution VI

	I	II	III	IV
A	~~0~~	[0]	~~0~~	2
B	~~0~~	~~0~~	[0]	2
C	[0]	~~0~~	~~0~~	~~0~~
D	~~0~~	1	3	[0]

Solution VII

	I	II	III	IV
A	~~0~~	~~0~~	[0]	2
B	[0]	~~0~~	~~0~~	2
C	~~0~~	[0]	~~0~~	~~0~~
D	~~0~~	1	3	[0]

Solution VIII

	I	II	III	IV
A	~~0~~	~~0~~	[0]	2
B	~~0~~	[0]	~~0~~	2
C	[0]	~~0~~	~~0~~	~~0~~
D	~~0~~	1	3	[0]

Solutions to the problem are

(1) A to II, B to III, C to IV, D to I or
(2) A to III, B to II, C to III, D to I or
(3) A to I, B to II, C to III, D to IV or
(4) A to I, B to III, C to II, D to IV or
(5) A to II, B to I, C to III, D to IV or
(6) A to II, B to III, C to I, D to IV or
(7) A to III, B to I, C to II, D to IV or
(8) A to III, B to II, C to I, D to IV

Total cost in all these cases = 20

questions and exercises

1. What is the basis of assignment model?
2. Give any two cases where assignment model is applied?
3. What is Hungarian method of solution to assignment problem?
4. What are the uses of HAM?
5. State the procedure in HAM?
6. Find the optimal solution for the assignmen problem with the following cost matrix.

7. Solve the following assignmen problem.

task	1	2	3	4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

8. A department head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ n their intrinsic difficulty. The estimate of time each subordinate would like to perform a task is given in the effectiveness matrix . How should the task be allocated, one to person, so as to minimize the total m an hours?

	Subordinate			
Tasks	I	II	III	IV
A	9	27	18	12
B	14	29	5	27
C	39	20	19	16
D	20	27	25	11

9. Find optimal solution fo

Following assignment problem.

	Area			
	w	x	y	z
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12
D	14	10	12	11

10. A shop manager has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ n their intrinsic difficulty. His estimate of the time each man would take to perform each tasks is given in the matrix below. How should the tasks be allotted so as to minimize the total man hours.

	Men			
Tasks	E	F	G	H
A	8	26	27	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

11. Make optimal assignment of the Following machine – worker Problem.

	L	M	N	O	P
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

n

UNIT 12

ASSIGNMENT- MAXIMISATION & OTHER SPECIAL CASES

Normally, an assignment problem is formulated for the purpose of finding the lowest cost of assigning jobs to persons. However, the objective of some assignment problems is to maximize the effectiveness , like maximizing profit. Such problems can be converted into minimization problem, and be solved as usual. For this, convert the effectiveness matrix to an opportunity loss matrix by substracting each element from the highest element of the matrix. Minimization of the resulting matrix has the effect of maximization of the original matrix.

Ex.10: Given below is a matrix showing the profit for different jobs done through different machines. Find an assignment programme which will maximize total profit.

	Machines			
	M1	M2	M3	M4
J1	51	53	54	50
J2	47	50	48	50
J3	49	50	60	61
J4	63	64	60	61

Ans: Since the given problem is a maximization problem. Convert it into a minimization problem For this, subtract all the elements from the highest element 64. Then proceed with usual procedure

	M1	M2	M3	M4
J1	13	11	10	14
J2	17	14	16	14
J3	15	14	4	3
J4	1	0	4	3

Subtracting the smallest element of each row from all the elements of that row. Then subtracting the smallest elements of each column from the all the elements of that column. Then making zero assignments.

	M1	M2	M3	M4
J1	2	1	0	4
J2	2	0	2	0
J3	11	11	1	0
J4	0	0	4	3

Solution is J1-M3, J2-M2, J3-M4, J4-M1

Total profit = 54+50+61+63=228

Ex. 11 : A department chairman has the problem of providing instructions for the courses offered. He has arrived at the following relative ratings regarding the ability of each instructor to each of the four courses.How should he assign instructors to courses to maximize quality

		Courses			
		A	B	C	D
Instructors	1	6	4	6	7
	2	2	6	5	7
	3	2	3	4	6
	4	3	1	3	4

Ans: This is a maximization of problem. So convert it into a minimization problem. Highest value is 7. So subtract every element from 7 and then minimize.

	A	B	C	D
1	1	3	1	0
2	5	1	2	0
3	5	4	3	1
4	4	6	4	3

Subtracting the smallest element of every row from all the elements of that row and then subtracting the smallest element of every column from all the elements of that column.

	A	B	C	D
1	0	2	[0]	0
2	4	0	1	0
3	3	2	1	0
4	0	[2]	0	0

There are two solutions for the problem.

Solution 1

	A	B	C	D
1	0	2	0	0
2	4	0	1	0
3	3	2	1	0
4	0	2	0	0

Solutions
1)1-A, 2-B, 3-D, 4-C
2)1-C, 2-B, 3-D, 4-A
In both the cases maximum educational quality=6+6+6+3=21

Ex. 12: Five different machines can do any of the five required jobs with different profits resulting from each assignment as shown below. Find out maximum profit possible through optimal assignment.

		Machines				
		A	B	C	D	E
JOB	1	30	37	40	28	40
	2	40	24	27	21	36
	3	40	32	33	30	35
	4	25	38	40	36	36
	5	29	62	41	41	39

This is maximization problem. So we have to convert it into a minimization problem. For this, prepare an opportunity loss matrix. Highest of the elements is 62 so subtract all elements from 62.

Subtracting the smallest element of each row from every elements of that row and subtracting the smallest element of each column from every element of that column and then making zero assignment.

	A	B	C	D	E
1	32	25	22	34	22
2	22	38	35	41	26
3	22	30	29	32	27
4	37	24	22	26	26
5	33	0	21	28	23

Now the assignment is not complete. So draw minimum number of lines after marking √ against the relevant rows and columns.

	A	B	C	D	E
1	10	3	0	8	0
2	0	16	13	15	4
3	0	8	7	6	5
4	15	2	0	0	4
5	33	0	21	24	23

Smallest among those uncovered by lines is 4. Subtract or add 4 (as the case may be) with the respective elements and then make zero assignments.

	A	B	C	D	E
1	14	3	0	8	0
2	0	12	9	11	0
3	0	4	3	2	1
4	19	2	0	0	4
5	37	0	21	24	23

Now the assignment is complete and the solution is 1 to C, 2 to E, 3 to A, 4 to D, 5 to B.
Maximum profit = 40+36+40+36+62=214

Travelling Salesman Problem

Travelling sales man problem is a special type of routing or assignment problem. The routing problems are those where we have to select a route, from an origin to a destination, which yeidls minimum cost.

Suppose a salesman has to visit n cities. He wishes to start from a particular city, visits each city once, and then returns to his starting point. The objective is to select the sequence in which the cities are visited in such a way that his total travelling time is minimized. If there are 4 cities A< B< C< and D, then the solution can be , for example, A to C, C to D, D to B, and B to A.

Travelling salesman problem is very similar to the assignment problem, except that in the former there is an additional restriction. The additional restriction is, choosing a sequence which can minimize cost. This is the route condition. For solving a travelling salesman problem, first solve it like an assignment problem. If the solution does not satisfy the additional restriction, then, use the method of enumeration..

Ex. 13: A company has four territories open, and four salesmen available for assignments. The territories are not equally rich in their sales potential. It is estimated that a typical salesman operating in such territory would bring in the following annual sales.

Territories	I	II	III	IV
Annual sales (000Rs.)	126	105	84	63

The four salesmen are also considered to differ in ability. It is estimated that, working under the same conditions their yearly sales would be proportionally as follows:

Salesman	A	B	C	D
Proportion	7	5	5	4

If the criterion is maximum expected total sales, the intuitive answer is to assign the best sales man to the second richest, and so on, verify this answer by the assignment technique.

Ans: Dividing the total sales in each territory in the ratio 7:5:5:

Annual sales of each salesman in each territory is

	Territory			
Slesman	I	II	III	IV
A	7/21 x 126	7/21 x 105	7/21 x 84	7/21 x 63
B	5/21 x 126	5/21 x 105	5/21 x 84	5/21 x 63
C	5/21 x 126	5/21 x 105	5/21 x 84	5/21 x 63
D	4/21 x 126	4/21 x 105	4/21 x 84	4/21 x 63

42	35	28	21
30	25	20	15
30	25	20	15
24	20	16	12

The problem is to determine the assignments (of salesman to territories) which make the total sales maximum. So it is a maximization problem.

Therefore convert the problem into a minimization problem. Write the difference between every element and 42 (42 being the highest element)

Now this is a minimization problem. Apply usual procedure for getting solutions. We find that two solutions exist.

Solution I: Assign salesman A to territory I, B to territory II, C to territory III, D to territory 4.

Solution II: Assign salesman A to territory I, B to territory III, C to territory II, D to territory IV.

	I	II	III	IV
A	0	7	14	21
B	12	17	22	27
C	12	17	22	27
D	18	22	26	30

Ex. 14: A company has four machines to do three jobs. Each job can be assigned to one only one machine. The cost of each job on each machine is given in the following table.

What are job assignments which will minimize the cost ?

Ans:

The given problem is unbalanced since the number of rows is one less than the number of columns. So introducing a fictitious (dummy) row, we have the matrix

	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22
4	0	0	0	0

Subtracting the least element of every row from all the elements of that row and then subtracting the least element of every column from all the elements of that column and making zero assignment.
As all the rows and columns do not have assignments, we proceed further

	A	B	C	D
1	0	6	10	14
2	0	5	9	11
3	0	5	9	12
4	0	0	0	0

Making √ mark against the appropriate rows and columns and drawing minimum number of lines.

	A	B	C	D
1	0	6	10	14
2	0	5	9	11
3	0	5	9	12
4	0	0	0	0

Least element uncovered by the lines is 5 . subtracting or adding 5 with respective elements. Then making assignment.

	A	B	C	D
1	1	3	1	0
2	5	1	2	0
3	5	4	3	1
4	4	6	4	3

Again assignment is not complete

Making √ mark against the appropriate rows and columns and drawing minimum number of lines.

	A	B	C	D
1	1	3	1	0
2	5	1	2	0
3	5	4	3	1
4	4	6	4	3

Least element uncovered by the lines is 4. Subtracting or adding 4 with respective elements. Then making assignment, we get two solutions

	A	B	C	D
1	0	2	0	0
2	4	0	1	0
3	3	2	1	0
4	0	2	0	0

Thus the two solutions are

1) A to 1, B to 2, C to 3, D to 4
2)A to 1, B to 3, C to 2, D to 4
With minimum cost = 50

Unbalanced assignment priblem

An assignment problem is called an unbalanced assignment problem, whenever the number of tasks or jobs is not equal to the number of facilities or persons. Thus the cost matrix, of an unbalanced problem is not a square matrix. For the solution of such problems, we add dummy rows or columns to the given matrix to make it a square matrix. The cost in the dummy rows or columns are taken to be zero. Now the problem reduces to a normal balanced assignment problem , and can be solved by Hungarian assignment algorithm.

Prohibited assignment

In some assignment problems, it may not be possible to assign a particular task to a particular facility due to space or size of the task or other restrictions. In such situations, we can assign a very big cost , to the corresponding cells, so that it will be automatically excluded in minimizing process of assignment. It is also called restricted or constrained assignment..

Review Questions and Exercises

1. What are assignment problems?
2. State the assumptions of assignment problem.
3. What are the requirements of assignment problem?
4. State its applications in business
5. Explain sources and destinations
6. Distringuish between assignment problem and transportation problem
7. Explain methods of solving assignment problems.
8. Explain Hungarian Assignment method.
9. What is unbalanced assignment model
10. How a maximization problem can be solved?
11. Explain the nature of travelling salesman problem
12. What are prohibited assignment problems.
13. An automobile dealer has to put four repairman to four different jobs. Following are the manhours that would be required for each job. Find optimal assignments that will result in minimum man hours needed.

	A	B	C	D
1	1	3	4	6
2	5	1	2	6
3	5	4	3	1
4	4	6	4	3

14. Solve following assignment problem and find optimal allocation and total cost.

	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	41	39

15. Four men are available to do four different jobs. Following matrix shows time taken for each job. Find the assignment of men to jobs, so that it will minimize the total time taken

	Machines			
	M1	M2	M3	M4
J1	8	26	27	11
J2	13	28	4	26
J3	38	19	18	15
J4	19	26	24	10

16. Solve the following problem so as to travelling salesman minimise cost.

	A	B	C	D	E
1	-	3	6	2	3
2	3	-	5	2	3
3	6	5	-	6	4
4	2	2	6	-	6
5	3	3	4	6	-

UNIT 13 NETWORK ANALYSIS

Present days are the days of projects. Large scale projects involve large number of linterrelatd activities or tasks which must be completedwithin a specified time, in a specified sequence or order and require resources such as personnel,lmoney, materials, faciltities and space. The main obljective before starting any project is to schedule the required activities in an efficient manner so as to -

- Complete the project on time fixed
- Minimize total time
- Minimize time for a prescribed cost
- Minimise total cost
- Minimize cost for a given time
- Minimize idle resources etc.

Management of large scale projects require net work analysis technique for planning, scheduling and executing them within time. A project is composed of a number of jobs, activities, or tasks that are related to one another, and all of these should be completed in order to complete the project. Sometimes, an activity of a project can start only at the completion of many other activities. A network is a graphic representation of combination of activities and events of a project in a sequential order.

Programme Evaluation and Review Technique or PERT and Critical Path Method or CPM are network techniques or models which are widely used in project management. These techniques are very useful for planning, scheduling and excuting large time bound projects which involve careful coordinatin of variety of complex and interrelated activities, estimating resource requirements and time for each activity and establishing inter-relationship amongst the activities. Scheduling requires the details of starting and finishing date or timelof each activity. Generally PERT and CPM are two popular quantitative analysis techniques that help managers to plan, schedule, monitor and control large and complex projects.

Evolution of PERT and CPM

Prior to the devlopment of PERT and CPM, the most popular technique for project scheduling was the Bar Chart or Gantt Chart. These charts show a graphic representation of

work on a time scale. The main limitation of this technique is its inability to show the interraltionship and inter dependence among the different activities. To over come such limitations, PERT and CPM were developed in the late 1950s.

PERT was developed in 1950s by the U.S Navy, Special Project Office, in cooperation with Booz, Allen, and Hamiltan, a management consulting firm. It was directed in planning and controlling Polaris Missile Programme. At about the same time in 1957, Critical Path Method was developed by J.E. Kelly of Remington Rand and M.R. Walker of Dupont, in England.

Network analysis is one of most promising strategies which has come to the forefront in the recent years for analysis and solving complex and large scale projects ,with interwoven network of operations and activities.

Objectives of network analysis

Almost any large project can be subdivided into series of smaller activities that can be analysed with PERT or CPM. Following are the main objectives of the networks.

1. Planning – network analysis is powerful tool for planning, scheduling and controlling large scale projects involving a number of interrelatd sequential activities.
2. Interrelation – network analysis identifies interrelationship and inter dependence of various activities of project or a programme. This relationship helps in bringing out the technological interdependence of activities.
3. Cost control – in certain cases we can measure cost of delay in the completion of the project. This cost can be compared to the cost of the resources required to carry out various activities at various speeds. Their total cost can be calculated and minimzed.
4. Reduction of time – some times, we have to rearrange existing resources with a view to reducing the total time for the project, rather than reduing cost.
5. Maintainence – network analysis helps the management to minimize the total maintaenance time. If the cost of production overhead is vey high then it may be economically justifiable to minimize the maintenance time, regardless of high rsesource costs.
6. Idle resources – network analysis also helps to control idle resources . One should adhere to scheduled cost and time, avoiding waste.
7. Delays – network techniques devlop discipline and systematic approach in planning, scheduling etc. This is not the case in traditional methods. Network techniques help managers to avoid delays, interruptions in production and control of large and complex projects.

Network techniques

Government and industrial establishments always plan new projects for future development and expansion. A project is a collection of well defined tasks called activities and when all these activities are carried out, the project is said to be completed.

Many projects are of great size and complexity and require huge expenditure and time. For example - constructing bridge. Thus, there is need of careful study and anlaysis of entire project. It is essential that the planning and control on the project is precise.

There are many techniques to study and analyze such projects and the main objective of all such techniques is to find some strategy so that the project finishes in time with lowest cost.

Network technique is a major advancement in managment science. This technique is based on the basic characteristics of all projects, that all work must be done in well defined steps. For example, for completing a foundtion, the various steps areclearing , leveling, lay out, digging, placing side board and concreting.

Different network techqniques are PERT , CPM, UNETICS, LESS, TOPS and SCANS. However, these and other systems have emerged from the two major network systems PERT and CPM. These two network techniques help managers to plan, schedule, monitor and control large and compex projects.

Uses of network techniques for management

1. Network techniques help management in planning complicated projects, controlling working plan and also keeping heplan uptodate
2. Network tecyhniques provide a number of checks and safefuards against going astray in developing theplan for the project.
3. Network techniques help the management in reaching the goal with minimum time and least cost and also in forecasig probable project duaration and associated costs.
4. They have resulted in better managerial control, better utilisatin of resources and better decision making.
5. Network techniques have resulted in saving of time or early compeltion of projects which in turn results in earlier return of revenue and increase in profit.

Application of network technqieus

Network techbniques are widely used in following areas :

1. Construction of buildings, bridges, factories and irrigation projects
2. Administration of large offices,
3. Manufacturing industrial plants
4. Maintainence planning
5. Research & development
6. inventory planning and control
7. Marketing management

Phases in application of network technique

Important managerial functions for any project relating to the application of CPM and PERT are planning, scheduling and controlling

Planning

Planning is the most important project management function, in which the jobs or activities to be performed, are estimated and formalized. Gross requirements of material, equipment and manpower in n addition to the estimates of costs and duration of various activities of the project, are also determined in this phase.

Scheduling

Scheduling is the determinatin of the time required for executing each operation and the order in which each operation has to be carried out to meet the planned objectives. It is the mechanical process of formalizing the planned function, assigning starting and completing date to each part of the project , in such a manner that the whole peoject proceeds in a logical sequence and in an orderly and systematic manner. In this phase, men,

material , and other requirments for each activity at each stage of the project are also determined.

Controlling

Controlling is the process in which difference or deviation between the plan and the actual performance are reviewed after the project has started. The analysis and correction of these deviations from the basic aspect of control are performed, whenever major changes are made in the schedule, or the network is revised accordingly and a new schedule is computed . In other words, this strategy calls for updating of the network, and montoring the progess of the project. If necesry, changes are to be made in schedules to ensure completion of the project. In CPM, controlling is required not only in respect of physical progress of work, but also in respect of cost.

Resource allocation and updating in newtok technique

Resources in general include labour, finance, equipment and space. Allocation of these resources to various activities are to be perfomed in a network technique to achieve desired objectives. When these resources are limited, a systematic method for allocation of resources becomes necessary.

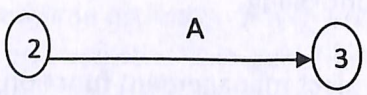
Once these scheduled plan has ben prepared and execution commenced, control over the progress of work has to be executed, in order to complete the work by the stipulated date. Based on the the progress of the work and the revised duration of unfinished activities due to delay, the network diagram has to be redrawn, and this process is known as updating.

Basic concepts

When network is constructed following concepts and relations must be clearly understoodand and followed

Activity

An activity is a task associated with a project. It is a physically identiafiable part of a project which consumes time and resources. Activity is the work to be undertaken to materialise a specific event. Thes an activity is the actual performance of a task. Example - Lay pipe line is an activity. An activity is denoted by a capital letter or by two numbers. It is represented by an arrow, the tail ofwhich represents its start and the head, its finish.



Here A or 2-3 represents an activity. Number 2 represents initial node or start and 3 represents terminal node or finish.

Start and terminal activities

Activities which have no predecessors, but initiate a process, are called start activities. Activities which have no successors and finish a process, are called terminal activities.

Dummy activity

Usually an activity or a task requires time and cost. But there are certain activities which do not take time or resources. They are known as dummy activities. These are used to represent a situation, where one event cannot take place until a previous event has taken place, although this requires no time resource. Tlhey are used to maintain a proper precedence relation between two events and is denoted by dotted arros.

Consider the example : A is the first activity, followed by B and C. D commences only after finishing B nd C. so a dummy activity or relation is necessary to make B and C meet. Arrow with dots is used to show dummy activity.

Event

Event represents instant in time when certain activities have been started or completed. In other words, en event describes start or completion of a tasks. An event or node isn a network disaram is a junctin of two or more arrows representing activities. Event is a point in time and does not consume any resource.

Example Pipe Line laid is an event. Events are represented by circled numbers.

Tail event – A tail event is the one which marks the beginning of an activity. Example, if an activity A is denoted as 2-3, then 2 is the tail event.

