MATHEMATICAL TOOLS FOR ECONOMICS - I

I SEMESTER

COMPLEMENTARYCOURSE

BA ECONOMICS

(CUCBCSS - 2014 Admission)

UNIVERSITY OF CALICUT

SCHOOL OF DISTANCE EDUCATION

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STUDY MATERIAL

Complementary Course for BA Economics

I Semester

MATHEMATICAL TOOLS FOR ECONOMICS - I

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MODULE – I

Basic Mathematical Concepts :– Arithmetic and Algebra

1.1 Arithmetic

Arithmetic or arithmetics (from the Greek word *arithmos*meaning "number") is the oldestand most elementary branch of mathematics. It consists in the study of numbers, especially the properties of the traditional operations between them — addition, subtraction, multiplication and division. Arithmetic is an elementary part of number theory, and number theory is considered to be one of the top-level divisions of modern mathematics, along with algebra, geometry, and analysis.

Thus, arithmetic is a branch of mathematics that deals with properties of the counting (and also whole) numbers and fractions and the basic operations applied to these numbers.

Difference with mathematics…

"Arithmetic is to mathematics as spelling is to writing."

The dictionary definitions of these two bodies of learning are:

arithmetic

(1) the branch of mathematics that deals with addition, subtraction, multiplication, and division,

(2) the use of numbers in calculations

mathematics

(1) the study of the relationships among numbers, shapes, and quantities,

(2) it uses signs, symbols, and proofs and includes arithmetic, algebra, calculus, geometry, and trigonometry.

Thus the most obvious difference is that arithmetic is all about numbers and mathematics is all about theory.

Rules of Arithmetic

Rule 1: Addition and Subtraction

When two numbers are added together, the result is called a sum. When one number is subtracted from another, the result is called a difference.

Exercises

(a) $6 + 4 = 10$
(b) $6 + (-4) = 2$
(c) $6 - 4 = 2$
(d) $6 - (-4) = 10$
(e) $10 + (-3) - (-6) = 13$
(f) $15 - (-4) + (-6) = 13$
(g) $(-3) + 5 - (-3) + (-5) = 0$
(h) $(-16) - 8 - (-12) + (-4) = -16$
(i) $(-30) - (-18) - (16) = -28$
(j) $(-8) + 8 - (4) + (-4) = 0$
(k) $73 + (-54) - 35 + 18 + 15 = 17$
(l) $73 + (-54) + 35 + (-18) + (-15) = 21$
(m) $(-73) + 54 + 35 + (-18) + 15 = 13$
(n) $73 + (-54) + (-35) + (-18) + 15 = -19$

(o) $33 + 0 + (-15) + 3 = 21$

Rule 2: Multiplication and Division

Multiplication

If a number is multiplied by positive sign (eg. +1), the sign remains unchanged. If a number is multiplied by negative sign (eg. −1), the sign is reversed. That is, if it was previously positive, it becomes negative. If it was previously negative, it becomes positive.

$(+8) \times (+1) = +8$: multiplication by +ve does not change sign

$(-8) \times (+1) = -8$: multiplication by +ve does not change sign

$(+8) \times (-1) = -8$: multiplication by –veresults in change of sign

$(-8) \times (-1) = +8$: multiplication of two –ve results in change of sign, it becomes +ve.

Note that multiplication of any number by 0, results in 0. Thus

$(+8) \times 0 = 0$, $(-8) \times 0 = 0$

Division

Division reverses multiplication. So the same sign rules as multiplication applies here.

$(+10) \div (+2) = +5$

$(-10) \div (-2) = +5$

$(-10) \div (+2) = -5$

$(+10) \div (-2) = -5$

Note that division of a number by itself gives 1. Thus $10 \div 10 = 1$

Division of a number by 0 is undefined.

Order of Operations

When there is more than one operation (multiplication, division, exponents, brackets, subtraction, addition)involved in a mathematical problem, it must be solved by using the correct order of operations.In Mathematics, the order in which mathematical problems are solved is extremely important.

A number of acronyms are used to help students to retain the order. The most popular are BODMAS (Bracket, of, Division, Multiplication, Addition, Subtraction) and BEDMAS (Bracket, Exponents, Division, Multiplication, Addition, Subtraction).

Example 1

$20 - [3 \times (2 + 4)]$ Do the inside bracket (parenthesis) first.

$= 20 - [3 \times 6]$ Do the remaining bracket.

$= 20 - 18$ Do the subtraction.

$= 2$

Example 2

$(6 - 3)2 - 2 \times 4$ Do the bracket (parenthesis)

= (3)2 − 2 × 4 Calculate the exponent.

= 9 − 2 × 4 Now multiply

= 9 − 8 Now subtract

= 1

Example 3

= 22 – 3 × (10 – 6) Calculate inside the bracket (parenthesis).

= 22 – 3 × 4 Calculate the exponent.

= 4 −3 × 4 Do the multiplication.

= 4 − 12 Do the subtraction.

= − 8

Example 4

12 ÷ 4 + 3^2Exponent first

= 12 ÷ 4 + 9Divide

= 3 + 9Add

= 12

1.2 Algebra

Algebra and arithmetic are two related areas of mathematical study, though there are some differences. Arithmetic focuses mainly on the use of numbers in calculations, while algebra is a generalization of arithmetic where letters are used in place of numbers to better understand equation principles. Algebra explores several areas by study according to the rules of arithmetic. Simple algebra, also known as classical algebra, involves letters that represent numbers, which are then combined according to the laws of arithmetic.

Thus Arithmetic is a branch of mathematics usually concerned with the four operations (adding, subtracting, multiplication and division) of positive numbers.

eg:- 2 + 2 = 4, 3 – 2 = 1, 5 x 5 = 25, 49/7=7.

Thus Algebra is the branch of mathematics that uses letters, symbols, and/or characters to represent numbers and express mathematical relationships. Those symbols are called variables.

eg:- 5m + 6m = 11m.

One can do arithmetic without doing algebra, but one cannot do algebra without doing arithmetic.

Rules for addition and subtraction is much the same for algebra as arithmetic.

Rules of Algebra

Rule 1: Addition and Subtraction

(a) a + b = a + b
(b) a – (– b) = a + b
(c) a + (–b) = a – b
(d) a – (+b) = a – b

Example

(1) a+ a + a + a + b + b + b + c + c = 4a + 3b + 2c
(2) 4a + (–6b) – a – (–2) = 3a – 6b + 2
(3) 10u + v + (–2v) –3u + (–5u) = 2u – v
(4) – 7 + 3q + (– 4p) –(– 5p) + 8q = 11q + p – 7
(5) a + 2b – 3b + 6 – 5a – b = −4a − 2b + 6
(6) –9p – 4p + 8p – p= –6p
(7) 14x – 5y – x + 6y – 5x + 2y = –6x
(8) 5a + 2b – 3c + a – 2b – 2c + 2a + b – 5c = 8a + b– 10c
(9) 5xy – 2xy + 3 yx – 5 xy = xy

Rule 2: Multiplication and Division

(a) a × b = ab

(b) –a × –b = ab

(c) a × –b = –ab

(d) –a × b = ab

(e) a ÷ b = a/b

(f) –a ÷ –b = $-a/-b = a/b$

(g) a ÷ –b = $a/-b = -a/b$

(h) –a ÷ b = $-a/b = -a/b$

Examples

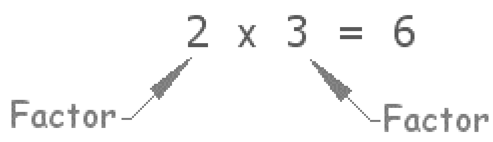
1. $\frac{1}{3} \times \frac{3}{5} = \frac{3}{15} = \frac{1}{5}$
2. 3 × −6 = −18
3. −8 ÷ −2 = 4.
4. −8 × −2 = 16
5. 8 ÷ −2 = −4
6. −8 × 2 = −16
7. −8 ÷ 2 = −4
8. $\frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$