Head event – All activities have an ending marked by an event. Such an event is known as head event.example, if an activity is 2-3, then 3 is the head event.

Successor event – events that follow an event are called successor event. When several activities are connected as 1-2-3-4, then events 2, 3, and 4 are successor events of 1.

Predecessor event – the events that occur before an event are called predecsssor events. In the above example , the event 2 is the immdediate predecessor of 3

Representation of activities & events

Activities are represeneed by simple arrows in a net work diagram. Length of arrow does not represent either the magnitude of work or the time required for its completion. The lengthof the arrows is chosen to suit the convenience.

Events are representd by numbered circles. Here numbers assigned to the events, are marked within the circles. Thus anctivity can be denoted by a capital letter or by two numbers. Activity is represented by arrow. That arrow starts from a circle and ends in a circle. The two circles are respectively tail and head events.

Netwrork diagram

It is possible to break up any project in to a number of distinct and well defined jobs called activities. The beginning or end of each activity constitutes an event of the project.

A graphical picture drawn connecting the various activities and events of a project, is a nework disgram. Each event is represented in it by a circle called node. And each activity by an arrow. The arrow denotes the sequence of activities and shows which activity follows which.

Types of networks

Nework disgrams are of two types - evnt oriented diagrams and activity oriented diagrams.

Event oriented diagrams are also known as PERT network diagrams. Here emplahasis is given to the events of the project. The events that are to be included in that plan are first selected. The events in such network fall in a logicl sequence.

Activity oriented diagrams are also known as CPM network diagrams. Here emphasis is given to activities of the project. The activities are arrangd in a logical order. A network , whether event oriented or activity oriented, wil include both events and activities.

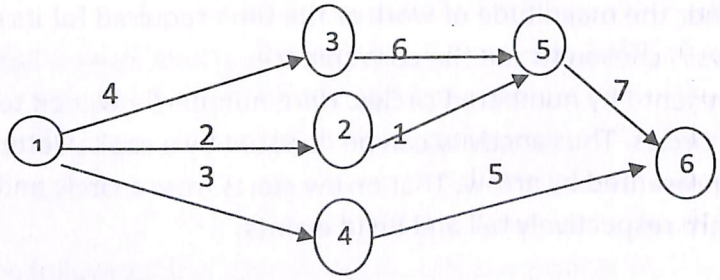
Rules for constructing netwrok diagram

Following rules are to be followed while constructing network digrams:

1. Each activity is represented y one and only one arrow in the network.
2. No two activities can be identified with same head and tail events.
3. Except for the nodes at the beginning and at the end evry node must have at least one ativity preceding it and at least one following it
4. Only one activity may connect any two nodes.
5. The activities and project must flow from left to right
6. Activities should not be drawn back.
7. Whereve necessary, dummy arrows may be drawn.

Ex. 14.1. Draw network diagram to thefollowing activities

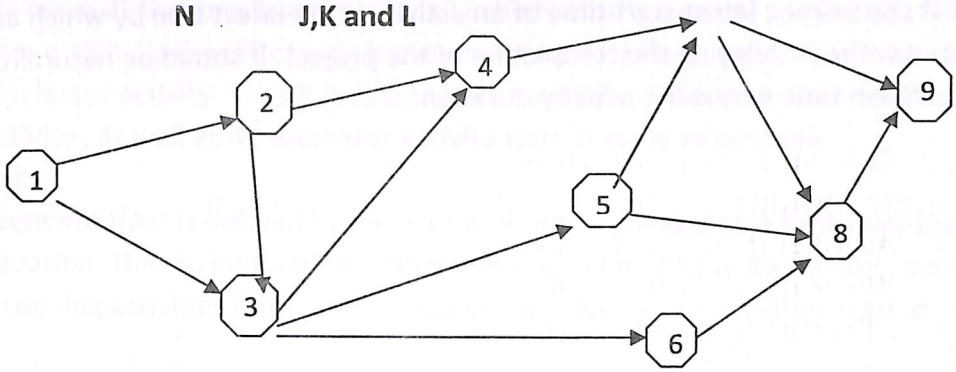
Activity	duration
1 – 2	2 weeks
1 – 3	4
1 – 4	3
2 -5	1
3 – 5	6
4 – 6	5
5 – 6	7



Ex 14. 2 Following activities and timings relating to construction of a bridge, are given. Draw network diagram for the following set of activities

Actoivities	predecessor
A	-
B`	-
C	A
D	A
E	B and C
F	B and C
G	B and C
H	D and E
I	F
J	F
K	G
L	H and I
M	H and I

N J,K and L



Network diagram of a bridge

Numbering of events

The numbering of events is necessry in a nework. Every activity has two events, known as tail and head events. These two events are identified by the numbers given to them. Suppose an activity D has tail event numbered 2, and head event numbered 3, then the activity D can be known as 2-3.

For numbering the events, following steps may be adopted :

1. Initial event of the network diagram is numbered 1
2. The arrows emerging from the event 1 are then considred. Those arrows end in new events. Treat them as initial events and number them 2,3,4 etc.
3. From these, new initial arrows emerge, which end in new events. They may be treated as new initial events. Number them as 5,6,7 etc.
4. Follow step 3 until last event which has no merging arrows.

Earliest and Latest event times.

An activity can be started at various times. Accordingly, there are earliest event time, latest event ime and again they may be classified as earliest start time, earliest finish time, latest finish time and latest start time.

Earliest event time: the earliest occurrence time or earliest event time is the earliest at which an event can occur. Earliest occurrence of an event say 2 is denoted by E2.

Latest event time : the latest allowable occurrence time or the latest event time is the latest time by which an event must occur to keep the project on shcedule. Latest occurrence of an event, say, 2 is denoted by L2.

Earliest start time : the earliest start time of an activity is the erliest time by which it can commence. This is naturally equal to the earliest event time associated with the tail event of an activity.

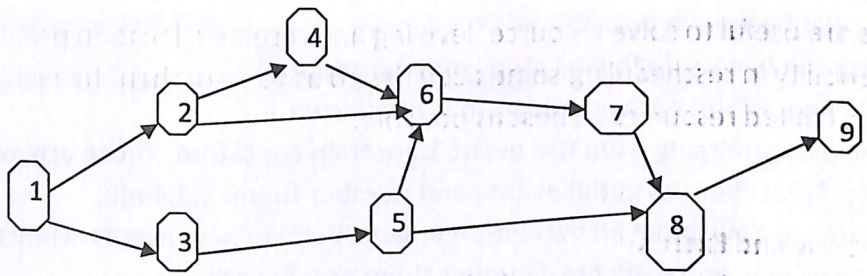
Earliest finish time : if an activity proceeds at its early time and takes the estimated duration for completion, the activity wil have an early finish. Hence earliest finish time for an activity is dined as the arliest time by which it can be finished. This is evidently equal to the earliest start time plus estimated dduration of the activity.

Latest finish time ; the latest finish time of an activity is the latest time by which an activity can be finished without delaying the completion of the project.Naturally, the latest finish time for an activity will be equal to the latest allowable coccurence time of the head event.

Latest start time ; latest start time of an activity is the latest tme by which an activity can be started without delaying the completion of the project. It shoud be naturally be equal to the latest finish time minus the activity duration.

Ex 14.3 draw network for the plant instllation project whose acivities and their precedence relationships are as givn below

Activity	A	B	C	D	E	F	G	H	I
Predecessor	-	A	A	_	D	B,C,E	F	E	G,H



Network - plant installation project

Slack & float

Slack is a term associated with events. It denotes the flexibility range within which an event can occur, ie, slack of an event is the difference between the erliest event time and the latest event time .

The term float is associated with the activity time. Float denotes the range within which an activity start time or its finish time may fuluctuate without affecting the completion of the project.

Floats are of following types - total float, free float, independent float and intrrering float.

Total float

Total float is the time spent by which the starting or finishing af an activity can be delayed without delaying the completion of the project . In certain activities, it will be found that there is a difference between maximum time available and the actual time reaquired to perform the activity. The difference is known as the total float. Total float of an anctivity is the excess of maximum available time over the activitytime.

Free float

Free float is that portion of positive total float that can be used by an activity without delaying any succeeding activity. The concept of free float is based on the possibility

that all the events occur at their earliest time, ie, all activity strarts as early as possible. Hence free float for activity is the difference between its earliets finish time and the earliest start time of its sucfessor activity. Thus it is excess of the availabe time over the required time when the activity , as well as its successor activity start as early as possible.

Independent float

The independent float is defined as the excess of minimum available time over the required acivity duarion. That is, indepndnent float is the amount of time an activity could be delayed if preceeding activities finish at their latest and subsequent activities start at their earliest.

Independent float is equal to the free float minus tail event slack. If the tail event slack is zero, free float and indepndent float are equal . It is to be noted that if a negative value of indepndnet float is obtained,t hen independent float is taken as zero.

Interfering float

Interfering float is just another name given to the head event slack, especially in CPM networks, which are activity oriented. Interfering float is equal to the difference between total float and the free float.

Uses of floats

Floats are useful to solve resource leveling and resorce allocation problems. Floats give some flexibiiity in rescheduling some activities so as to smoothen the level of resources or allocate the limited resources as best as possible.

Review Questions and Exercises

1. State the objective of netrok analysis
2. What necessitated network analysis
3. What are the uses of networks
4. Explain areas of application of network techniques
5. Highlight the difficulties in using netwrk techniques
6. Define an event and activity
7. Explain the concept of float
8. What is independent float
9. Explain free float
10. What is meant by interfering float
11. Distinguigh between slack and float
12. What are the application areas of network techniques.
13. What is dummy activity
14. How is cost control effected through networks
15. Draw network for the following project

Activity	Predecessor
A	-
B	A
C	B
D	B
E	D

F	B
G	E
H	G, E
J	D,F,H
K	C,J
L	K

16.A project schedule has following charctristics

Activity	Time
1 6	4
1 – 3	1
2 – 4	1
3 – 4	1
3 – 5	6
4 – 9	6
5 – 6	4
5 – 7	8
6 – 8	1
7 – 8	2
8 – 10	5
9 – 10	7

Construct a network diagram.

17. Following is a list of of activities and description for a project XYZ. Draw the network.

Activity	Predecessor
A	None
B	None
C	A
D	A
E	A
F	C
G	C
H	C
J	B, D
K	F,J
L	E,H,G,K
M	E,H
N	L.M

18. Draw complete CPM network according to the following table :

Activity	Starts at Event	Ends at Event
1 – 2	1	2
1 – 3	1	3

1 – 4	1	4
2 – 3	2	3
2 – 5	2	5
3 – 4	3	4
3 – 5	3	5
4 – 5	4	5

19. Draw the following logic network :

Activities C and D both follow A Activity E follows C

Activity F follows D E and F precedes B.

2.0n putting a job together to run at a data-processing centre, certain steps need to be taken. These jobs can be described as follows:

Job	Immediate predecessors	Description
A	-	Design flow chart and write FORTRAN statements
B	A	Punch control cards
C	A	Punch comment cards
D	A	Punch program cards
E	B,C,D	Obtain brown folder
F	B,C,D	Put deck together
G	E,F	Submit deck

Draw complete CPM network.

21. Draw a network for the simple project of steel works for a shed. The various elements of a project are as next.

Activity code	Description	Prerequisites
A	Erect site workshop	-
B	Fence site	-
C	Bend reinforcement	A
D	Dig foundation	B
E	Fabricate steel work	A
F	Install concrete plant	B
G	Place reinforcement	C,D
H	Concrete foundation	G,F
I	Paint steel work	E
J	Erect steel work	H,I
K	Give finishing touch	J

UNIT 14 CRITICAL PATH METHOD

While analyzing a network of activities, it is often necesary to estimate the total project time. The total project time is the maximum of the lapsed time among all paths originating from the initial event and terminating at the terminal event, indicating completion of the project.

Therefore critical path is that sequence of activities which determines the total project time in a project network. There may be a number of paths starting from the initial event and ending in terminal event. These paths connect various activities from the tail event to head event. Among these paths that path which is longest on the basis of duration is called critical path.

For example a network has three paths ;

1. 1- 2- 5- 6 with duration 10 days
2. 1- 3 – 5 - 6 with duration 17 days
3. 1 - 4 - 6 withdurtion 8 days.

Then 1 – 3 – 5 – 6 is the critical path. Thus a critical path is the one which gives longest total activity timings within a network. It can be also obtained by connecting activities having zero float.

Critical activity

An activity whose float is zero is called critical activity. so any delay in the start of critical activity will cause a further delay in the completion of entire project. Activities lying on the critical path arecalled critical activities.

Critical path method

The Criticl Path Method, shortly known s CPM, is a netwrk technique. It was originally discovered for applications to industrial situations like constructin, manufacturing, maintenance etc. Since then it has found wide acceptane by construction industry with application to bridges, dams, tunnels, buildings, highways, power plants.etc.

CPM is a network technique which consists of planning the sequence of activities to be performed in a net work, scheduling the timings and resources to various operations and controlling the performances so that they are not deviating from the plans.

CPM is generally used for repetitive type of projects or for those projects for whilch fairly accurate estimate of time for completion of activitiy can be made and for which cost estimation can be made with fair degree of accuracy. The critical path method can be used effectively in productin planning, road systems and traffic schedules, communication

network etc. CPM emphasizes the relationship between applying more resources to shorten the duration of given jobs in a project and increased cost of these additional resources.

Steps in critical Path Method

1. List all the activities and draw a network diagram
2. Find the earliest event time and latest event time of each event and show in the network disgram
3. Calculate earliest start time, earliest flinish time, latest start time and latest finish time for each activity.
4. Determine the float for each activity
5. Identify the critical activities , having zero floats.
6. Draw double lines in the netwrk diagram passing through critical activities. The double lines show the critical path.
7. Calculate the total project duration which is the sum of duration ofall critical activities

Alternatively, critical path may be identified, easily, as per following method.

1. Draw the network
2. Starting from the tail event , to the head event, identify different paths through the diagram.
3. Calculate total time taken through different paths
4. Select that path with the longest duration, as the critical path.
5. The activities on the criticl path will be critical activities.

CPM analysis

CPM is a deterministic model. It assumes that both the time to complete each activity and the cost of doing so is known with certainty. This is known as CPM as it focuses directly on critical path and critical activities. Scheduling of activities is done in such a way that critical activities causes no delay to project, rather time requirement to these activities is reduced by inducing resources to complete the project before normal times.

CPM was developed in 1957 by J.E.Walker of Du-Pont, to help schedule maintenance of chemical plant. The fundamental departure of CPM from PERT is that CPM brings more prominently into the planning and control process, the concept of cost where the time can be estimated very accurately in advance. Similarly cost can be calculated accurately in advance. CPM may be superior to PERT. But when there is extreme degree of uncertainty and when control over time out weight control over cost, PERT will be a better choice.

Time estimate in cpm

A CPM network is drawn like a PERT network. For CPM only one time estimate is taken instead of three time estimates, as in PERT. Besides the crash estimate of activities or thew project can also be made. Crash time is the minimum time in which the activity can be completed in case additional resources are inducted. Crash cost is the cost of completing an activity in crash time. For the purpose of simplicity,the relationship between normal time cost and crash time cost for an activity is generally assumed to be linear.

The objective of project crash cost analysis is to reduce the total project completion time, while minimizing the cost of crashing. Since the project completion time can be shortened only by crashing critical activities. It follows that not all project activities should be crashed. However, when activities are crashed, the critical path may change, requiring further crashing of previously non-critical activities in order to further reduce the project completion time. In a nut shell, crashing means adding extra resources and Managers are usually interested in speeding up project at the least additional cost.

Limitations of CPM

The CPM sufferes from following limitations:

1. It operates on the assumption of a precisely known time for each activity which may not be true in real life situations.

2. It does not make use of statistical analysis in the determination of the time estimates for each activity.

3. It requires repetition of the evaluation of the entire project each time a change is introduced to the network. This is a very difficult and cumbersome process.

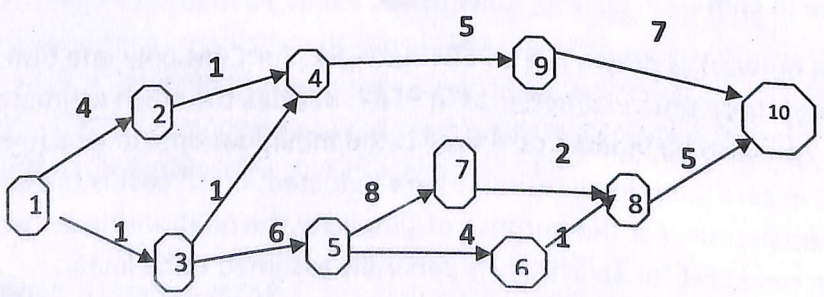
4. It cannot serve as a dynamic controlling device as it was introduced as a static planning model.

Ex .14.1

A project schedule has following characteristics

Activity	Time	Activity	Time
1 – 2	4	5 – 6	4
1 – 3	1	5 – 7	8
2 – 4	1	6 – 8	1
3 – 4	1	7 – 8	2
3 – 5	6	8 – 10	5
4 – 9	5	9 – 10	7

1. **Construct network diagram**
2. **Find EST, LST, EFT, and LFT values of all activities**
3. **Find critiacal path and project duration.**

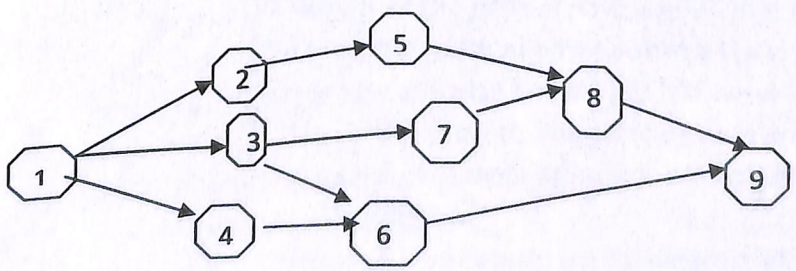


Activity	Time	Earliest time		Latest time		Total float
		EST	EFT	LST	LFT	LST – EST
1-2	4	0	4	5	9	5
1-3	1	0	1	0	1	0
2-4	1	4	5	9	10	5
3-4	1	1	2	9	10	10
3-5	6	1	7	1	7	0
4-9	5	5	10	10	15	5
5-6	4	7	11	12	16	5
5-7	8	7	15	7	15	0
6-8	1	11	12	16	17	5
7-8	2	15	17	15	17	0
8-10	5	17	22	17	22	0
9-10	7	10	17	15	22	5

Activities with zero float = 1-3, 3-5, 5-7, 7-8, and 8-10
Project duration = 22

Ex 14.2 A project has the following time schedule. Construct the net work and find critical path. Also find project duration.

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Time	2	2	1	4	8	5	3	1	5	4	3



Critical path 1 – 3 – 6 – 9
Critical path duration = 2 + 8 + 5 = 15 days.

Review Questions and Exercises

1. Explain project analysis techniques
2. What is CPM technique
3. How dies CPM help in large projects
4. What are dummy activity in networks
5. Explain Earlist starting time
6. What are critical activities
7. How to draw a CPM network
8. Explain steps in ascertaining critical path
9. What is project duration
10. What are the objectives of CPM analysis
11. Define an event and activity
12. Draw network and project duration.

Activity	1-2	2-3	2-4	3-5	3-6	4-6	4-7	5-8	6-8	7-8
Time	4	6	10	8	2	12	4	15	14	8

13. For a small project having 12 activities, draw network and find critical path.

Activity	A	B	C	D	E	F	G	H	I	j	k	L
Dependence	-	-	-	B,C	A	C	E	E	D,F,H	E	IJ	G
Time	9	4	7	8	7	5	19	8	6	9	10	2.

14. From the following details find critical path and project duration

Activity :	1-2	1-3	1-4	2-5	4-6	3-7	5-7	6-7	5-8	6-9	7-10	8-10	9-10
Time:	10	8	9	8	7	16	7	7	6	5	12	13	15

UNIT 15 PROGRAM EVALUATION AND REVIEW TECHNIQUE

PERT is a network analysis technique in which we try to exercise logical disciplnie in planning and controlling projects. It is a network technique which uses a network diagram consisting of events. The successive events are joined by arrows.

PERT analysis is preferred for those projects or operations which are of non repetitive nature or for those projects in which precise time determination for various activities can not be made. In such projects, management cannot be guided by past experience. For example, the project of launching a space craft involves the work never done before. For such research and development projects, the time estimates made for use may be little more than guesses. PERT system is best suited for such projects.

PERT is useful technique in project planning and conrol, where projects are non repetitive in nature. It gives the planner a perfect idea about the sequence of activities and their times. It is a method of minimizing delay and interruptions. It helps in coordinating various parts of the overall job and seeing that every predecessor activity is finished in time for the following activity to commence. It shows the way how a project can be finished earlier than the original schedule. Fo this, resources may be re- allocated from activities with spare time to activities that have no spare time.