9. $2\frac{1}{4} \times 3\frac{1}{6} = 7\frac{1}{8}$
10. $\frac{3}{8} \div \frac{1}{3} \times 1\frac{1}{5} = 1\frac{7}{20}$

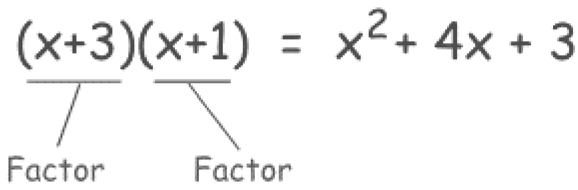
FACTORING IN ALGEBRA

A composite number is a number that can be written as the product of two positive integers other than 1 and the number itself. For example: 14 is a composite number because it can be written as 7 times 2. In this case, 7 and 2 are called factors of 14.

Numbers have factors:

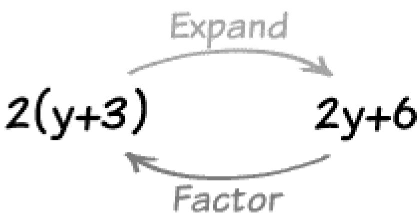


And expressions (like x^2+4x+3) also have factors:



Factoring (called "Factorising" in the UK) is the process of finding the factors. It is like "splitting" an expression into a multiplication of simpler expressions.

Factoring is also the opposite of Expanding:



Example:

Factor 2y + 6

Both 2y and 6 have a common factor of 2:

- 2y is 2 × y

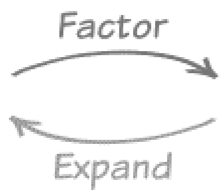
- 6 is 2 × 3

So you can factor the whole expression into:

$$2y + 6 = 2(y+3)$$

So 2y + 6 has been "factored into" 2 and y + 3

Here is a list of common "Identities" It is worth remembering these, as they can make factoring easier.



$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$

$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3+3a^2b+3ab^2+b^3 = (a+b)^3$$

$$a^3-3a^2b+3ab^2-b^3 = (a-b)^3$$

A prime number is a number greater than 1 which has only two positive factors: 1 and itself. For example, 11 is a prime number because its only positive factors are 1 and 11.

EXPONENTS

Exponents are shorthand for repeated multiplication of the same thing by itself. For instance, the shorthand for multiplying three copies of the number 5 is shown on the right-hand side of the "equals" sign in $(5)(5)(5) = 5^3$. The "exponent", being 3 in this example, stands for however many times the value is being multiplied. The thing that's being multiplied, being 5 in this example, is called the "base".

This process of using exponents is called "raising to a power", where the exponent is the “power”. The expression “5^3” is pronounced as “five, raised to the third power” or “five to the third”. There are two specially-named powers: “to the second power” is generally pronounced as “squared”, and “to the third power” is generally pronounced as “cubed”. So “5^3” is commonly pronounced as “five cubed”.

Thus, the exponent of a number says how many times to use the number in a multiplication.

In 8^2 the ‘2’says to use 8 twice in a multiplication,

so $8^2 = 8 \times 8 = 64$

In words: 8^2 could be called ‘8 to the power 2’or ‘8 to the second power’, or simply ‘8 squared’.

So in general:

a^n tells you to multiply **a** by itself, so there are **n** of those **a**'s:

$$a^n = \underbrace{a \times a \times \dots \times a}_{n}$$

Some more examples:

Example: $5^3 = 5 \times 5 \times 5 = 125$

- In words: 5^3 could be called "5 to the third power", "5 to the power 3" or simply "5 cubed"

Example: $2^4 = 2 \times 2 \times 2 \times 2 = 16$

- In words: 2^4 could be called "2 to the fourth power" or "2 to the power 4" or simply "2 to the 4th"

Exponents make it easier to write and use many multiplications

Example: 9^6 is easier to write and read than $9 \times 9 \times 9 \times 9 \times 9 \times 9$

A negative exponent means how many times to divide one by the number.