The main assumption in PERT is that activity durations are independent. That is , time required for one activity has nothing to do with the time for another activity.Besides, time estimates of activities are not so precise and accurate but probabilisitic.

Time estimate in PERT

Time is the most essential and baisic variable in project management. Once activities have been specified and management has decided which activity must proceed and follow others and the network has been drawn, the next step is to assign estimates of times required to complete each activity. The time is usually given in units of weeks or days. The degree of success attained in the network planning process depends upon the accuracy of time estimates . Providing time estimates is not always an easy task. Without solid historical data, managrs are often uncertain as to activity times. For this reason, if the time estimates are not deterministic in nature, then the usual way of expressing this uncertainty is to

employ a proaability distribution based on three time estimates for each activity. These estimates are :

Optimistic time (t_o)

This is the shortest possible time in which an activity can be completed,under ideal conditions. This particular time estimate represents the time in which the activity or job can be completed if every thing goes well with no problems or adverse conditions.

Pessimistic time (t_p)

It is the maxmum time that would be required to complete the activity. This particular time estimate represents the time it might take to complete a particular activity, if everything went wrong and abnormal situations prevailed.

Most likely time (t_m)

It is the time which the activity will take most frequently, if performed a number of times. This time estimate lies between the optimistic and pessimistic time estimates. The time estimate reflects a situation, where conditions are normal, things are as usual and there is nothing exciting.

In PERT, probabilistic approach is followed for time estimation, and expected time for each activity is calculated. . Using the above three time estimates, expected time is calculatd for each activity using the formula -

$$Te = \frac{to+tp+4tm}{6}$$

These expected times are used in the network diagram for finding project duration and probability of completing projects within scheduled dates.

Steps in PERT

1. Identify activities and times and draw network.
2. Obtain various time estimates and compute expected time for each activity.
3. $Te = \frac{to+tp+4tm}{6}$
4. Using the exected time estimats, determine the critical path
5. Compute floats and activities with Zero float are critical activities.
6. Obtain total expected duration of the project
7. Find the variance of time estimates of all activities and standard deviation.
8. Find the probability of finishing the project within a target date by using the principle of normal probability distribution.

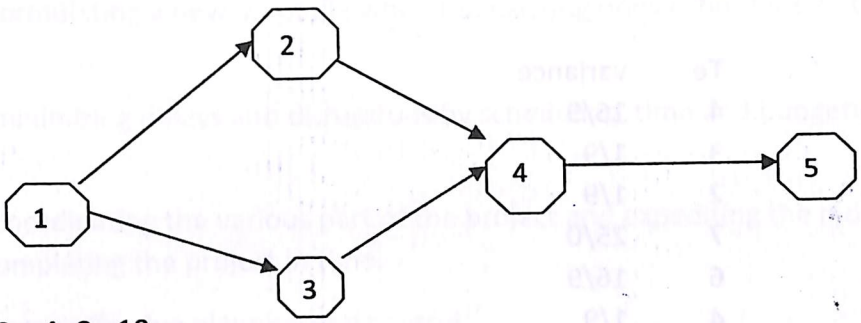
Ex 15 . 1 Calculate average expected time and draw network for a project with the following activity times. Find critical path.

Activity	to	tp	tm
2 – 4	1	5	3
2 – 6	1	7	4

4 – 8	4	16	7
6 – 8	1	5	1.5
8 -10	1.5	14.5	3.5

Ans. $Te = = \frac{to+tp+4tm}{6}$

Te
3.0
4.0
8.0
2.0
5.0

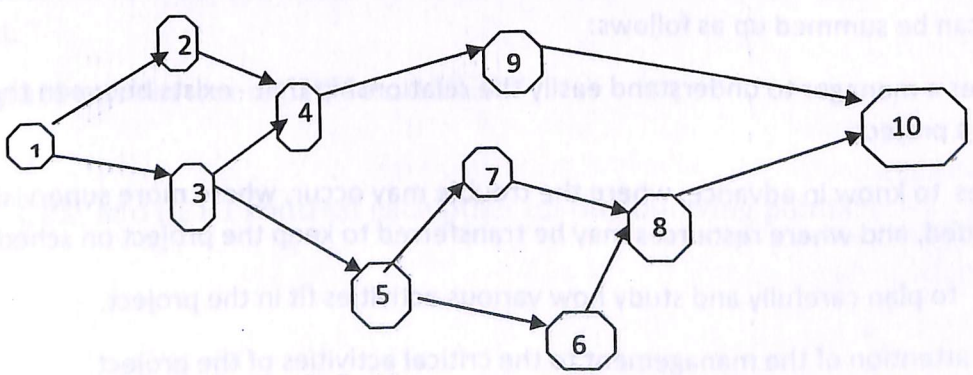


Critical path = 2 – 4- 8 - 10
Project duration = 16 days

Ex 15.2 The characteristics of project schedule ae as given below.

Activity	time	activity	time
1 -2	4	5 – 6	4
1 – 3	1	5 – 7	8
2 – 4	1	6 – 8	1
3 – 4	1	7 – 8	2
3 – 5	6	8 – 10	5
4 – 9	5	9 -10	7

Construct Pert network and find critical path and project duration.



Critical path = 1 – 3 – 5 – 7 – 7- 8 -10
Project duration = 22 days

Ex 15.3 For a project following time estimates are givn. Prepare network and find project duration. Also lfind variance of theproject..

Activity	preceeding	to	tp	tm
A	- -	2	10	3
B	- -	2	4	3
C	A	1	3	2
D	A	4	14	6
E	B	4	12	5
F	C	3	5	4
G	D,E	1	7	1

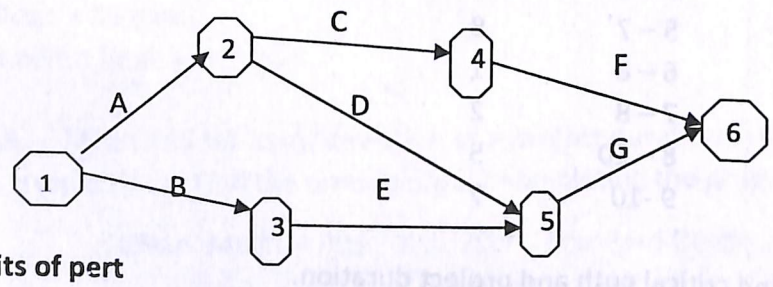
Ans.

Te	variance
4	16/9
3	1/9
2	1/9
7	25/0
6	16/9
4	1/9
2	1

Critical path A – D - G

Project duration = 13 days

Project variance == variance of critical path = 16/9 + 25/9 + 1 + 50/9



Merits of pert

Having analysed calculations , various merits of the PERT as a Quantitaive Technique can be summed up as follows:

(1) It enables a manager to understand easily the relationship that exists between the activities in a project.

(2) It enables to know in advance, where the trouble may occur, where more supervision may be needed, and where resources may be transferred to keep the project on schedule.

(3) It assists to plan carefully and study how various activities fit in the project.

(4) It draws attention of the management to the critical activities of the project.

(5) It suggests areas of increasing efficiency, decreasing cost and maximizing profits.

(6) It enables the use of statistical analysis.

(7) It makes possible a forward looking type of control.

(8) It compels the management for taking necessary action at the right time without any delay.

(9) It provides upto date information through frequent reporting, data processing and program analysis.

(10) It helps in formulating a new schedule when the existing ones cannot meet the situation.

(11) It helps in minimizing delays and disruptions by scheduling time and budgeting the resources.

(12) It helps in coordinating the various part of the project and expediting the mode of operation for completing the project in time.

(13) It permits more effective planning and control.

Demerits

(1) It does not lay any emphasis on the cost of a project except on the time only.

(2) It does not help in routine planning of the recurring events.

(3) Errors in time estimates under the PERT make the network diagram and the critical path etc. meaningless.

(4) In the calculation of the probabilities under the PERT it is assumed that a large number of independent activities operate on critical path and that the distribution of total time is normal. This may not hold good in peculiar situation.

(5) For effective control, the PERT requires, frequent up-to date information and revision in calculation which may be quite costly for the management.

(6) It does not consider the matter of resources required for various type of activities of a project.

Comparison between CPM and PERT

CPM and PERT contrast each other on the following points:

CPM	PERT
(1) It is a deterministic model under which the result is ascertained in a manner of certainity.	(1) It is a probabilistic model under which the result is estimated in a manner of probability.
(2) It deals wit the activities of precise well known time.	(2) It deals with the activities of uncertain times.
(3) It is used for repetitive jobs like residential construction.	(3) It is known for non-repetitative jobs like planning and scheduling of research programmes.
(4) It is activity oriented in as much as its results are calculated on the basis of the activities.	(4) It is even oriented in as much as its results are calculated on the basis of events.
(5) It does not make use of dummy activities.	(5) It make use of dummy activities to represent the proper sequencing of the activities.
(6) It deals with costs of a project schedules and their minimization.	(6) It has nothing to do with cost of a project.
(7) It deals with the concept of crashing.	(7) It does not deal with concept of crashing.
(8) Its calculation is based on one type of time estimation that is precisely known.	(8) It finds out expected time of each activity on the basis of three types of estimates.
(9) It cannot be used as a control devices as it requires repetition of the entire evaluation of the project each time the changes are introduced to the network.	(9) It is used as an important control device as it assists the management in contolling a project by constant review of th edelays in the activities.
(10) It does not make use of the statistical devices in the determination of the time estimates.	(10) It make us eof the statistical devices.

Comparison - CPM and PERT

Inspite of the above differences both PERT and CPM have following common points:

1. All significant activities and tasks are defined in the project.
2. Relationship among the activities is developed. This relationship decides which activities must precede and which follow others.
3. Network is drawn connecting all of the activities.
4. Time and cost estimates are assigned to each activity.
5. Longest path through the network is computed and this is called the Critical path.
6. Network is used to help management to plan, schedule, monitor and control the project.

Review Questions and Exercises

1. What are project analysis techniques?
2. How does network analysis help in large projects
3. What are dummy activities in PERT network
4. Differentiate between CPM and PERT
5. What is PERT technique
6. Explain optimistic time

7. What is pessimistic time
8. What is the significance of most likely time
9. What is slack and float
10. How to find critical path as per PERT
11. Explin time estimates in PERT
12. Following details of a project are given. Draw netwrk, identify critical path , calculate project duration.

Activity	to	tm	tp
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

13. The time estimate(in weeks) for the activities of a PERT network are given below:

Activity	t_0	t_m	t_p
1 – 2	1	1	7
1 – 3	1	4	7
1 – 4	2	2	8
2 – 5	1	1	1
3 – 5	2	5	14
4 – 6	2	5	8
5 – 6	3	6	15

a. Draw the project network and identify all the paths.
b. Determine the expected project length.
c. Calculate the standard deviation and variance of the project.
d. Find the probability of completing the project within 5 days.

13. During a slack period, part of an assembly line is to be shut down for repair of a certain machine. While the machine is turned down the area will be painted. Construct a network for this machine rebuilding project based on the activity list furnished by the line foreman as follows :

	Activity	Restriction
A	Order new parts	A < B

B	Reassemble machine	B < E, I
C	Tear out foundation	C < G
D	Dismental foundation	D < C, F, A
E	Paint area	E < I
F	Delivery parts to be Repaid	F < H
G	Build new foundation	G < B
H	Pick up repaired parts	H < B
I	Clean up	-

14. If the scheduled completion date is the earliest expected time for the end event, draw the network, identify the critical path for the following project :

Activities	Duration (days)
0 – 1	3
1 – 3	16
1 – 2	6
2 – 3	8
1 – 4	10
3 – 4	5
4 – 5	3

15. A researcher gives the following information regarding activities and sequencing requirement along with expected time for various activities related to his thesis.

Activities	A	B	C	D	E	F	G	H	I	J	K	L	M
Pre-requisitic Activities	-	-	B	C	A,D	D	A,D	E	G,H	I	G	J,K	L

Expected Time	6	5	2	2	2	1	6	2	6	2	4	3	1

(i) Draw the Network diagram and trace CPM

(ii) What is the minimum time to complete the thesis ?

16. The Madras Construction Company is bidding on a contract to install a line of microwave towers. It has the following identifical activities, along with their predecessor activities.

Activity	Predecessor	Durations
A	None	4
B	None	7
C	A	3
D	A	3
E	B	2
F	B	2
G	D,E	2
H	F,G	3

Draw the network, show clearly the Critical Path. Determine E_S & L_F by following the backward pass & forward pass technique within the network diagram itself.

17. A project consists of nine jobs (A, B, C, ... I), with the following precedence relations and time estimates :

Job	Predecessor	Time (days)
A	-	15
B	-	10
C	A, B	10
D	A, B	10
E	B	5
F	D, E	5

G	C, F	20
H	D, E	10
I	G, H	15

e.

f. (i) Draw the project network.

g. (ii) Determine the earliest and latest starting and completion times of jobs.

h. (iii) Identify the critical path.

i. (iv) Determine the total float of jobs.

j.

Q.18 . Draw the network for the following project and compute the earliest and latest time for each event and also find the critical path.

Activity	Immediate predecessors	Time (days)
1 – 2	-	5
1 – 3	-	4
2 – 4	1 – 2	6
3 – 4	1 - 3	2
4 – 5	2 – 4 & 3 – 4	1
4 – 6	2 – 4 & 3 – 4	7
5 – 7	4 – 5	8
6 – 7	4 – 6	4
7 – 8	6 – 7 & 5 - 7	3

Q.19. The activities with duration and dependence of a small project are given below :

Activities	A	B	C	D	E	F	G	H

Duration (days)	7	5	3	8	7	5	6	9
Dependence	-	-	A, B	A, B	C, D	C, D	E	F, G

(i) Find out the project completion time.

(i After 15 days from the start of the project, it was found out that activities E, F, G and H would require 3, 10, 3, 3 days respectively. As a result, what is the % change in the project complete time ?

Q. 12. Given is the following information regarding a project :

Activity	Required Preceding Activities	Durations (Days)
A	None	3
B	None	4
C	None	2
D	A, B	5
E	B	1
F	B	3
G	F, C	6
H	B	4
I	E, H	4
J	E, H	2
K	C, D, F, J	1
L	K	5

Draw the network for the above project.

Determine the critical path and the duration of the project.

Find the three types of floats (viz., total, free and independent) for each

UNIT 16 PROJECT COST AND TIME ANALYSIS

The main objective of network analysis is to examine cost and time aspects of projects. Most activities can be completed within reduced time if extra resources are assigned to them. Thus, with reduction in time duration or crashing ,their cost goes up and on the other hand,the project duration comes down.If there is no special reason to reduce the time duration, the activities can be completed in normal time and at normal cost. In fact, there is no requirement of reduction in the duration of all activities, as the over all time can be reduced only if the activities on the critical path are crashed. Activities on non critical pathned need not be crashed.

This unit attempts to analyse projects in terms of time reduction and cost reduction.

Probability of project completion by a target date

Sometimes the management would like to know the probability of completing the project by a particular date.

It is worth mentioning here that PERT make two assumptions:

1. Total project completion time follow a normal probability distribution.

2. Activity time are statistically independent.

With these assumptions, the bell shaped Normal Distribution Curve can be used to represent project dates. It means that there is 50% chance the entire project will be completed in less than expected project duration, and 50% chance that it will exceed project time.

The above discussion can be summarized as below.

(a) In case the Z factor is positive (+), the probability of completion of project by scheduled date is more than 50%.

(b) In case the Z factor is negative (-), the probability completion of project by scheduled date is less than 50%.

(c) In case the Z factor is zero, then the probability of completion of project by schedule date is 50%. This will happen in case the expected completion date and schedule date are equal.

Say, we are given 14 weeks in order to find the probability that this project will be finished on or before 14 weeks deadline, we need to determine appropriate area under the normal curve. The standard normal equation can be applied as follows:

$$Z = \frac{Due\ Date - Expected\ Date}{\sigma}$$

project variance

PERT uses the variance of critical path activity to help in determining the variance of the overall project. Project variance is computed by summing variance of just critical activities.

Project variance = Total of variances of activities on critical path.

$$\text{Variance } (\sigma^2) = ((t_p - t_0)/6)^2$$

$$\text{Project standard deviation} = \sqrt{\text{Project variance}}$$

Continuing with Example 13 ; Let the due date is 14 week, where as

$$Z = (X - \dot{X})/\sigma$$

X = due date = 14 weeks

$\dot{X}$ = Expected date of completion = 13 weeks

σ = Standard derivation of activities on the critical path

$$\sqrt{\sigma^2} = \sqrt{50/9} = 2.357$$

Now $Z = (14-13)/2.357 = 0.4243$

The value of Z from normal distribution curve table is .1643.
Therefore, the required probability of completing the project within 14 weeks is
= 0.5 + 0.1643 = 0 .6643 i.e., 66.43%

(i) Suppose we are interested in finding the probability of completing the project within 11 weeks.

$$Z = (X - \dot{X})/\sigma = (11 - 13)/2257 = -0.85$$

The value of Z from normal distribution curve is = 0.3023

Therefore, the required probability of completing the project within 11 weeks is 0.5 - 0.3023 = 0.1977 OR 19.77%

(ii) Supposing the manager wants 95% surety to complete the project when should he start.

$$Z = (X - \dot{X})/\sigma$$

For 95% probability read the value from the normal distribution table. It is 1.64 [The value is read from the table at 0.4500 (i.e. 0.95 - 50)].

$$164 = (X - 13)/2.357 = \text{So } X = 1.64 * 2.357 + 13 = 16.86 \text{ weeks}$$

Hence, the manager should start 16.86 weeks before the stipulated date for 95 probability or 95 surity.

Example 16.1 The time estimate(in weeks) for the activities of a PERT network are given below:

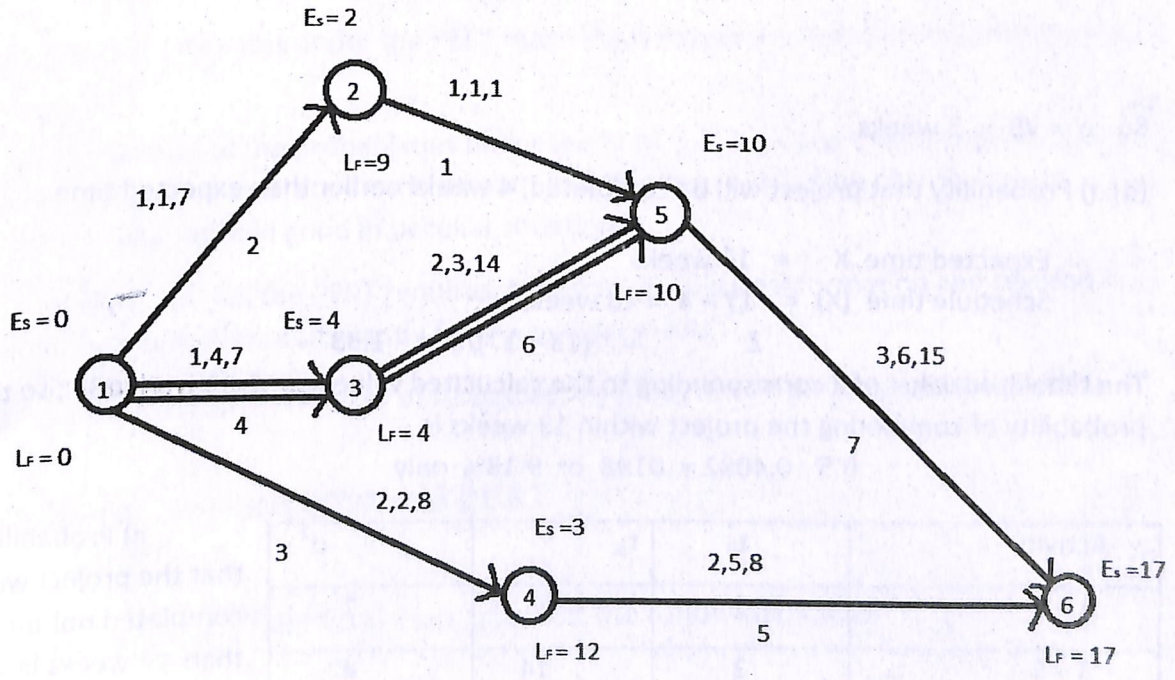
Activity	t_o	t_m	t_p
1 – 2	1	1	7
1 – 3	1	4	7
1 – 4	2	2	8
2 – 5	1	1	1
3 – 5	2	5	14

4 – 6	2	5	8
5 – 6	3	6	15

a. Draw the project network and identify all the paths.
b. Determine the expected project length.
c. Calculate the standard deviation and variance of the project
d. What is the probability that the project will be completed.
   i) At least 4 weeks earlier than expected time.
   ii) No more than 4 weeks later than expected time
e. If the project due date is 19 weeks, what is the probability of not meeting the due date ?
f. The probability that the project will be completed on schedule if the schedule completion time is 20 weeks.
g. What should be the scheduled completion time for the probability of completion to be 90%.

Solution :

(a) The network from the given information is shown below :



Expected activity times and variances are calculated as under :

Activity	t_0	t_m	t_p	t_e	σ^2 (Variance)
1 – 2	1	1	7	2	1

1 – 3	1	4	7	4	1
1 – 4	2	2	8	3	1
2 – 5	1	1	1	1	0
3 – 5	2	5	14	6	4
4 – 6	2	5	8	5	1
5 – 6	3	6	15	7	4

(b) Length of path 1 – 2 – 5 – 6 = 2 + 1 + 7 = 10

Length of path 1 – 3 – 5 – 6 = 4 + 6 + 7 = 17*

Length of path 1 – 4 – 6 = 4 + 3 + 5 = 8

Since 1 – 3 – 5 – 6 has the longest duration, it is critical path of the net work.

So, the expected project length = 17 weeks

(c) Variance on the critical path is

So $\sigma = \sqrt{9} = 3$ weeks

(d). i) Probability that project will be completed, 4 weeks earlier than expected time :

Expected time $\dot{X}$ = 17 weeks

Schedule time (X) = 17 – 4 = 13 weeks

$Z = (13 - 17)/3 = -1.33$

The tabulated value of Z corresponding to the calculated value ie., - 1.33 is 0.4082 , so the probability of completing the project within 13 weeks is

0.5 - 0.4082 = .0198 or 9.18% only

Activity	t_0	t_p	σ^2
1 – 3	1	7	1
3 – 5	2	14	4
5 – 6	3	15	4

ii) Probability that the project will be completed not more than 4 weeks later than expected time :

Expected time = 17 weeks

Therefore, Scheduled time = 17 + 4 = 21 weeks

$$Z = (21 - 17)/3 = 1.33$$

Calculated value of Z is 0.40824 , so the probability is

$$0.5 + 0.4082 = 0.9082$$

ie., 90.82% that project will be completed on 21 weeks.

(e) When the project due date is 19 weeks.

$$Z = (X - \dot{X})/\sqrt{\sum\sigma^2} = (19 - 17)/\sqrt{9} = 2/3 = 0.67$$

The tabulated value of Z corresponding to the calculated value ie., 0.67 is 0.2486 for the performance of the project within the scheduled time. Hence the probability of the project within 19 week is

$$0.5 + 0.2486 = 0.7486$$

Therefore, the probability of not completion will be 1 - 0.7486 = 0.2514 or 25.14%.

(f) When schedule time is 20 weeks.

$$Z = (20 - 17)/3 = 1$$

for which corresponding value of Z is 0.3413%

So, the probability that the project will be completed in 20 weeks is

$$0.5 + 0.3413 = 0.8413 \text{ or } 84.13\%$$

(g)Scheduled completion time when probability of completion is 90%.

Probability = 0.50 + 0.40

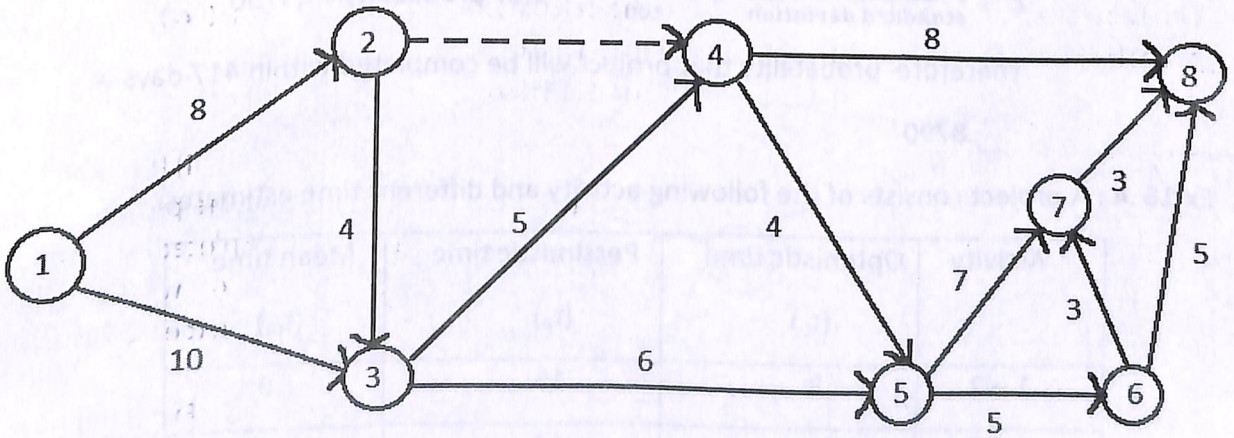
$Z = (X - \dot{X})/\sigma$ (The corresponding value of Z at 0.40 is 1.28)

$Z = 1.28$; $1.28 = (X - 17)/3$

$X = (1.28 * 3) + 17 = 20.84$ weeks OR 21 weeks

Ex 16.2 :

From the following network of a plant installation, calculate all the floats and ascertain critical path and project duration.



Solution : In order to determine all the three floats, we compute the earliest and latest times in respect of each node points. These are given in the following table:

Activity (i , j)	Normal time	Earliest time		Latest time		Float		
		Start	Finish	Start	Finish	Total	Free	Independ-ent
(1-2)	8	0	8	0	8	0	0	0
(1-3)	10	0	10	2	12	2	2	2
(2-3)	4	8	12	8	12	0	0	0
(2-4)	0	8	8	17	17	9	9	9
(3-4)	5	12	17	12	17	0	0	0
(3-5)	6	12	18	15	21	3	3	3
(4-5)	4	17	21	17	21	0	0	0
(4-8)	8	17	25	28	36	11	11	11
(5-6)	5	21	26	26	31	5	5	5
(5-7)	7	21	28	21	28	0	0	0
(6-7)	3	28	31	28	31	0	0	0
(6-8)	5	31	36	31	36	0	0	0
(7-8)	3	28	31	33	36	5	5	5

Total float = 30 days
Free Float = 30 days
Indepnedent float = 30 days

Ex 16.3 Mean and standard deviation of a project duration are 300 and 100 days respectively. Find the probability for completing the project within 417 days.

Given Mean = 300, X = 417, Standard Deviation = 100

$$Z = \frac{X - Mean}{standard\ deviation} = \frac{417 - 300}{100} = 1.17 \text{ probability} = .3790$$

Therefore probability that project will be completed within 417 days = .8790

Ex 16 .4 : A project consists of the following activity and different time estimates.

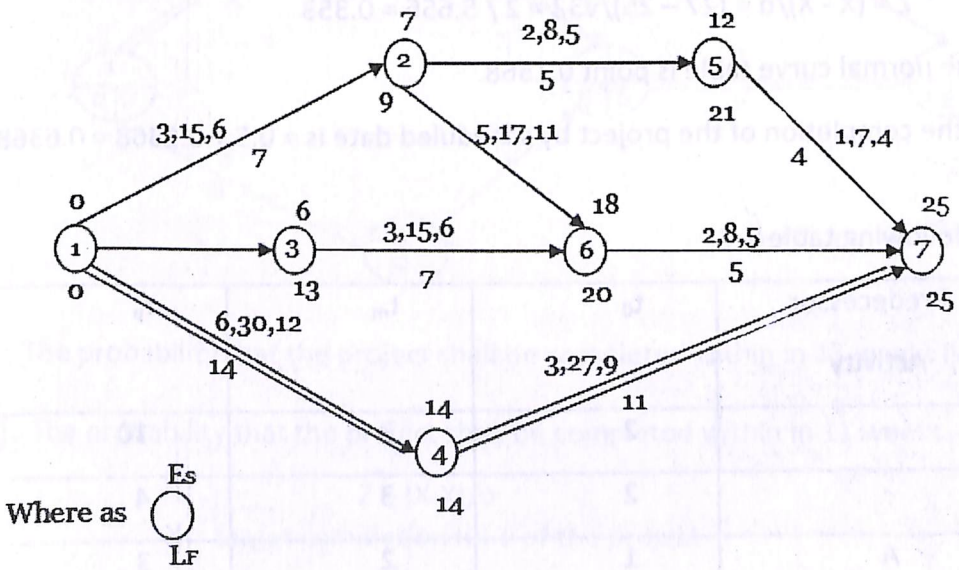
Activity	Optimistic time (t_0)	Pessimistic time (t_p)	Mean time (t_m)
1 – 2	3	15	6
1 – 3	2	14	5

1 – 4	6	30	12
2 - 5	2	8	5
2 – 6	5	17	11
3 – 6	3	15	6
4 – 7	3	27	9
5 – 7	1	7	4
6 – 7	2	8	5

(a) Draw network.
(b) Determine the critical path and their variances.
(c) Find the earliest and latest expected times to reach each node.
(d) What is the probability that the project will be completed by 27th day.

Solution :

(a) Network for the project is as under.



(b) Critical path 1 – 4 – 7

$t_e = (t_0 + 4t_m + t_p)/6$; Variance $(\sigma^2) = ((t_p - t_0)/6)^2$

(c) Earliest expected time for events are obtained by taking the sum of expected times of all the activities leading that event.

Latest finish time for different events are obtained moving backwards from the last event, the expected activity time from the latest expected time of head event

Activity	Activity Times			t_e	s_2	E_s	L_F	Slack
	t_o	t_m	t_p					
1-2	3	6	15	7	4	7	9	2
1-3	2	5	14	6	4	6	13	7
1-4	6	12	30	14	16	14	14	0
2-5	2	5	8	5	1	12	21	9
2-6	5	11	17	11	4	18	20	2
3-6	3	6	15	7	4	13	20	7
4-7	3	9	27	11	16	25	25	0
5-7	1	4	7	4	1	16	25	9
6-8	2	5	8	5	1	23	25	2

(d) The expected completion time for the project is 25 days while time scheduled completion time is fixed at 27 days. So Z factor for the project completion is

$$Z = (X - \dot{X})/\sigma = (27 - 25)/\sqrt{32} = 2 / 5.656 = 0.353$$

The value under the normal curve table is point 0.1368.

The probability of the completion of the project by scheduled date is = 0.5 + 0.1368 = 0.6368 or **63.68%.**

Ex 16.5: Consider following table

Activity	Predecessor Activity	t_0	t_m	t_p
A	-	2	3	10
B	-	2	3	4
C	A	1	2	3
D	A	4	6	14
E	B	4	5	12
F	C	3	4	5
G	D,E	1	1	7

1. What is the probability that the project shall be complete within a period of 13 weeks.

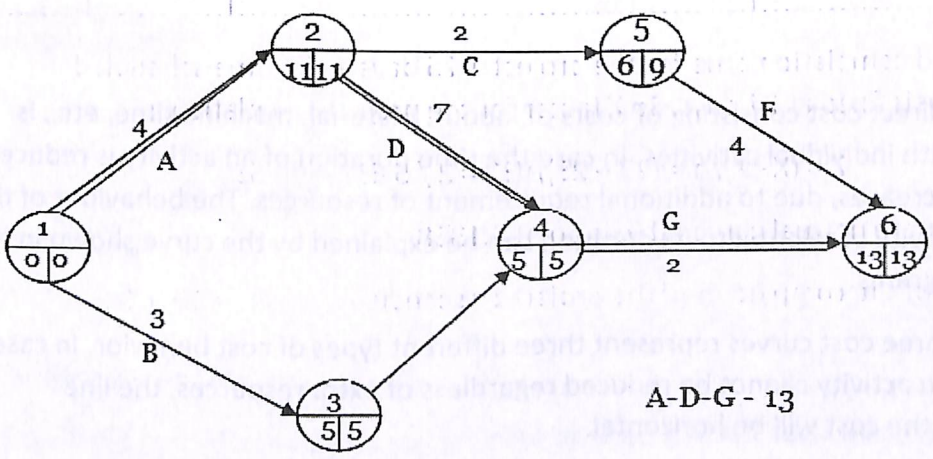
2. What is the probability that the project is completed within 11 weeks.
3. What is the probability that the project is completed within 16 weeks.

Solution.

Activity	Predecessor Activity	t_o	t_m	t_p	$t_e = (t_o + 4t_m + t_p)/6$	$s^2 = ((t_p - t_o)/6)^2$
A	-	2	3	10	4	16/9
B	-	2	3	4	3	1/9
C	A	1	2	3	2	1/9
D	A	4	6	14	7	25/9
E	B	4	5	12	6	16/9

Standard deviation of the activities on critical path

$$\sigma^2 = \sqrt{(16/9) + (25/9) + 1} = 2.357$$



A-D-G - 13

(i) The probability that the project shall be completed within in 13 weeks is 50%.

(ii) The probability that the project shall be completed within in 11 weeks.

$$Z = (X - \dot{X})/\sigma$$

X = target completion time of the project

$\dot{X}$ = Mean/Expected completion time

Z = (11 - 13)/2357 = -0.85

Value of the Z factor at 0.85 in the normal distribution table = 0.3023

Probability = 0.5 – 0.3023 = 0.1977 i.e. 19.77%

(iii) Probability that project shall be completed within 16 weeks

$$Z = (16\text{-}13) / 2357 = 1.27$$

Value of Z at 1.27 in the normal distribution table = 0.398

Add it to 0.5 i.e. 5 + 0.398 = 0.898 i.e. 89.8%

Project cost analysis

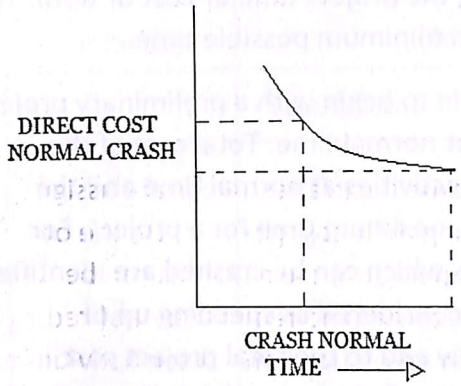
Most activities can be completed in reduced time if extra resources are assigned for them. Thus, with reduction in time duration or crashing their cost goes up. On the other hand, if there is no special reason to reduce the time duration, the activities can be completed in normal time and at normal cost. In fact, there is no requirement of reduction in the duration of all activities, as the over all time can be reduced only if the activities on the critical path are crashed. Activities on non-critical path need not be crashed.

The total costs associated with a project broadly consist of direct costs and indirect costs.

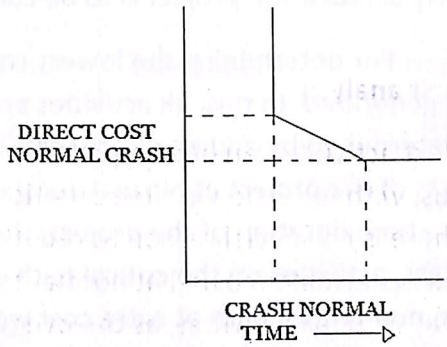
Direct costs:

The direct cost consisting of costs of labour, material, machine time, etc., is associated with individual activities. In case the time duration of an activity is reduced the direct cost increases, due to additional requirement of resources. The behaviour of direct cost of an activity in relation to its duration can be explained by the curve shown in the following diagrams.

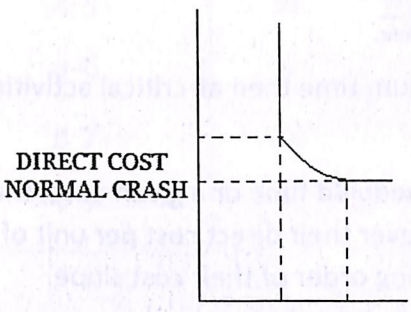
The three cost curves represent three different types of cost behavior. In case the duration of an activity cannot be reduced regardless of extra resources, the line representing the cost will be horizontal.



(a)



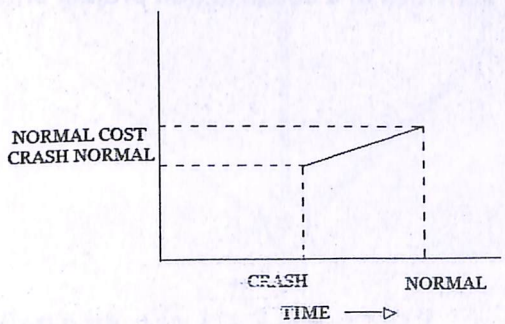
(b)



(c)

Indirect costs:

The indirect costs are associated with the project, and not with the activities. In case the duration of the project is reduced or crashed the indirect costs decrease. The indirect costs consist of general administrative overhead, rent of equipment, depreciation of plant, insurance charges, etc. The behavior of indirect costs in relation to project duration can be explained by the following figure.



Lowest cost schedule

CPM lays great stress upon time cost trade off. This provides a systematic method for determining a project schedule minimizing the project time or cost or both. This is particularly useful if the project is to be completed in minimum possible time.

For determining the lowest cost schedule to begin with a preliminary project schedule is generated. In this, all activities are shown at normal time. Total cost of the project is determined by adding direct costs of all the activities at normal time and the indirect costs of the project at normal duration. This is maximum time for a project. For reducing the time duration of the project, the activities which can be crashed are identified. In this process, activities on the critical path alone are considered as speeding up of activities on non-critical path at extra cost would merely add to the total project cost without reducing the project duration. The increase in the direct costs of activities due to reduction of their duration or the cost slope is determined by the equation :

$$\text{Cost / Time} = \frac{crash\ cost - normal\ cost}{normal\ time - crash\ time}$$

(a) Now if the object is to complete the project in minimum time then all critical activities are crashed, whatever their direct cost per unit of time.

(b) In case the object is to complete the project by a scheduled time or a given date, then all critical activities upto that date alone are crashed, whatever their direct cost per unit of time. Activities are considered one by one in the ascending order of their cost slope.

(c) But in case the object is to complete the project at optimum cost, then all activities on critical path whose direct cost per unit of time is less than the indirect cost of the project per unit of time alone are crashed. In the process of step by step crashing the activities on the critical path are considered in the ascending order of their cost slopes.

If in the process of crashing, the activities on the critical path, some other path or paths become critical, the activities on this new critical are also crashed in the same way. In case there are parallel critical paths then one activity each from the critical path is selected at one time for crashing and so on. This process is continued as long as further shortening of the project time is possible or until the increase in the direct cost per unit of time is less than the indirect cost per unit of time for the project.

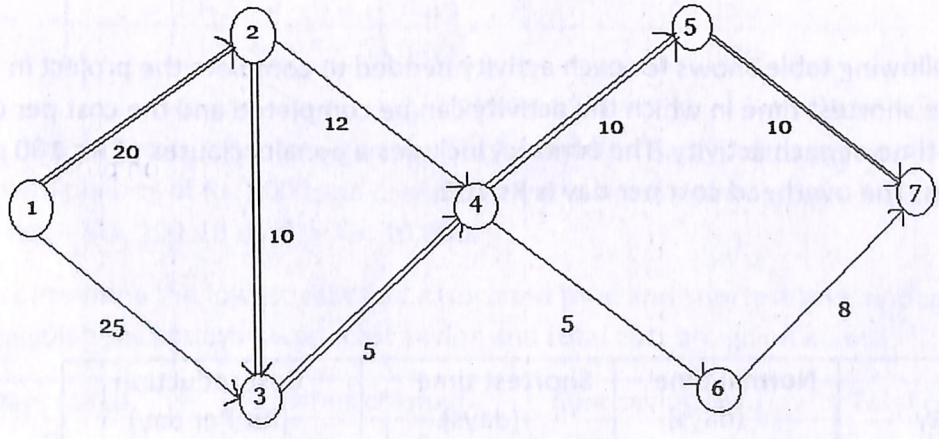
Ex 16.6 : Following table contains details of activities in a construction project and other relevant informations:

Activities	Normal		Crash	
	Time(days)	Cost(Rs.)	Time(days)	Cost(Rs.)
1-2	20	600	17	720
1-3	25	200	25	200
2-3	10	300	8	440
2-4	12	400	6	700
3-4	5	300	2	420
4-5	10	300	5	600
4-6	5	600	3	900
5-7	10	500	5	800
6-7	8	400	3	700

(a) Draw activity network of the project.

(b) Using above information crash the activities step by step until all paths are critical.

Solution: Network for normal time duration is given below:



Initial

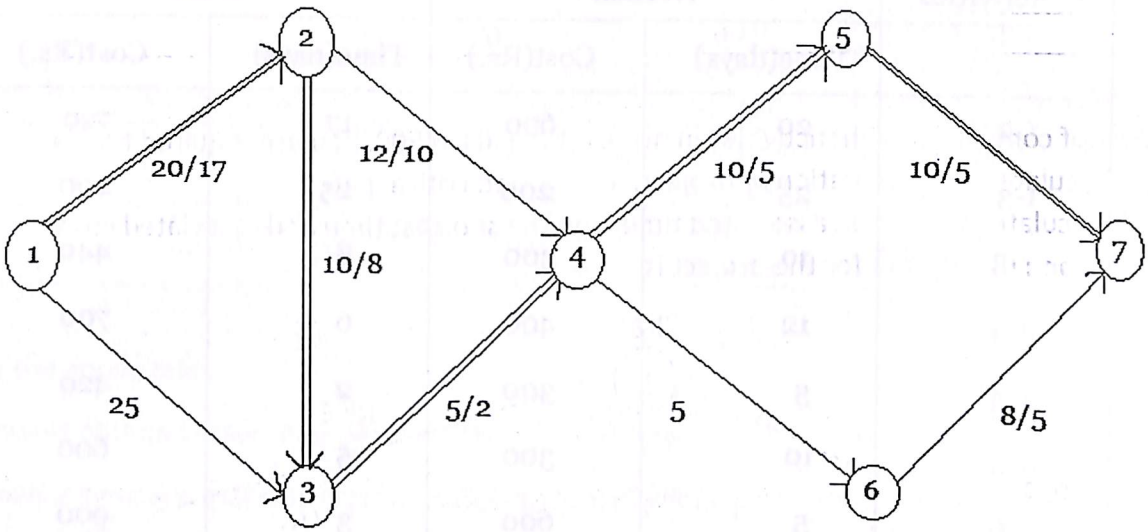
Critical path: 1 -> 2 -> 3 -> 4 -> 5 -> 7

Project time an critical path = 55 days

Total cost = Rs. (600 + 200 + 300 + 400 + 300 + 300 + 600 + 500 + 400) = Rs. 3,600

For step by step crashing of activities, slope of various activities is determined as under:

To shorten the activity time, activities lying on critical path are crashed one by one starting with the lowest cost slope



Total Cost Rs.

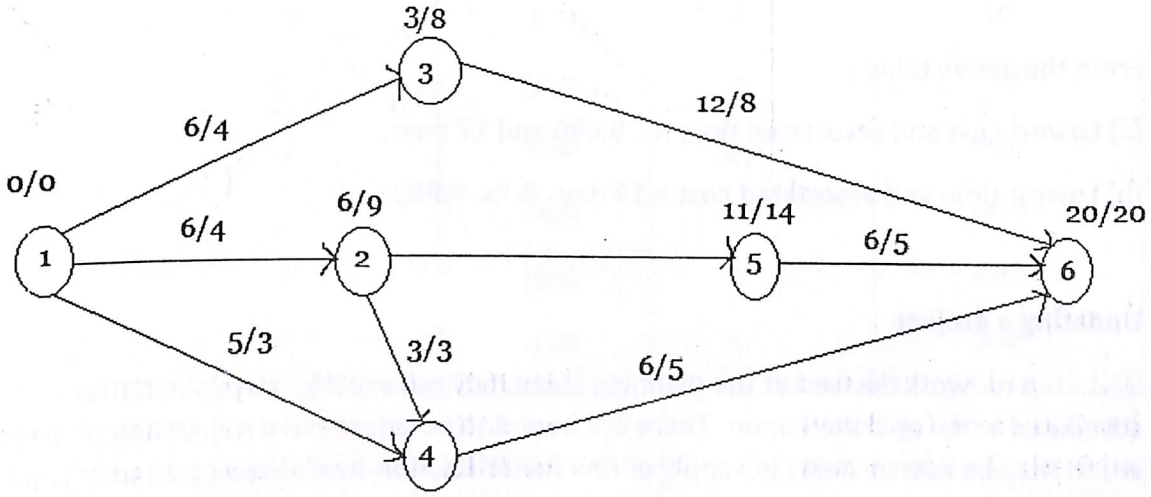
First Crash 1 - 2 for 3 days : Rs. 3600 + (Rs. 40 x 3 days) = 3720
2nd Crash 3 - 4 for 3 days : Rs. 3720 + (Rs. 40 x 3 days) = 3840
3rd Crash 4 - 5 for 5 days : Rs. 3840 + (Rs. 60 x 5 days) = 4140
4th Crash 5 - 7 for 5 days : Rs. 4140 + (Rs. 60 x 5 days) = 4440
5th Crash 2 - 3 for 2 days : Rs. 4440 + (Rs. 70 x 2 days) = 4580
Now the total duration of on 1-2-4-5-7 (non critical path) is 39 days, hence
6th Crash 2 - 4 for 2 days : Rs. 4580 + (Rs. 50 x 2 days) = 4680
7th Crash 6 - 7 for 3 days : Rs. 4680 + (Rs. 60 x 3 days) = 4860
Hence, the project cost = Rs. 4860
Total duration = 37 days **Ans.**

Ex. 16.7 : The following table shows for each activity needed to complete the project in normal time, the shortest time in which the activity can be completed and the cost per day of reducing the time of each activity. The contract includes a penalty clauses of Rs. 100 per day over 17 days. The overhead cost per day is Rs.160.

Activity	Normal time (days)	Shortest time (days)	Cost reduction (Rs. Per day)
1 – 2	6	4	80
1 – 3	8	4	90
1 – 4	5	3	30
2 – 4	3	3	–

2 – 5	5	3	40
3 – 6	12	8	200
4 – 6	8	5	50
5 – 6	6	6	–

Cost of completing eight activities in normal time Rs. 6,500. You are required to :
(i) Calculate normal duration of project, its cost and critical path.
(ii) Calculate lowest cost associated time, and the shortest time and associated cost.
Solution : (i) Network for the project is



Now we can crash the critical activities to find the lowest cost under :

Normal time paths	Shortest time paths
1 – 3 – 6 = 20 days	12 days
1 – 2 – 5 – 6 = 17 days	13 days
1 – 2 – 4 – 6 = 17 days	12 days
1 – 4 – 6 = 13 days	8 days

The project in its normal time will be completed 3 days after the contract period. Thus, with penalty of Rs. 300 total cost of project in normal time will be Rs. 6,500 + (Rs. 160 x 20 days) + (Rs. 100 x 3 days) = Rs. 10,000.

To determine the lowest cost and associated time and shortest time and associated cost, calculations of days saved, cost saving and total cost are given below :

Days saved	Activities crashed	Cost saving (Rs.)	Total cost (Rs.)
0			10,000
1	1 – 3	Rs. 100 + 160 – 90 = 170	9,830
2	1 – 3	Rs. 100 + 160 – 90 = 170	9,660

3	1 – 3	Rs. 100 + 160 – 90 = 170	9,490
4	1 – 3 & 1 – 2	Rs. 160 – 90 – 80 = -10	9,500
5	1 – 2 & 3 – 6	Rs. 160 – 80 – 200 = -120	9,620
6	3 – 6 ; 2 – 5 & 4 – 6	Rs. 160 – 200 – 40 - 50 = -130	9,750
7	3 – 6; 2 – 5 & 4 – 6	Rs. 160 – 200 – 40 -50 = 130	9,880

From the above table :

(a) Lowest cost and associated time Rs. 9,490 and 17 days.

(b) Lowest time and associated cost = 13 days & Rs. 9,880.

Updating a project

A network devised at the planning stage may not exactly follow the pattern as scheduled when operated upon. There are bound to be unexpected delays and difficulties which may be due to delay in supply of raw materials, non-availability of some machines due to breakdown, non-availability of skilled workers or some natural calamities, etc. In such cases, it may be necessary to review the progress of the network planning.

When such changes are being made, the original network diagram is no longer valid. The arrow diagram should always be kept up-to-date by incorporating changes occuring due to replanning. The updated network diagram warns against the effects of unexpected problems will create if nothing is due. Moreover, the updated arrow diagram can suggest the ways and means to overcome the new problems.

Thus, any adjustment to the network diagram which becomes necessary owing to departure from the project schedule laid down earlier is called the updating. It is the process of incorporating in the network the changes which have occured due to replanning and rescheduling.

The frequency of updating depends on the size of the project as well as the period when updating is made. The frequency of updating is more in case of those projects whose overall duration is small, because few slippages in detecting the progress will affect the project as a whole as the time for absorbing such slippages is less. In large projects, the frequency may be less at the initial stages, because a few initial slippages can be absorbed later in the project. However, more frequent updating is necessary as the project approaches completion.

Disadvantages of network techniques

In spite of several advantages, following difficulties are faced by the management while using the network techniques :

1. The difficulty arises while securing the realistic time estimates. In the case of new and non-repetitive type of projects, the time estimates produced are often mere guesses.
2. It is also sometimes troublesome to develop a clear logical network. This depends upon the data input and thus the plan can be no better than the personnel who provides
3. The natural tendency to oppose changed results in the difficulty of persuading the management to accept these techniques.
4. Determination to the level of network detail is another troublesome area. The level of detail varies from planner to planner and depends upon the judgement and experience.
5. The planning and implementation of networks require personnel trained in the network methodology. Managements are reluctant to spare the existing staff to learn this technique or to recruit trained personnel.

Review questions and exercises

1. How does net work analysis helps in managing large complex projects ?
2. What purpose is served by including dummy activities in net work diagram?
3. Define an 'event' and 'activity' in network diagram.
4. What is the ultimate objecive of network analysis
5. Explain project time analysis
6. What is the purpose of cost analysis
7. Explain following terms
   (i) Optimistic Time
   (ii) Norml Time
   (iii) Pessimistic Time
   (iv) Expected Time
   (v) Variance in relation to activities.
7. Descirbe the Term 'Crashing' in network analysis
8. What is Resource Leveling in network analysis ?
9. Explain following terms :
   (i) Slack
   (ii) Float
   (iii) Type of Floats
   (iv) Crash Time & Crash Cost.
10. What is project variance?
11. What are the three time estimates needed for PERT
12. Explain PERT and its importance in network analysis.
13. What do you understand by the term Direct Cost in networks
14. What are Indirect costs in PERT.
15. Explain the uses of floats in the CPM network.
16. What are the principal difficulties with PERT? How can they be overcome?
17. How is the probability of finishing a project in schedule time determined ? Explain with example ?

18. (i) What is Critical Path ?
(ii) Can a critical path change during the course of a project ?
(iii) Can a project have multiple critical path.
19. State the circumstances where CPM is a better technique of project analysis than PERT.
20. Explain the various assumptions of PERT & CPM.
21. 'Time' is the significant factor in the PERT analysis, comment.
22. Mean and standard deviation of a project duration are 42 and 3 days respectively. Find the probability for completing the project within 45 days.
23. A project is expcted to take 15 months along the critical path having a standard dviatin of 3 months. What is theprobability of comleting the project on the due date, if the due datae fixed is 18 minths?
24. From the following table, find critical path and project duration. Assuming overhead cost per Rs 100, calculate revised time if project is to be comleted 2 days earlier. Cost of completing eight activities in normal time Rs. 6000. You are required to :

Activity	Normal time (days)	Shortest time (days)	Cost reduction (Rs. Per day)
1 – 2	6	4	80
1 – 3	6	4	90
1 – 4	5	3	30
2 – 4	3	3	–
2 – 5	5	3	40
3 – 6	12	8	200
4 – 6	8	5	50
5 – 6	6	6	–

UNIT 17 QUEUING (WAITING LINE) THEORY

Introduction

Long queues are generally seen in front of railway booking offices, post offices, bank counters, bus stations, cinema ticket windows etc. Similarly we find automobiles waiting at service stations, airplanes waiting for landing, patients waiting for doctors and so on. Thus queues may be of persons waiting at doctor's clinic or parcels at railway booking office, may be of machines waiting to be required, or may be of letters arriving at the clerk's table for despatch. Queues are formed by units requiring services.

Some customers wait when the total number of customers exceeds the number of facilities. Some service facilities stand idle when the number of service facilities exceeds the number of customers. However queues are very common phenomena of modern civilized life.

A queue is formed when the demand for a service exceeds the capacity to provide that service.

Queuing theory is a quantitive technique which consists in constructing mathematical models of various types of queuing systems. These models can be used for making predictions about how the system can behave and adjust with the demands on it. Queuing theory deals with analysis of queues and queuing behaviour.

Some of the areas where queues are quite common

1. Business: In front of banks, super market, booking offices etc.
2. Industries: In servicing of machines, storage etc.
3. Engineering: In the field of telephony, electronic, computers etc.
4. Transportation: In postal services, airports, harbours, railway etc
5. Other field: Cinema ticket window, barbershop, restaurant etc.

In all these fields queues are quit common. Queuing theory can be applied to the problems associated with these queues.

Objective of Queuing theory

Customers wait for services. The time thus lost by them is expensive. The costs associated with waiting in Queue are known as waiting time costs. Similarly if there are no customers, service station will be idle. Costs associated with service or the facilities are known as service costs. The object of queuing theory is to achieve an economic balance between these two types of costs. That is study of queuing theory helps in minimising the total waiting and service costs.

Queuing theory does not directly solve the problem of minimizing total waiting and service costs. But it provides the management with information necessary to take relevant

decisions for the purpose. Queuing theory can be used to estimates the different characteristics of the witting line such as

(1) Average arrival rate

(2) Average service rate

(3) Average waiting time

(4) Average time spent in the system etc.

Applications of Queuing Theory

Queuing theory can be applied to a wide variety of operational situations. In particular, the technique of queuing theory is applied for solution of a large number of problem such as

1. Scheduling of air craft at landing and takeoff from busy air ports.
2. Scheduling of issues and return of tools by workmen from tool cribs in factories.
3. Scheduling and distributing of scarce war material.
4. Scheduling of work and jobs in production control.
5. Minimization of congestion due to traffic delay at tool booths
6. Scheduling of components to assembling lines.
7. Scheduling and routing of salesmen.

Role of Queuing theory in management

Queuing theory plays a very important role in the management. Decision regarding the amount of capacity required, must be made frequently in industry and elsewhere. These decisions are often difficult ones. It is impossible to predict accurately when units will arrive to seek services, how much will be required to provide that service etc. Providing very much service facility would result in excessive costs. If enough service facilities are not provided, that will cause long waiting lines to form. Excessive waiting is also costly. Therefore ultimate goal of queuing theory is to achieve an economic balance between the cost of service and cost associated with waiting. Based on probability theory it attempts to minimize the extent and duration of investment costs. Queuing theory is able to provide with the estimated average time and intervals under sampling method and helps in taking decision about optimum capacity so that the cost of investment is minimum, keeping the extent of queue within tolerance limits.

Definition of terms

Queue: A group of items, (may be people, machine, letters etc) waiting for service in a service station, known as Queue (or waiting line). A Queue may be finite or infinite.

Customer: Customer are those persons or items waiting in a queue or receiving service. 'Customers' may be people, machines, ships, letters etc. customers form queues.

Server: A server is a person by whom service is rendered.it is the service facility to be availed by customers in the queue

System: The queue plus the service. System includes the customers forming the queue and the service facility.

Time spent by a customer: Time spent for waiting in the queue and service time. It includes both waiting time in the serice centre and the queue.

Queue length: Number of customers waiting in the queue. The queue length is determined by the number of customers waiting for service.

Queuing system: System consisting of arrival of customers, waiting in Queue, picked up for service, being served and the departure of customers.

Average length of Queue: Number of customers in the queue per unit time.

Waiting time: The time up to which a customer has to wait in the Queue before taken to service. It does not include the service time

Traffic intensity: The ratio between mean arrival rate and mean service rate. It shows how much is the arrival per unit service. It is the utilization factor. It is the rte of business at a service facility system.

Idle period: Idle period of a server is the time during which theservice syustem remains free. It is the time where ther is no customer.

Arrival pattern (input): Customers arrive in a random fashion. So their arrival pattern can be described in terms of probabilities. Commonly assumed probability distribution of arrival pattern is 'Poisson".

Service pattern: The service pattern followed by the service stations follows some probability distribution. Commonly assumed probability distributions service pattern are Exponential and Erlang.

Single channel and Multi channel: When there is only one counter in a station so that only one customer can be served at a time, the service mechanism is a single channel. Eg: a cinema ticket window.

When there are more than one service counter in a service station so that more than one customer can be served at a time, the service mechanism is of multiple channel. Eg: the barber shop with three barbers.

Behaviour in a Queue:

1. *Balking:* A customer's behaviour of leaving the queue, when he does not like to wait in the queue due to take of time or space.
2. *Reneging:* A customer's behaviour of leaving the queue due to impatience.
3. *Jockeying:* A customer's behaviour of jumping from one queue to another.

4. *Collusion:* Customer's collaboration so that only one joins the queue as in the case of cinema ticket window where one purchases the ticket for himself and for his friend.

Queuing Process

A Queuing process is centred around a service system which has one or more service facilities. Customers requiring services are arriving at different times by an input source. The customers arrivingat the service system may or may not enter the system (on the basis queue conditions). Any customer entering service system joins a queue for service facilities. Customers are selected for service by some rule, known as service discipline. After the service is completed the customer leaves the service system.

Characteristics of Queuing system (Elements of the Queuing system)

A queueing system is characterized by following factors. They are also called the elements of queueing system.

The input process or the arrivals: The input source may be finite or finite. The input describes the way in which the customers arrive and join the system. Generally customers arrive in a more or less random fashion. For example, the arrival times of customers in a restaurant are distributed more or less randomly and cannot be predicted.

Arrivals may occur at regular intervals also. In a clinic, patients are given appointments in such a manner that they arrive at the clinic at specified equal intervals of time. Arrivals mat occur at a constant rate or may be in accordance with some probability distribution like Poisson distribution. Thus the arrival pattern can best be described in terms of probabilities.

Service mechanism: Service mechanism concerns with the service time and service facilities. Service can either be fixed or distributed in accordance with some probability distribution. Service facilities can be

(a) One queue-one service station (single channel facility)

(b) One queue-several service stations.

(c) Several queue-one service station.

(d) Many queues and many service stations (multi channel facilities).

Queue discipline (or service discipline): If any of the service facilities is free, the incoming customer is taken into service immediately. If however, all the service facilities are busy, the customers in the queue may be handled in a number of ways as service becomes free. Some of these being:

(i) First come first served (FCFS): Here the customers are taken into service in the order in which they arrive. This is known as the 'first - come - first served' service discipline. This is may, for instance, be found at airports, where taxicabs queue, while waiting for passengers.

(ii) Last come - first served: This discipline may be seen in big go down where the items which come last are taken out and served first.

(iii) Random service: The customers are selected for service at random. This is known as the 'random' service discipline. This is found in many operational situation where the customers do not wait in a well organized line.

(iv) Priorities: The customers may be assigned priorities. When the service facilities becoming free, service on the customer commence with the highest priority. If there is more than one customer of the same priority in the queue the service facilities may select a customer from among these either on the 'first -come, first served' or random basis.

Queue discipline also refers to the manner in which the customers form into queue and the manner in which they behave while being in the queue. A customer may decide to wait no matter how long the queue becomes. Some customers may decide not to enter the queue because of its huge length. Some customers after entering the queue and waiting for some time lose patience and leave the queue. When there are more than one queue the customers may move from one queue to another.

Output of queue: In a single channel facilities, the output of the queue does not pose any problem for the customer who leaves after getting service. But it is important when the system is multistage channel facilities, because a service station break down can have repercussion on the queues.

Arrival time and service time distributions

The periods between arrival of individual customers may be constant or following some probability distribution. Many of these distributions can of the distributions like poisson, exponential and Erlang.

Various States of the Queuing system

The state of Queuing systems may be Transient or steady. In transient state, the operating characteristics like waiting time, servicing time etc are dependent on time. In steady state, the operating characteristics of the system are independent of time.

Let $P_n(t)$ stand for probability that there are n units in the system at time t, then the acquires steady state as tends to ∞ ie $P_n(t) \rightarrow P_n$

Explosive state: If the arrival rate of a system is more than its servicing rate, the length of the Queue goes on increasing with time and tends to infinity at t tends to infinity. This state is called explosive state.

Notations:

1. t stands for inter arrival time between two successive customers
2. n stands for number of customers in the queuing system
3. P_n (t) stands for probability that there are 'n' units in the system at any time, 't'.

4. P_n stands for probability that there are exactly 'n' units in the system.

5. λ_n stands for mean arrival rate of customers when there are 'n' units in the system.

6. μ_n stands for mean service rate when there are 'n' units in the system.

7. λ stands for mean arrival rate of customers (independent of n)

8. μ stands for mean service rate (independent of n)

10. ρ stands for traffic intensity = $\frac{\lambda}{\mu}$

11. $E(L_s)$ stands for expected number of customers in the system.

12. $E(L_q)$ stands for expected number of customers in the queue.

13. $E(W_s)$ stands for average waiting time of a customer in the system.

14. $E(W_q)$ stands for average waiting time of a customer in the queue.

15. $E(L_q/L_q > 0)$ stands for expected length of non-empty queue

16. $E(W_s/W_s > 0)$ stands for expected waiting time of a customer, who has to wait (non - empty queue)

Relationship between $E(L_s)$, $E(L_q)$, $E(W_s)$, and $E(W_q)$

1. $E(L_q) = E(L_s) - \frac{\lambda}{\mu}$

2. $E(W_q) = \frac{1}{\lambda} E(L_q)$

3. $E(W_s) = \frac{1}{\lambda} E(L_s)$

Classes of Queuing system

Four important classes of Queuing system are

1. Single queue - single service point.(single channel facility)
2. Multiple queues - multiple service points.(multi channel facilities)
3. Single queue - multiple service points.
4. Multiple queues - single service point.

These classes of Queuing systems are studied through various models.

Classification of Queuing models

Model I	:	(M/M/1)	:	(∞ /FCFS)
Model II	:	(M/M/1)	:	(∞ /FCFS) with long queue
Model III	:	(M/M/1)	:	(N/FCFS)

Model IV : (M/M/S) : (∞ /FCFS)

Model V : ($M/E_k/1$) : (∞ /FCFS)

Model VI : ($M/E_k/1$) : (1 /FCFS)

Model VII : (M/M/R) : (K / GD) K < R

Model VIII : Power supply model

Here first M stands for Poisson arrival and second M stands for Poisson departure

FCFS stands for first come first served

I - stands for single service

∞ - stands for infinite capacity of the system

N - stands for finite number of channels (capacity)

S - stands for several services

R - stands for number of channels

k - stands for number of phases

K - stands for number of machines

GD - stands for General Queue Discipline

Model I MMIFCFS

This is the queuing model, with

(1) arrival rate following poisson distribution and service rate of the following negative exponential distribution,

(2) Single channel with infinite capacityand ,

(3) the service discipline; first come first served.

These are single channel problems. They are simplest queuing problems

Derivation of various formulae:

Let λ stand for mean arrival rate and μ stand for mean service rate.

Then $\frac{\lambda}{\mu}$ is the traffic intensity.

1. Probability that there are 'n' units in the system at any time

$P_n = (\frac{\lambda}{\mu})^n (1 - \frac{\lambda}{\mu})$

2. P (Number of units in the Queue is at least N) = P (size ≥ N)

$= P_n - (P_0 + P_1 + ...P_{N-1}) = (\frac{\lambda}{\mu})^N$

3. $E(L_s) = \Sigma n . P_n = \frac{\lambda}{\mu-\lambda}$

4. $E(L_q) = \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu-\lambda)}$

5. $E(L_q/L_q > 0)$ = Average length of non - empty que $= \frac{\mu}{\mu-\lambda}$

6. $E(W_q) = \frac{\lambda^2}{\mu(\mu-\lambda)} \div \lambda = \frac{\lambda}{\mu(\mu-\lambda)}$

7. $E(W_s) = \left(\frac{\lambda}{\mu} \quad \lambda\right) \div \lambda = \frac{1}{\mu-\lambda}$

8. $E(W_q/W_q > 0) = \frac{1}{\mu-\lambda}$

9. Probability that the time a unit spends in the system ≥ T

$= \left[e^{-(\mu-\lambda)t}\right]_T^{\infty}$

10. Probability that waiting time of an arrival in the queue ≥ T

$= \frac{\lambda}{\mu}\left[e^{-(\mu-\lambda)t}\right]_T^{\infty}$

LIST OF FARMULAE IN MODEL I

Let λ stand for mean rate and μ stand for mean service rate

1. Probability that the service facility is not idle $= \frac{\lambda}{\mu}$

2. Probability that service facility is idle = $P_0 = 1 - \frac{\lambda}{\mu}$

3. P ('n' units in the system) = $P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 \quad \frac{\lambda}{\mu}\right)$

4. P (the number of units in the Queue is at least n) $= \left(\frac{\lambda}{\mu}\right)^n$

5. Average number of units in the system = $E(L_s) = \frac{\lambda}{\mu-\lambda}$

6. Average number of units in the queue = $E(L_q) = \frac{\lambda^2}{\mu(\mu-\lambda)}$

7. Average time a unit spends in the system = $E(W_s) = \frac{1}{\mu-\lambda}$

8. Average time a unit spends waiting in the queue = $E(W_q) = \frac{\lambda}{\mu(\mu-\lambda)}$

9. Average length of non empty queue = $E(L_q / L_q > 0) = \frac{\mu}{\mu-\lambda}$

10. Average waiting time an arrival who has to wait = Average waiting time in a non empty Queue $E(W_q / W_q > 0) = \frac{\mu}{\mu-\lambda}$

11. P (time a unit spends the system ≥ T) = $\left[e^{-(\mu-\lambda)t}\right]_T^{\infty}$

12. P (time a unit spends in a queue ≥ T) = $\frac{\lambda}{\mu}\left[e^{-(\mu-\lambda)t}\right]_T^{\infty}$

PROBLEMS

Ex.17.1: A repair shop attended by a single machine has an average of four customers an hour who bring small appliances. The mechanic inspects them for defects and often can fix them right away or otherwise render a diagnosis. This takes him six minutes, on the average. Arrivals are poisson and service time has the exponential distribution. You are required to (a) Find the probability that the shop is empty. (b) Find the probability of finding at least one customer in the shop. (c) What is the average number of customers in the system (d) Find the average time spent, including service.

Ans: Mean arrival rate, $\lambda = 4$ customer per hour

Mean service rate, $\mu = \frac{1}{6} \times 60 = 10$ per hour

(a) Probability that the shop is empty = Probability that the facility is idle

$= 1 \quad \frac{\lambda}{\mu} = 1 \quad \frac{4}{10} = .6$

(b) Probability of at least one customer in the shop = Prob. that the service facilities is not idle. $= \frac{\lambda}{\mu} = 0.4$

(c) Average number of customers in the system $= \frac{\lambda}{\mu - \lambda} = \frac{4}{10-4} = \frac{2}{3}$

(d) Average time spent in the system

$= \frac{\lambda}{\mu - \lambda} = \frac{4}{10-4} = \frac{1}{6}$ hours = 10 minutes

Ex.17.2:In a railway marshalling yard, goods trains arrive at a rate of 30 train per day. Assuming that the inter - arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following:

(a) Average length of non empty queue

(b) The probability that the queue size exceeds 10

Ans: Mean arrival rate, $\lambda = 30$ trains / day

Mean service rate, $\mu = \frac{1}{36} \times 60 \times 24 = 40$ per day

(a) The average length of non empty queue $= \frac{\mu}{\mu - \lambda} = \frac{40}{40-30} = 4$ trains per hour

(b) Prob. for queue size exceeds 10 = P (n ≥ 11) $= \left(\frac{\lambda}{\mu}\right)^{11} = \left(\frac{3}{4}\right)^{11} = 0.042$

Ex.17.3: Customers arriving at a booking office window, being manned by a single individual at the rate of 25 per hour. Time required to serve customer has exponential distribution with a mean of 120 seconds. Find the average waiting time of customers.

Ans: $\lambda = 25$ per hours

$$\mu = \frac{60 \times 60}{120} = 30 \text{ per hour}$$

$$\left.\begin{matrix}\text{Average waiting} \\ \text{Time of a customer}\end{matrix}\right] = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{25}{30\,(30\,25)} = \frac{1}{6} \text{ hour} = \underline{10 \text{ minutes}}$$

Ex.17.4: A TV repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If the repairs set in the order in which they came n, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is repairman's expected idle time each day ? How many jobs are on an average, ahead of the set just brought in ?

Ans: Here $\lambda = 10$ per day and $\mu = \frac{1}{30} \times 60 \times 8 = 16$ per day

Prob. that facility is idle = $P_0 = 1 \quad \frac{\lambda}{\mu} = 1 \quad \frac{5}{8} = \frac{3}{8}$

$\therefore$ 1) Repaireman's expected idle time in 8 hour day $= \frac{3}{8} \times = 3$ hr

2) Average number of jobs ahead = expected number units in the system

$$= \frac{\lambda}{\mu-\lambda} = \frac{10}{16-10} = \frac{10}{6} = 1\frac{2}{3} \text{ jobs.}$$

Ex.17.5: The belt snapping for conveyers in open cast mine occur at the rate of 2 per shift. There is only one hot place available for vulcanising and it can vulcanise on an average 5 belts snap per shift.

1. What is the probability that when a belt snaps, the plate is readily available?
2. What is the average number in the system?
3. What is the waiting time of an arrival?
4. What is the average waiting time plus vulcanising time?

Ans: $\lambda = 2$ belts per shift; $\mu = 5$ belts per shift.

(1) P (hot plate is readily available) = P (there is idling)

$= P_0 = 1 \quad \frac{\lambda}{\mu} = 1 \quad \frac{2}{5} = \frac{3}{5}$

(2) Average number in the system $= \frac{\lambda}{\mu-\lambda} = \frac{2}{5-2} = \frac{2}{3}$

(3) Average waiting time of an arrival $= \frac{\lambda}{\mu(\mu-\lambda)} = \frac{2}{5(5-2)} = \frac{2}{15}$

(4) Average waiting time plus vulcanising time = Average waiting time in the system

$$= \frac{1}{\mu - \lambda} = \frac{1}{5-2} = \frac{1}{3}$$

Ex.6: The mean rate of arrival of plans at an airport during the peak period is 20 per hour, and the actual number of arrivals in any hour follows a Poisson distribution. The airport can land 60 planes per hour on an average in good weather and 30 planes per hour in bad weather, but the actual number landed in any hour follows a Poisson distribution with these respective averages. When there is congestion, the planes are forced to fly over the field in the stack awaiting the landing of other planes that arrived earlier.

(i) How many planes would be flying over the field in the stack on an average on good weather and in bad weather?

(ii) How long a plane would be in the stack and the process of landing in good and in bad weather?

Ans: In this problem, we are given

λ = 20 planes / hour

$$\mu = \begin{cases} 60 \text{ planes /hour in good weather} \\ 30 \text{ planes /hour in bad weather} \end{cases}$$

(i) Average number of planes i the stock

=Average number of units in the Queue

$$= \frac{\lambda^2}{\mu(\mu - \lambda)} \begin{cases} 20^2 \div [60(60-20)] = \frac{1}{6} (\text{in good weather}) \\ 20^2 \div [30(30-20)] = \frac{4}{3} (\text{in bad weather}) \end{cases}$$

(ii) Average time a plane would be in the stock and the process of landing = Average waiting time in the system

$$= \frac{1}{\mu - \lambda} \begin{cases} 1 \div (60 - 20) = \frac{1}{40} \text{ hrs} = 1.5 \text{ hours (in good weather)} \\ 1 \div (30 - 20) = \frac{1}{10} \text{ hrs} = 6 \text{ hours (in bad weather)} \end{cases}$$

Ex.7: Customer arrive at a one window drive in bank according to Poisson Distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window, including that for serviced car can accommodate a maximum of three cars. Other cars wait outside this space.

(a) What is the probability that arriving customer can drive directly to the space in front of the window?

(b) What is the probability that arriving customer will have to wait outside the indicated space?

(c) How long is an arriving customer expected to wait before starting service?

Ans: Hereλ = 10 per hour and μ = 60/50 = 12 per hour.

(a) The probability that can arriving customer can drive directly to the space in front of the window.

= Prob that 2 or less cars in the system = $P_0 + P_1 + P_2$

$$= \left(1 \quad \frac{\lambda}{\mu}\right) + \frac{\lambda}{\mu}\left(1 \quad \frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^2 \left(1 \quad \frac{\lambda}{\mu}\right)$$

$$= \left(1 \quad \frac{5}{6}\right) + \frac{5}{6}\left(1 \quad \frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 \left(1 \quad \frac{5}{6}\right)$$

$$= \frac{1}{6} + \frac{5}{36} + \frac{25}{216} = \frac{91}{216} = 0.412$$

(b) The probability that an arriving customer has to wait outside the indicated space.

=Prob. that more than 3 in the system $= \left(\frac{\lambda}{\mu}\right)^4 = \left(\frac{10}{12}\right)^4 = 0.48$

(c) Average waiting time of a customer in queue

$$= \frac{\lambda}{\mu\,(\mu - \lambda)} = \frac{10}{12\,(12-10)} = 0.47$$

Ex.8: A bank has two letters working on saving accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distribution for both deposits and withdrawals are exponential with mean service time 3 minutes per customer. Deposits are found to arrival in a Poisson fashion throughout the day with mean arrival rate 16 per hours. Withdrawals also arrive in a Poisson fashion with mean arrival rate 14 per hour. What would be the effect on the average waiting time for depositors and withdrawals?

Ans:

Mean arrival rate of depositors = $\lambda_1 = 16$ per hr

Mean arrival rate of withdrawers = $\lambda_2 = 14$ per hr

Mean service rate for both tellers = $\mu = \frac{1}{3} \times 60 = 20$ per hr.

Average waiting time for depositors

$$= \frac{1}{(\quad - 1)} = \frac{16}{20\,(20-16)} = \frac{1}{5} \text{ hour} = 12 \text{ minutes.}$$

Average waiting time for withdrawers

$$= \frac{2}{(\quad - 2)} = \frac{14}{20\,(20-14)} = \frac{7}{60} \text{ hour} = 7 \text{ minutes.}$$

Ex.9: A repairman is to be hired to repair machines which break down at an average rate of 16 per hour. The breakdown follows Poisson distribution. The productive time of machine is considered to cost Rs. 20 per hour. The repairmen, Mr. X and Mr. Y have been interviewed for this purpose. Mr. X charges Rs. 10 per hour and he services breakdown machines at the rate of 8 per hour. Mr Y demands Rs. 14 per hour and he services at an

average rate of 12 machines per hour. Which repairman should be hired? (Assume 8 hour shift per day)

Ans: $\lambda = 6$ per hour

$\mu = 8$ per hour for X

$= 12$ per hour for Y

Mr. X

Average number of machines under repaire and waiting $= \frac{}{-} = \frac{6}{8-6} = 3$ per hour

$\therefore$ Machine hours lost in 8 hour $= 3 \times 8 = 24$ hrs.

Cost of the productive time of a machine = 20 Rs.

$\therefore$ The cost for 24 hrs $= 24 \times 20 = 480$ Rs.

For a day (8 hrs), the charge for the repair man X, $= 8 \times 10 = 80$ Rs.

$\therefore$ Total cost = 480 + 80 = 560 Rs.

Mr. Y

$$\text{Average number of machines under repaire and waiting} = \frac{}{} = \frac{6}{12 \quad 6} = 1$$

$\therefore$Machine hours lost in 8 hours $= 1 \times 8 = 8$ hrs.

Total productive cost for these 8 hrs $= 8 \times 20 = 160$ Rs.

Charge for repairing per day = 14 Rs.

Charge for the whole day $= 8 \times 14 = 112$ Rs.

$\therefore$ Total cost = 112 + 160 = 272 Rs.

$\therefore$ Repairman Y should be preferred as the total cost is least.

Ex.10: Arrivals of machines at a tool crib are considered to be Poisson distribution at an average rate of 6 per hour. The service at the tool crib is exponentially distributed with an average of 3 minutes. (1) What is the probability that a machine arriving at the tool crib will have to wait? (2) What is the average number of machinists at the tool crib? (3) The company will install a second tool crib when convinced that a machinist would have to wait atleast three minutes before being served. By how much the flow of machinists to the tool crib must increased, to justify the addition of a second tool crib?

Ans: $\lambda = 6$ per day

$\mu = \frac{1}{3}$ per minute $= \frac{1}{3} \times 60 = 20$ per hour.

(1) Prob that the tool crib will have to wait

= Prob that tool crib is not idle = $- = \frac{6}{20} = \underline{0.3}$

(2) Average number of machines at tool crib

= Average number of units in the system = $\frac{}{-} = \frac{6}{20-6} = 0.43$

(3) Let λ_1 be the new arrival rate. It is given that the average waiting in the queue = 3 min = .05 hour.

$\therefore \frac{1}{(- _1)} = .05$ or $\frac{1}{20(20 - _1)} = .05$

$\therefore \lambda_1 = .05 \times 20 (20 \quad _1)$

$\therefore \quad _1 = 1(20 \quad _1) \qquad \lambda_1 = 20 - \lambda_1$

$\therefore \quad _1 + \quad _2 = 20$

$\therefore 2 \quad _1 = 20$

$\therefore \quad _1 = 10$per hour So arrival should be 10 per hour

$\therefore$ Increased in arrival rate = 10 - 6 = 4 per hour.

$\therefore$ There should be an increase by 4 per hour to justify the additio of a second tool crib.

Ex.12: On an average 96 patients per 24 hour day require the servi:e of an emergency clinic. Also on an average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes and such that each minute of decreased in this average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decreased the average size of the queue from - patients to - patient.

Ans: Arrival rate $= \frac{96}{24} = 4$ per hour.

Service rate = $= \frac{1}{10}$per min = 6 per hour.

Average number of patients in the queue = $\frac{^2}{(-)} = \frac{4^2}{6(6-4)} = 1\frac{1}{3}$ patients

We want to reduce this to $\frac{1}{2}$

Let μ be the new service rate, then

$\therefore \frac{^2}{(\quad)} = \frac{1}{2} \therefore \frac{4^2}{(\quad 4)} = \frac{1}{2}$

$\therefore \quad = 8$ per hour

Service rate required is 8 for reducing the average size of queue

Time required for one service = $\frac{1}{8}$ hour = 7.5 minutes

$\therefore$ Decrease in the average time of treatment = 10 - 7.5 minutes

Cost of the clinic before decrease = 100 Rs.

Cost of the clinic after decrease = 100 + (2.5 × *10*) = 125 Rs.

$\therefore$ The budget should be Rs. 125 per patient.

Ex.13: In a Bank every 15 minutes one customer arrives for cashing the queue. The staff in only payment counter takes 10 minutes for serving a customer on an average. State suitable assumptions and fine (1) the average queue length (2) increase in the arrival rate in order to justify a second counter (when the waiting time of a customer is atleast 15 minutes the management will increase one more counter).

Ans: Assumptions: (a) Arrival pattern follows Poisson distribution

(b)Service time follows an exponential distribution.

Arrival rate = $\lambda = \frac{1}{5}$ per minute = 4 per hour

Service rate = $\mu = \frac{1}{10}$ per minute = 6 per hour

(1) Average queue length (Average number of units in the queue)

$$= \frac{^{2}}{(\ -\ )} = \frac{4^2}{6(6-2)} = 1.33$$

(2) The second counter is justified when the average waiting time of an arrival in the queue should be atleast 15 minutes:

Let the new arrival rate of this be λ

$$\frac{}{(\ -\ )} = \frac{15}{60}$$

$$\therefore 60 \quad = 15 \quad (\qquad)$$

$$\therefore 60 \quad = 15 \times 6\,(6 \qquad)$$

$$\therefore 60 \quad = 540 \quad 90$$

$\therefore 150 \quad = 540$ or $\lambda = \frac{540}{150} = 3.6$

For increasing the number of counters minimum arrival rate

$= \lambda = 3.6$ per hour.

Present arrival rate = λ = 4 per hour.

Increase in the present arrival rate = 4 - 3.6 = 0.4 per hour.

So even now a second counter is justified.

Ex.14: Arrivals at a telephone booth are considered to be poisson with an average time of 10 minutes between one arrival and the next. The duration of a phone call assumed to be distributed exponentially with mean = 3 minutes.

1) What is the probability that a person arriving at the booth will have to wait?

2) Fraction of the time the phone will be in use

3) The telephone company will install a second booth when convinced that an arrival would expect to have to wait atleast three minutes for the phone. By how much must the flow of arrivals be increased in order to justify a second booth ?

4) Find the average number of unit in the system.

5) What is the probability that it will take, and arrival, more than 10 minutes altogether to wait for the phone and complete his call.

6) What is the probability that an arrival will have to wait more than 10 minutes before the phone is free.

7) Estimate the fraction of a day that the server is busy

Mean service rate (μ) $= \frac{1}{3} \times 60 = 20$ per hour

1) P [an arrival will have to wait] =P [system is not handle]

$= - = \frac{6}{20} = .3$

2) Fraction of the time the phone is in use $= - = \frac{6}{20} = .3$

3) Average waiting time of an arrival in the Queue $= \frac{\quad}{(\ -\ )}$ Given, this is equal to 3 minutes

ie $\frac{\quad}{(\ -\ )}$ = 3 minutes = .05 hour $\mu = 20, \lambda = ?$

$\therefore \frac{\quad}{20\,(20-\ )} = .05$ ie $\frac{\quad}{400-200} = 0.05$

$\lambda = .05\,(400 - 20\lambda\ )$

$\lambda = 20 - \lambda$ $\therefore \quad + \quad = 20$

$\therefore 2 \quad = 20$ $\therefore \quad = 10$ per hour

Increase in mean arrival rate = new value of λ -old value of λ

= 10 - 6 = 4 per hour

$\therefore$ Second booth is justified when the increase in arrival rate of customers is 4 per hour

4) Average number of units in the system

$$= \frac{}{} = \frac{6}{20-6} = \frac{3}{7}$$

5) P [time a unit spends in the system $\geq \frac{10}{60}$] = $\left[\quad -(\ - \ ) \right]_{1/6}^{\infty}$

$$= \left[0 \quad -(20-6)^{\frac{1}{6}} \right] = \frac{-14}{6} = -2.33 = .10$$

6) P [time an arrival in queue has to wait $\geq \frac{10}{60}$] = $-\left[\quad -(\ - \ ) \right]_{1/6}^{\infty}$

$$= \frac{6}{10}\left[\ 0 \quad 2.33 \right] = .3\left[\quad -2.33 \right] = .3 \times .1 = .03$$

7) Fraction of a day that the server will be busy $- = \frac{6}{20} = .3$

Limitations of Queuing Theory

1. Most of the Queuing models are complex and cannot be easily understood. There is always the element of uncertainty in all queuing situations. There, the probability distribution to be applied for arrival or servicing may not be clearly known.

2. Queue discipline also imposes some limitations. We assume first come first service discipline. If this assumption is not true, Queuing analysis becomes more complex.

3. In multichannel queuing, several times the departure from one queue forms arrival for another. This makes the analysis more complex.

Review questions and exercises

1. What do you understand by queue? Give some important applications of queuing theory ?

2. Give the essential characteristics of the queuing process.

3. What do you understand by (a) queuing model (b) queue discipline.

4. Explain the constituents of a queuing model.

5. Explain the object of Queuing theory.

6. Explain in brief what are queuing problems? How does the queuing theory apply to these problems.

7. Define the terms (a) Queue (b) input (c) output (d) FCFS

8. What are single and multiple channel facilities ?

9. What are transient and steady states of queuing system ?

10. Weavers in a Textile Mill arrive at a Department Store Room to obtain spare parts needed for keeping the looms running. The store is manned by one attendant. The average arrival rate of weavers per hour is 10 and service rate per hour is 12. Both arrival and service rates follow Poisson process. Determine.

(i) Average length of Waiting Line.

(ii) Average time a machine spends in the system.

(iii) Percentage idle time of Departmental Store Room (attendant)

11. People arrive at a theatre ticket booth, in a Poisson distribution arrival rate of 25 per hour. Service time is exponentially distributed with an average time of two minutes. Calculate:

(i) the mean number in the waiting line,

(ii) the mean waiting time,

(iii) the utilization factor

12. There is congestion of platform of a railway station. The trains arrive at the rate of 30 trains per day. The waiting time for any train to humps is exponentially distributed with an average of 36 minutes. Calculating the following (1) mean queue size (2) The probability that queue size exceeds 9

13. Consider a box office ticket window being manned by a single individual. Customers arrive to purchase tickets according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with mean of 90 seconds. Find the following.

(i) Expected line length.

(ii) Expected queue length.

(iii) Expected waiting time in the system.

(iv) Expected waiting time in the queue.

14. The workers come to a tool store room to enquire about the special tools [required by them] for a particular job. The average time between the arrivals is 60 seconds and the arrivals are assumed to be in Poisson distribution. The average service time is 40 seconds. Determine. [a] average queue length [b] average length of non empty queue [c] average number of workers in the system including the workers being attended. [d] mean waiting time of an arrival [e] average waiting time of an arrival [worker] who waits.

15. In bank cheques are cashed at a single letter counter. Customers arrive at the counter in Poisson manner at an average rate of 30 customers per hour. The letter takes on an average a minute and a half of cash cheque. The service time has been shown to be exponentially distributed.

(1) Calculate the percentage of time the letter is busy

(2) Calculate the average time a customer is expected to wait.

16. Problems arriving at a computer centre is Poisson fashion with a mean arrival rate of 25 per hour. The average computing job requires 2 minutes of terminal time. Calculate the following:

(1) Average number of problems waiting for the customer's use

(2) Percent of times an arrival can walk right in without having to wait.

17. A postal clerk can service a customer in 5 minutes, the service time being exponentially distributed with an average of 10 minutes during the early morning slack period and an average of 6 minutes during the afternoon peak period.

Assess the (a) average queue length and (b) the expected waiting time in the queue during the two periods.

18. A One man barber shop, customers arrive according to poisson distribution with a mean arrival rate of 5 per hour and his hair cutting time was exponentially distributed with an average hair cut taking 10 minutes. It is assumed that because of his excellent reputation, customers were always willing to wait. Calculate the following:

(i) Average number of customers in the shop and the average number of customers waiting for a haircut.

(ii) The percentage of time an arrival can walk right in without having to wait.

(iii) The percentage of customers who have to wait prior to getting into the barber's chair.

19. A ticket window of a cinema theatre is manned by a single individual. Customers arrive to purchase tickets in a poisson fashion with a mean rate of 30 per hour. The time require to serve a customer has an exponential distribution with a mean of 30 seconds. Find (i) expected queue length (ii) expected waiting time.

20. Workers come to a tool store room to enquire about the special tools (required by them) for a particular job. The average time between two arrivals is 60 seconds and the arrivals are assumed to be in Poisson distribution. The average service time is 40 seconds. Determine (i) average queue length (ii) average length of non-empty queue (iii) average number of workers being attended (iv) mean waiting time of an arrival (v) average waiting time of arrival who waits

21. A repair shop attended by a single machine has an average of four customers an hour who bring small appliances for repair. The machine inspects them for defects and quit often can fix them six minutes, on the average. Arrivals are poisson and service time has the exponential distribution. You are required to (i) find the proportion of time during which the shop is empty (ii) find the probability for atleast one customer in the shop (iii) what is the

average number of customers in the system (iv) find the average time spent including service.

22. A fertilizer company distributions its products by truck loaded at its only loading station. Both company tricks and contractor's truck are used for this purpose. It was found that on an average every 5 minutes one truck arrived and the average loading time was 3 minutes. 40% of the trucks belonging to the contractors. Making suitable assumption, determine:

(i) the prob that a truck has to wait

(ii) the waiting time of a truck that waits.

(iii) the expected waiting time of a contractor's truck per day.

23. At a tool service centre the arrival rate is 2% per hour and the service potential is 3 per hour. The hourly wage paid to the attendant at the service centre is Rs. 50 per hour and the hourly cost of a machinist away from his work is 120 Rs. (i) calculate the cost of the operating the system for an 8 hour day (ii) calculate the cost of the system if there were two attendants working together as a team, each paid Rs. 50 per hour and each able to serve as an average two customers per hour.

24. A bank has only one typist. Since the typing work various in length and number of copies required, the typing rate is randomly distributed, approximately as a poisson distribution with mean service rate of 8 letters per hour during entire 8 - hour work day. If the time of the typist is valued at Rs.15 per hour, determine the following (i) equipment utilization (ii) the present time that an arriving letter has to wait (iii) average system time (iv) average cost due to waiting and operating type writer

25. An over head crane moves jobs from one machine to another and must be used every time a machine requires loading or unloading. The demand for service is at random. Data taken by recording the elapsed time between service cells every 30 minutes. In a similar manner, the actual service tome of loading or unloading took an average of 10 minutes. If the machine time is valued at Rs. 8.50 per hour how much does the down time cost per day

26. Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes.

(a) What is the probability that a person arriving at the booth will have to wait?

(b) Find the average number of units in the system.

(c) The telephone department will install a second both when convinced that an arrival would expect waiting for atleast 3 minutes fir phone. By how much should the flow of arrivals increase in order to justify a second booth?

27. A repairman is to be hired to repair machines that breakdown at an average rate of 4 per hour. Breakdowns are distributed randomly in time. Non productive time of any machine is considered to cost the company Rs. 9 per hour. The company has narrowed the choice to two repairman, one slow but cheap, the other fast but expensive. The slow but cheap repairman asks Rs. 3 per hour, in terms he will service breakdown machine at an average rate of 5 per hour. The fast but expensive repairman demands Rs. 6 per hour and will repair machines at an average of 7 per hour. Which repairman should be hired?

28. A repairman is to be hired to repair machines which breakdown at an average rate of 3 per hour. Breakdowns are distributed in time in a manner that may be regard as Poisson. Non productive time on any one machine is considered to cost the company Rs. 5 per hour. The company has narrowed the choice to two repairman one slow but cheap, the other fast but expensive. Thus slow cheap repairman asks Rs. 3 per hour, in return he will service break down machines exponentially at an average rate of 4 per hour. The fast expensive repairman demands Rs. 5 per hour and will repair machines exponentially at an average rate of 6 per hour. Which repairman should be hired?

29. The average rate of arrivals at a service store is 30 per hour. At present there is one cashier who an average attends to 45 customers per hour. The store proprietor estimates that each extra minute of system process time per customer means a loss of Rs. 0.50. An assistant can be provided to the cashier and in that case the service unit can deal with 75 customers per hour. The wage rate of the assistant is Rs. 15 per hour. Is it worth employing assistant?

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UNIT 18 OTHER QUEUING MODELS

Waiting line theory or quenuing models play a very significant role in the management decision regarding the amount of capacity required, utilization of capacity, idle time, etc. These decisions are often critical for business firms offering service instead of products. Besides MMI model,with single line and single facility, rThere are other models, which are described below.

Model II → (M / M / I) : (∞ / FCFS)

(General single station Queuing model or General Erlang model)

This is same as Model I except that Mean arrival rate and Service rate are not constant. Both are dependent on 'n'.

Here arrival rate = λ_n service rate = μ_n

Three cases are possible

Case 1 : $\lambda_n = \lambda$, and $\mu_n = \mu$. Then this similar to Model I

Case 2 : $\lambda_n = \frac{}{+1}$ and $\mu_n = \mu$

Case 3 : $\lambda_n = \lambda$ and $\mu_n = n\mu$

In cases (2)and (3), E (L_S) = ρ = –

We can find E (L_W), E (W_q) and E (W_S) from the value of E (L_S) using the interrelation between them.

(1) E (L_q) = E (L_S) – – (2) E (W_q) = $\frac{1}{}$ E (L_q)

(3) E (W_S) = $\frac{1}{}$ E (L_q) + $\frac{1}{}$

Ex. 1 : A shipping company has simple unloading birth with ships arriving in a poisson fashion at an average rate of 3 per day. The unloading time distribution for a ship with n unloading crews is found to be exponential with mean unloading time 1/2n days. The company has a large labour supply without regular working hours and to avoid long waiting lines the company has a policy of using as many unloaded crews as there are ships waiting in line or being unloaded. Under these conditions find (a) the average number of unloading crews working at any time (b) probability that more than four crews will be needed.

Ans: Mean arrival time = λ = 3 shops per day

Mean service rate is not constant. It depends on the waiting line

$\mu_n = 2n$

For one unloading crew, μ = 2

(a) Average number of unloading crews working at any time

E (L_S) = ρ = – = $\frac{3}{2}$ = 1.5

Probability for n units = $P_n = \frac{-}{!}$

(b) P (ship entering service will need more than four crews)

= P ($n \geq 5$) = 1- [P_0+P_1+P_2+P_3+P_4]

= 1- [$^{-}$ + $\frac{-}{1!}$ + $\frac{-\ 2}{2!}$ + $\frac{-\ 3}{3!}$ + $\frac{-\ 4}{4!}$]

= 1- $^{-1.5}$ [1+1.5+$\frac{(1.5)^2}{2}$ + $\frac{(1.5)^3}{6}$ + $\frac{(1.5)^4}{24}$]

= 1 - .2231 [1 + 1.5 + 1.25 + .5625 + .2109] = 1 - .981 = .019

Ex.2: Problems arrive at computing centre in s poisson fashion at an average rate 0r 5 per day. The rules f computing centre are that any man waiting to get his problem solved must aid the man whose problem is being solved. If the time to solve a problem with one man has an exponential distribution with mean time of 1/3 day, find the expected number of persons working on the problem at any instant.

Ans: $\lambda = 5$

Here serve rate (μ) is not constant. Service rate increases with increase in the number of persons. Given $\mu = \frac{1}{3}$ day $= \frac{1}{1/3} = 3$persons

Expected number of persons working on the problem at any instant

$E(L_S) = \rho = - = \frac{5}{3}$ person.

Model III → (M / M / 1) : (N / FCFS)

This is queuing model with (a) Poisson arrival (b) Poisson service (c) single channel (d) finite capacity (e) first come first served

In this system $\left.\begin{matrix}\lambda_n \\ \lambda_n\end{matrix}\right\} = \lambda$ when $n < N$ (a fixed quantity)

0 when $n \geq N$ and

$\mu_n = \mu$

Probability for n units in the system ie P_n is given by

$P_n = \left(\frac{1}{1 \quad +1}\right)$ for $0 \leq n \leq N$ and $P_0 = \frac{1=}{1= \quad +1}$

Average number of units in the system = $E(L_S) = \sum_{=0} \quad = \quad {}_0\Sigma_{=0}$

$= \frac{1-}{1- \quad +1} [\quad +2^{\ 2} + \ldots\ldots\ldots \quad]$

$E(L_q)$; $E(W_s)$ and $E(W_q)$ can be obtained from $E(L_s)$ using the interrelation between them.

Ex. 3: If for a period of two hours in the day (8 - 10 AM) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period (a) the probability that the yard is empty (b) average queue length on the assumptions that the line capacity of yard is limited to 4 trains only.

Ans: Here $\lambda_n = \lambda$ for $n \leq 4$, $\lambda_n = 0$ for $n > 4$

$\lambda = \frac{1}{2} \times 60 = 3$ per hour

$\mu = \frac{1}{36} \times 60 = \frac{5}{3}$ per hour

$N = 4$ and $\rho = \frac{\lambda}{\mu} = \frac{3}{5/3} = \frac{9}{5} = 1.8$

(a) Prob. that the yard is empty $= P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-1.8}{1-(1.8)^5} = .04$

(b) Average queue length $= E(L_s) = \frac{1-\rho}{1-\rho^{N+1}} [\rho + 2\rho^2 + 3\rho^3 + 4\rho^4]$

$= .04 [1 + 1.8 + 2(1.8)^2 + 3(1.8)^3 + 4(1.8)^4] = 2.9 =$ 3 trains

Ex. 4: In a railway marshalling yard goods train arrive at a rate of 30 trains per day. Assuming that the interval arrival time follows an exponential distribution and the service time (the time taken to hump a train) distribution is also exponential with an average 36 minutes. The line capacity of the yard is to admit 9 trains only (there being 10 lines, one of which is earmarked for the shunting engine to reverse itself from the crest of the hump to the rear of the train). Calculate the following on the assumption that 30 trains, on an average, are received in the yard. (a) probability that the yard is empty (b) average queue length

Ans: Here N = 9, $\lambda = 30$ trains per day

$$\mu = \frac{1}{36} \text{ train per minute} = \frac{1}{36} \times 60 \times 24 = 40 \text{ per day}$$

$$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = .75$$

(a) Probability that the yard is empty $= P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-.75}{1-(.75)^{10}} = 0.28$

(b) Average queue length $= E(L_s) = \frac{1-\rho}{1-\rho^{N+1}} [\rho + 2\rho^2 \ldots\ldots + 9\rho^9]$

$= 0.28 [.75 + 2(.75)^2 + \ldots\ldots + 9(.75)^9] = .28 \times 9.07 = 2.5 =$ 3 trains

Model IV → (M / M / S) : (∞ / FCFS)

This is Queuing model with (i) Poisson arrival (ii) Service times exponentially distributed (iii) S channels (S > 1) (iv) same service rate (μ) at each channel (v) service discipline first come , first served.

If n < S , all customers will be served simultaneously. But s - n.

Service channels will remain idle.

If n = S, all the service channels will be busy and the rate of service = $\mu_n = S\mu$

If n > S, all the service channels will be busy. n - S customers will be waiting in the Queue.

'n' stands for number of customers and 'S' stands for number of channels.

In this model, $\lambda_n = \lambda$

$$\left.\begin{matrix}\mu_n \\ \mu_n\end{matrix}\right\} = n\mu \qquad \text{if } 0 \le n < S$$

$$S\mu \qquad \text{if } n \ge S$$

$$\therefore \rho = \frac{\lambda_n}{\mu_n} = \frac{\lambda}{n\mu} \qquad \text{for } 0 \le = n < S$$

$$\rho = \frac{\lambda_n}{\mu_n} = \frac{\lambda}{S\mu} \qquad \text{for } n \ge S$$

$$P_0 = \frac{1}{\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{(S\rho)^S}{S!(1-\rho)}}$$

$P_n = \frac{(n\rho)^n}{n!} P_0$ if $0 \leq n < s$

$P_n = \frac{S^s \rho^n}{S!} P_0$ if $n \geq S$ $\therefore P_s = \frac{(S\rho)^s}{S!} P_0$

$E(L_q) = \sum_S^\infty n \quad S P_n = \frac{\rho P_s}{(1-\rho)^2} = \frac{\rho (S\rho)^s}{S!(1-\rho)^2} P_0$ is expected queue length

$E(L_s)$, $E(W_s)$, $E(W_q)$, can be obtained from $E(L_q)$ using the relation between them.

(1) $E(L_s) = E(L_q) + \frac{\lambda}{\mu}$ (2) $E(W_q) = \frac{1}{\lambda} E(Lq)$ (3) $E(W_s) = \frac{1}{\mu} + \frac{1}{\lambda} E(L_q)$

(2) Expected length of non-empty queue = $E(L/L > 0) = \frac{1}{1-\rho}$

(3) $E(W/W > 0) = \frac{1}{s\mu - \lambda}$

(4) Probability that some customers have to wait = $P(n > S) = \frac{\rho P_s}{1-\rho}$

(5) Probability for channels to be busy = $P(n \geq S) = \frac{P_s}{1-\rho}$

(6) Average number of items served = $\sum_{n-1}^{s-1} nP_n + \sum_{n=s}^{\infty} P_n$

Ex. 6 : A super market has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the counter at the rate of 10 an hour. (a) What is the probability that an arrival will have to wait for service? (b) What is the expected percentage of idle time for each girl? (c) If a customer has to wait, what is the expected length of his waiting time?

Ans: This is a multi channel problem. $S = 2$

$\lambda = \frac{10}{60} = \frac{1}{6}$ people/ minute $= \frac{1}{6} \times 60 = 10$ people per hour

$\mu = \frac{1}{4}$ people minute $= \frac{1}{4} \times 60$ 2= 15 people per hour

$= \frac{\lambda}{\mu S} = \frac{10}{15 \times 2} = \frac{1}{3} \therefore S\rho = \frac{2}{3}$

$\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} = \sum_{n=0}^{1} \frac{(\frac{2}{3})^n}{n!} = 1 + (\frac{2}{3}) = \frac{5}{3}$

$\frac{(S\rho)^s}{S!(1-\rho)} = \frac{(\frac{2}{3})^2}{2!(1-1/3)} = \frac{4}{9} \times \frac{3}{4} = \frac{1}{3}$

$\therefore P_0 = \frac{1}{\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{(S\rho)^s}{S!(1-\rho)}} = \frac{1}{\frac{5}{3} + \frac{1}{3}} = \frac{1}{2}$

$P_n = \frac{S^s \rho^n}{S!} P_0$

$\therefore P_1 = \frac{2^2 (\frac{1}{3})}{2!} \times \frac{1}{2} = \frac{1}{3}$

(a) Probability of having to wait for service = $P(n \geq 2) = P_2 + P_3 + = \ldots\ldots$

$= 1 - (P_0 + P_1) = 1 - (\frac{1}{2} + \frac{1}{3}) = 1 - \frac{5}{6} = \frac{1}{6}$

(b) Expected % of idle time $= (1 - \rho) \times 100 = \left(1 \quad \frac{1}{3}\right) \times 100 = 67\%$

(c) Expected length of the customers waiting time

$= E(W_s) = \frac{1}{S\mu - \lambda} = \frac{1}{2 \times \frac{1}{4} - \frac{1}{6}}$ = units

Ex. 7: A bank has two letters working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time the distribution for both deposits and withdrawals are exponential with mean service time 3 minutes per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate 16 per hour. What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both with drawals and deposits?

Ans: This is a multi channel problem. S = 2

$\lambda_1 = 16$ per hour and $\lambda_2 = 14$ per hour

Mean arrival rate $= \lambda = \lambda_1 + \lambda_2 = 16 + 14 = 30$ per hour

Mean service rate $= \mu = \frac{1}{3} \times 60 = 20$ per hour

$\rho = \frac{\lambda}{2\mu} = \frac{30}{40} = \frac{3}{4} \therefore S.\rho = 2.\frac{3}{4} = \frac{3}{2}$

$\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} = \sum_{n=0}^{1} \frac{\left(\frac{3}{2}\right)^n}{n!} = 1 + \frac{3}{2} = \frac{5}{2}$

$$\frac{(S\rho)^s}{S!(1 \quad \rho)} = \frac{\left(\frac{3}{2}\right)^2}{2!(1 \quad \frac{3}{4})} = \frac{\left(\frac{9}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{9}{2}$$

$$\therefore P_0 = \frac{1}{\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{(S\rho)^s}{S!(1-\rho)}} = \frac{1}{\frac{5}{2} + \frac{9}{2}} = \frac{1}{7}$$

$$P_s = \frac{(S\rho)^s}{S!} P_0 = \frac{\left(\frac{3}{2}\right)^2}{2!} \times \frac{1}{4} = \frac{9}{56}$$

Average waiting time per customer

$$= E(W_q) = \frac{\rho P_s}{\lambda(1-\rho)^2} = \frac{\frac{3}{4} \times \frac{9}{56}}{30\left(1-\frac{3}{4}\right)^2} = \frac{9}{140} \text{hour} = \frac{9}{140} \times 60 =$$

3.86 minutes If both the tellers were handling deposits and withdrawals separately, then it will be model I question.

So average waiting time for depositors

$$= \frac{\lambda_1}{\mu(\mu - \lambda_1)} = \frac{16}{20(20-16)} = \frac{1}{5} \text{per hour} = 12 \text{ minutes}$$

Average waiting time for withdrawers

$$= \frac{\lambda_2}{\mu(\mu-\lambda_2)} = \frac{14}{20(20-14)} = \frac{7}{60}\text{per hour} = 7 \text{ minutes}$$

∴Effect when both are done together is that the average waiting time is per customer is reduced.

Model V → (M / E_k / 1) : (∞ / FCFS)

This is a queuing model with (1) poisson arrival (2) Erlang service time with k phases (3) single server (4) infinite capacity (5) service discipline. First come first served.

Here $\lambda_n = \lambda_1$ arrivals (of units) per unit time

$\mu_n = \mu k$ phases served per unit time.

$$\rho = \frac{\lambda}{k\mu}$$

$P_0 = 1 - \rho k$

$P_n = (1 - \rho k)\, \Sigma \rho^m (-1)^r (mC_r) \times (m + s - 1\ C_s)$

Expected number of units in the queue $= E(L_q) = \frac{k+1}{2k} \frac{\lambda^2}{\mu(\mu-\lambda)}$

$E(L_s)$; $E(W_s)$ and $E(W_q)$ can be obtained from their relation with $E(L_q)$

$$E(L_s) = E(L_q) + \frac{\lambda}{\mu} = \frac{k+1}{2k} \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu}$$

$$E(W_q) = \frac{1}{\lambda} E(L_q) = \frac{k+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)}$$

$$E(W_s) = \frac{1}{\lambda} E(L_q) + \frac{1}{\mu} = \frac{k+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu}$$

Note:

1) If k = 1, we get the same results as in Model I

2) If $k = \infty$, $\frac{1}{k} \to 0$ so that

$$\frac{k+1}{2k} = \frac{\frac{k}{2}+\frac{1}{k}}{\left(\frac{2k}{k}\right)} = \frac{1+\frac{1}{k}}{2} = \frac{1+0}{2} = \frac{1}{2}$$

Ex.8: A hospital clinic has a doctor examining every patient brought in for a general check up. The doctor averages 4 minutes on each phase of the check up although the distribution of time spent on each phase is approximately exponential. If each patient goes through four phases in the check up and if the arrivals of the patients to the doctor's office are approximately poisson at an average rate of 3 per hour, what is the average time spent by a patient waiting in the doctor's office ? What is the average time spent in the examination? What is the most probable time spent in the examination?

Ans: Here No. of phases = k = 4

Mean arrival rate = λ = 3/60 = 1/20 patient per minute = 3 per hour

Mean service time per phase = 4 minutes

Mean service time per customer = 4× 4 = 16 minutes

∴ Mean service rate = μ = 1/16 customer per minute

$= \frac{1}{16} \times 60$ per house = 3.75 per hour

(a) Average time spent by a patient waiting in the doctor's office

$= E(W_q) = \frac{k+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)} = \frac{4+1}{8} \frac{3}{3.75\,(3.75-3)}$ hour = 40 minutes

(b) Average time spent in the examination

$= \frac{1}{\text{Mean service rate}} \times 1 = \frac{1}{\mu} = \frac{1}{3.75}$ hour = 16 minutes

(c) Most probable time spent in the examination

$= \frac{k-1}{\mu k} = \frac{4-1}{3.75 \times 4}$ hour = 12 minutes

Ex. 9: Repairing a certain type of machine which breaks down in a given factory consists of 5 basic steps that must be performed sequentially. The time taken to perform each of the 5 steps is found to have an exponential distribution with mean 5 minutes and is independent of other steps. If this machines break down in a poisson fashion at an average rate of two per hour and if there is only one repair man, what is the average idle time for each machine that has broken down.

Ans: Here No. of phases = k = 5

Mean arrival rate = λ = 2/60 = 1/30 machines per minute = 2 per hour

Mean service time per customer = 5 × 5 = 25 minutes

Mean service rate = $\mu = \frac{1}{25}$ per minute = 2.4 per hour

Average idle time for each machine broken down = $E(W_s)$

$= \frac{k+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} = \frac{5+1}{10} \frac{2}{2.4\,(2.4-2)} + \frac{1}{2.4}$ hour

= 100 minutes

Ex. 10: At a certain airport it takes 5 minutes to land an airplane, once it is given the signal to land. Although in coming planes have scheduled arrival times, the wide variability in arrival times produces an effect which makes the incoming planes appear to arrive in a poisson fashion at a average rate of 6 per hour. This produces occasional stack - ups at the airport which can be dangerous and costly. Under this circumstances how much time will a pilot expect to spent circling the field waiting to land.

Ans: Here k = ∞ $\quad \therefore \frac{k+1}{2k} = \frac{1}{2}$

Mean arrival rate = $\lambda = 6/60 = 1/10$ airplanes per minute = 6 per hour

Mean service rate = $\mu = 1/5$ airplanes per minute = 12 per hour

Average waiting time = $E(W_q) = \frac{k+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1}{2} \times \frac{6}{12(12-6)}$ hr

=2.5 minutes.

Review questions and exercises

1. Consider a single server Queuing system with a poisson input, exponential service times. Suppose the arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and the maximum permissible number calling units i the system is 2. Derive the steady - state probability distribution of the number of calling units in the system and then calculate the expected number in the system.

2. There are 3 booking clerks at a railway ticket counter. Passengers arrive at a average rate of 200 per 8 hour day. The mean service time is 4 minutes and are served strictly on first cum first serve strictly on first cum first serve basic. Find the idle time of booking clerk.

3. A tax consulting firm has four stations (counters) in its office receive people who have problems and complaints about their income, wealth and sales taxes. Arrivals average 80 persons in an 8 hours service day. Each tax advisor spends an irregular amount of time servicing the arrivals which have been found to have an exponential distribution. The average service time is 20 minutes. Calculate the average number of customers in the system, average number of customers waiting to be serviced, average time a customer spends the system and average waiting time for a customer. Calculate how many hours each week does a tax advisor spend performing his job. What is the expected number of idle tax advisor at any specified time ?

4. A telephone exchange has two long distance operators. The telephone company finds that, during the peak load, load distance calls arrive in a poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponential distributed with mean length 5 minutes (a) what is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day ? (b) If the subscriber will wait and are serviced in turn, what is the expected waiting time ?

5. A component is produced in 5 stages in a machine shop. Each stage is independent is produced and follows exponential distribution with mean time as 10 minutes. The components are loaded in production at an average rate of one per hour following a poisson distribution. Find (a) average number of units in the system and (b) average time a production passes through the production time

6. The repair of a lathe requires four steps to be completed one after another in a certain order. The time taken to perform each step follows exponential distribution with a mean of 15 minutes and is independent of other steps. Machine break down per hour.

Which is the (a) expected idle time of the machine ? (b) expected number of broken down machines in the queue.

7. In a car manufacturing plant, a loading crane takes exactly 10 minutes to load a car into a wagon and again come back to poisson stream at an average of one every 20 minutes, , Model V Ans: 20.8, 45.8 calculate the average waiting time for a car.

8. A Colliery working one shift per day uses a large number of locomotives which break down at random intervals, on average one fails per 8 hour shift. The filter carries out a standard maintenance schedule on each family loco. Each of the five main parts of this schedule taken an average half an hour but the time varies widely. How much time will the filter have for the other tasks and what is the average time a loco is out of service.

9. A barber with one man takes exactly 25 minutes to complete one hair cut. If customer arrive in a poisson fashion at an average rate of one every 40 minutes, how long on the average must a customer wait for service.

UNIT 19 SIMULATION

Introudction

All problems like game theory, Linear programming problem, transportation etc are solved by using algorithms. In all these cases we first consider fomulatin and then developa naylyticl solution to the model. Hoever, in some of the reealworkd situatins, we cannot represent theproblem in thje mathematical model form, because of complication a nd complexity . in these cases, we can apply simulation approach.

To simulate is to try to lduplicate the features, appearance and characteristiscs of a real system in general terms simulatin involves de veloping a model of some real phenomenon and then performing an experiment on the model evolved. It is a descriptive and not optmisong technique. Infact in simulatin, given system is copied and thevariable and constant associated with it are manipulated in that artificial environment to examine thebehaviour of the system.

A familiar form of lsimulatin is analogye simulatinl, where an original phusical psystme is replaced by an analoguous physical system, whichcan use very esy to manipulate.inother words, to simulate is totry ltodupoicate thefeatures, appearances and characteristics of a real syste, so instead ofusing lunches and intuitions to dettrmien what may happe, the analyst using simulation cfan test and evaluate thecvrious alternatives and select the one whichgives thebest results.

thus simulation means deriving measures of performance about a complex system by conducting sampling experiments on a mathematical model of thsystem ovr period of imt. Usually the model is run on a computer in order to obtain operationsl formation.

There are many real life problems which cannot be adequately represented by a mathematical model or analytical methods. In such situations, simulation is a useful tool. A simulation model is a simplified representation of real life situations which helps to understand a problem and helps to find its solution by trial and error approach.

Simulation is a method of solving decision making problems by designing, constructing and manipulating a model of the real system. It is the action of performing experiments on a model of a given system. It duplicates the essence of a system or activity without actually obtaining the reality.

Simulation is a quantitative technique that can be used for determining alternative courses of action based on facts and assumptions. Simulation involves the division of the system into smallest component parts and combining them in their natural and logical order, analyzing the effects of their interactions on one another, studying various specific alternatives and choosing the best one.

Simulation is a way of representing one system such as a field operation or physical phenomenon by a model to facilitate its study. So under simulation technique some type of model is formulated which describes the system's operation. The system is divided into several elements and the interrelationships between these elements is studied. Also some predictable behavior in terms of probability distribution is studied about the various states of the system. The result is then used in connection with real life situations for which the simulated model has been developed.

Definitions of simulation

Shannon very lprecisely defined simuation as "a process of designing a model of a real system and conducting experiments with the model for thepurpose of understanding thebehaviousr for the operation of the system." This definition focuses on modeling and understanding of behavior and is veryu brief in nature.

Churchnman views simulation as certain mathematicalrelations which he expressed in the folloing words. X simulates Y is true if and only if Y iis taken to be thereal system and x is taken to be approximatin to real system and the rules of validity in X and non –error, free, otherwise X will become the real system.

According to Levin and Kirk Patrick, simulation is an "appropriate substitute for mathematical evaluation of a model inmany siutatins. Although it too involves assumptions, they ar manageable. The use of simulation enables us to pro vide insights into certain mmanagement problems where mathematical evaluation of a model is not possible.

Requirements for simulation

Simulation complex mathematical process involving several inter related active . There are two basic requirements for using simulation.

1. Construction of a model representing the essential characteristics of the system.
2. Development of a mechanism to simulate the model.
3. Establishment of provision for generating a stochastic process.

Phases of simulation model

A simulation model consists of two basic phases

(1) Data generation and

(2) Book keeping.

Data generation: Data can be generated from sample observations of a variable. The sample observations are collected,

(a) Using random number table

(b) Resorting to mechanical devices or

(c) Using electronic computers.

Book keeping phase: This phase deals with updating the system when new events occur. It also deals with monitoring and recording system.it involves

(a) Preparation of random number intervals
(b) Generatin of random numbers

Methods of simulation

There are two methods of simulation - system simulatin method and Monte carlo method

System simulation method.

Under this method, operating environment is produced and thesyustem allows for analyzing theresponse from the environment to alternative management actions. The method is complicated and costly.

Monte Carlo method.

Monte carlo lsimulation is quantitative techniuie based on probability theory and random tables. It is a combination of probabilitymethods and sampling techniques, proving solutions to complicated partial or integral disfferentia euations. Monte carlo method is concerned withexperimentas on random number and it provides solutoi to complicated management science problems.

Monte Carlo method is a substitution for the mathematical evaluation of a model. The basis of Monte Carlo technique is random sampling of a variable's possible values. For this technique some random numbers are required which may be converted into random variates whose behavior is known from past experience. Darker and Kac define Monte Carlo method as combination of probability methods and sampling techniques providing solutions

to complicated partial or integral differential equation. In short, Monte Carlo technique is concerned with experiments on random numbers and it provides solutions to complicated Operations Research problems.

General procedure of Monte Carlo methods

The method uses random numbers for originating some data by which a problem can be solved. The random numbers are used in creating a new set of hypothetical data of a problem from past experience. If no pattern can be assumed for the data, then randomness can be assumed. When past information is not available, it can be obtained by conducting a preliminary survey. The data collected are plotted on a graph from which a cumulative probability functions is obtained.

In Monte Carlo method, a sequence of random numbers is selected from the random number table. The random numbers obtained are taken as decimal numbers and also as the probabilities obtained at random from the parent population. These probabilities are plotted on the cumulative frequency curve of the given data. The value of x corresponding to each probability given by the random numbers, is the desired sample value.

Uses of Monte Caro simulation

Monte Carlo techniques are useful in the following situations :

1. Where one is dealing with a problem which has not yet arisen. ie. where it is not possible to gain any information from past experience.
2. Where the mathematical and statistical problems are too complicated and some alternative methods are needed.
3. To estimate parameters to a model.
4. To get the general idea of the system.

Steps in Monte carlo simulation

The main steps of Monte Carlo method are as follows:

(a) To get the general idea of the system, a flow diagram is drawn.

(b) Correct sample observations are taken to select some suitable model for the system. In this step, some probability distribution for the variables of our interest is determined.

(c) Probability distribution is converted to a cumulative distribution function.

(d) Using cumulative probabilities, probability intervals are prepared.

(e) sequence of random numbers is selected with the help of random number tables.

(e) Then a sequence of values of the variables of our interest is determined with the sequence of random numbers obtained.

(f) Finally, some standard mathematical function is applied to the sequence of values obtained.

Advantages of Monte Carlo Method:

(i) These are helpful in finding solution of complicated mathematical expressions which is not possible otherwise.

(ii) By these methods, difficulties of trial and error experimentation are avoided.

Disadvantages of Monte Carlo Method:

(i) These are costly way of getting a solution of any problem.

(ii) These methods do not provide optimal answer to the problems. The answers are good only when the size of the sample is sufficiently large.

Applications of Monte Carlo Simulation

Monte Carlo simulation is applied to a wide diversity of problems such as queuing problems, inventory problems, risk analysis concerning a major capital investment. Simulation is very useful in Budgeting. System flexible budgeting is an exercise in simulation. Simulation has made great contribution in quantitative analysis of complex systems.

Procedure in Monte Carlo

Monte Carlo method may applied when a system contains elements that exhibit chance in their behavior. Accordingly the technique is broken down into folling steps.

1. Probability distribution- in case of Monte carlo simulation, thevalues of thevariables have to be generated. There are number of variables in real world system that are probabilistic in nature and that we ant to simulate, e.g., the number of variables may be inventory demand on daily basis, lead time for inventory order to arrive, time between machine break down, time between arricals and user etc.
2. Cumulative probability – from the regular probabilitydistribution, we can arrive at the cumulative probabilitydistribution of ev ry job. Cumulative probability is thesum of numbers in theprobability column added to theprevious culilative probability.
3. Probability intervals – basing on cumulative probabilities, probabilityintervals with required sizes must be prepared. When a simulated an intem into such intervals, necessary value of variable is generated. Probabilityintercvals are really the data base to solve such sutuatins.
4. Random numbers – required numbers must be generated to simulate. If the simulation is small, thenumber may be selected that has 100 lots randomly from the random table. We can select numbers from anywhere in thetable. Thes e random numbers ar eselected in such a way that everynumber has an equal probability.

Applications of simulation

Simulatin model is applied externsively in business activities. It is recognized as a powerful tool for management decision making. A few application areas are:

1. Evaluating altrernative courses of investments.
2. Testing impact of various policy decisions
3. Examing government fiscal actions on the economy
4. Analyzing where to locate factories
5. Studying future courses of actions and alternatives
6. Testing market policies and studing their impacts
7. Inventory control
8. Production management
9. Capital budgfeting and investment

Advantages of simulation technique

1. Simulation technique can be used to solve the problems where values of the variables are not known, or partly known.

2. A model once constructed for a system can be employed again for analyzing different situations.

3. Simulation methods are handy for analyzing proposed system in which information can be presented well.

4. the effect using thesimulatin model can be studied without actually using it in thereal situatin.

5. from thesimulatin model, data for thrther analysis can be generated.

Disadvantages of simulation

1. The knowledge obtained from the part of the system, cannot be used for deriving thebehaviour of theentire system.
2. Simuation model does not give an analystical solution
3. It provides only statistical estimates rather thatn exact results.
4. It only compares thealternative rather than generating an optimal one.
5. It is slow and costly and complex

Ex 19.1 a tourist car operator find that during thepast 100 days , demand for car had been varied as shown below:

Trip/week	0	1	2	3	4	5
No od days	8	12	15	30	20	15

using random numbers – 09, 54, 01, 80, 06, 57, 79, 52. simulate demand for next 8 days

Ans.

Trips	Probability	Com. prob	Prob interval	Random no	Simu. demand

0	.08	.08	00 - 07	09	1 trip
1	.12	.20	08 - .19	54	3 trips
2	.15	.35	20 - 34	01	0 trip
3	.30	.65	35 - 64	80	4 trips
4	.20	.85	65 - 84	06	0 trip
5	.15	1.00	85 - 99	57	3 trips
				79	4 trips
				52	3 trips

Ex. 19. 2 mamtha Bakery sells cakes, on demand as detailed below. Simulate demand for next 6 days using random nimbners – 48, 78, 19, 51, 56, 77.

Daily demand	0	10	20	30	40	50
Probability	.01	.20	.15	.50	.12	.02

Ans.

demand	Probability	Cum. prob	Prob interval	Random no	Simu. demand
0	.01	.01	00	48	30 cakes
10	.20	.21	01 - .20	78	30 cakes
20	.15	.36	21 - 35	19	10 cakes
30	.50	.86	36 - 85	51	30 cakes
40	.12	.98	86 - 97	56	30 cakes
50	.02	1.00	98 - 99	77	40 cakes

Ex 19.3 Dr Shirin Shaheen Shahid schedule all their patients for 40 minutes appointments. Some of the patients take more or less than 4o minutes, depending on the ltype lof work, as below. Simulate the time of the clinic for five Hours starting from9.00 AM. You may use the random nos – 28, 72, 34,76,12, 67, 42, 82.

work	time	probability
Filling	45 min	.40
Crown	60	.15
Cleaning	15	.15
Extraction	30	.10

Check up	15	.20

Ans.

Work	probability	Cum prob	Prob interval	Ran No	Simu work
Filling	.40	.40	00 - 39	28	filling
Crown	.15	.55	40 - 54	72	extraction
Cleaning	.15	.70	55 - 69	34	Filling
Extraction	.10	.80	70 - 79	76	Extraction
Check up	.20	1.00	80 - 99	12	filling
				67	Cleaning
				42	crown
				82	Check up

If 40 minutes appointments are given for each patient –

1 patient 9.00 Am filling 45 minutes – leaves 9.45- no waiting

2 patient 9.40 extraction 15 minutes – leaves 10.00 – 5 minutes waiting

3 patient 10.20filling – 45 minutes – leaves 11.05 - no waiting

4 patient 11.00 extraction 30 minutes leaves 11.35 5 min waiting

5 patient 11.40 filling 45 minutes leaves 12.25 no waiting

6 patient 12.20 cleaning 15 minutes leaves 12.40 5 min waiting

7 patient 1.00 PM crown 60 minutes leaves 2.00 no waiting

8 patient 1.40 PM check up 15 minutes leaves 2.15 - 20 min waiting

Review Questons and exercises

1. What is simulatiomn
2. What are the uses of simulation
3. Explain system simulatin method
4. What is monte carlo lsimulation.

5. State the steps in monte carlo simulation
6. What are the applications of simulation
7. State the d=menrits of simulation
8. What are thedisadvalntages of simulation.
9. A bookstore wishes to carry Bib;le in strock. Follow ng is the demand position of The Book. Forecst demand for next 6 days. Use random nos – 8,96,45,34,24,21.

Demand	0	1	2	3	4
Probability	.05	.10	.30	.45	.10

10. Production of cars byan automobile company is described below;

Prod.	95	96	97	98	99	100	101	102	103	104	105
PROB	.03	.05	.07	.10	.15	.20	.15	.10	.07	.05	.03

Produced cars are sent by container ship. If the container can carry only 101 cars per trip, how many cars are waiting to be transported?

11. Demand for Rent A Car during the last 200 days is :

Demand	5	10	15	20	25	30
Days	2	40	30	100	24	4

Simulate the demand for next 8 days and find av erage demand per week.

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