Example:

(a) $8 - 1 = 1 \div 8 = 0.125$

(b) $5 - 3 = 1 \div 5 \div 5 \div 5 = 0.008$

In General

$$a^{-n} = \frac{1}{a^n}$$

Rules of Exponents

Here are the Rules of Exponents

Rule name	Rule	Example
Product rules	$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^4 = 2^{3+4} = 128$
	$a^n \cdot b^n = (a \cdot b)^n$	$3^2 \cdot 4^2 = (3{\cdot}4)^2 = 144$
Quotient rules	$a^m / a^n = a^{m-n}$	$2^5 / 2^3 = 2^{5-3} = 4$
	$a^n / b^n = (a / b)^n$	$4^3 / 2^3 = (4/2)^3 = 8$
Power rules	$(b^n)^m = b^{n \cdot m}$	$(2^3)^2 = 2^{3 \cdot 2} = 64$
	$b^{n^m} = b^{(n^m)}$	$2^{3^2} = 2^{(3^2)} = 512$
	$^m\sqrt{(b^n)} = b^{n/m}$	$^2\sqrt{(2^6)} = 2^{6/2} = 8$
	$b^{1/n} = {}^n\sqrt{b}$	$8^{1/3} = {}^3\sqrt{8} = 2$
Negative exponents	$b^{-n} = 1 / b^n$	$2^{-3} = 1/2^3 = 0.125$
Zero rules	$b^0 = 1$	$5^0 = 1$
	$0^n = 0$, for $n>0$	$0^5 = 0$

One rules	$b^1 = b$	$5^1 = 5$
	$1^n = 1$	$1^5 = 1$

Examples

1. $x^a(x^b) = x^{a+b}$
2. $\frac{x^a}{x^b} = x^{a-b}$
3. $(x^a)^b = x^{ab}$
4. $(xy)^a = x^a y^a$
5. $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
6. $\frac{1}{x^a} = x^{-a}$
7. $\sqrt{x} = x^{1/2}$
8. $\sqrt[a]{x} = x^{1/a}$
9. $\sqrt[b]{x^a} = x^{a/b}$
10. $x^{-(a/b)} = \frac{1}{x^{a/b}}$
11. $x^2(x^3) = x^5$
12. $\frac{x^6}{x^3} = x^3$
13. $(x^4)^2 = x^8$
14. $(xy)^4 = x^4y^4$
15. $\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$
16. $\frac{x^3}{x^4} = x^{3-4} = x^{-1}$
17. $\sqrt{x} = x^{1/2}$
18. $\sqrt[3]{x} = x^{1/3}$
19. $x^{-2/3} = \frac{1}{x^{2/3}}$

20. $\frac{x^3}{x^{-4}} = x^{3-(-4)} = x^7$

21. $2^{3^2} = 2^{(3^2)} = 2^{(3\cdot3)} = 2^9 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 = 512$

22. $2^3 \cdot 2^4 = 2^{3+4} = 2^7 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 = 128$

23. $3^2 \cdot 4^2 = (3\cdot4)^2 = 12^2 = 12\cdot12 = 144$

24. $2^5 / 2^3 = 2^{5-3} = 2^2 = 2\cdot2 = 4$

25. $4^3 / 2^3 = (4/2)^3 = 2^3 = 2\cdot2\cdot2 = 8$

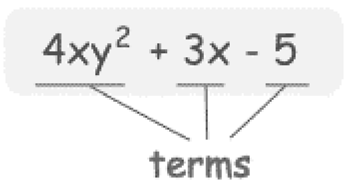
26. $(2^3)^2 = 2^{3\cdot2} = 2^6 = 2\cdot2\cdot2\cdot2\cdot2\cdot2 = 64$

27. $^2\sqrt{(2^6)} = 2^{6/2} = 2^3 = 2\cdot2\cdot2 = 8$

28. $2^{-3} = 1/2^3 = 1/(2\cdot2\cdot2) = 1/8 = 0.125$

Polynomials

Polynomial comes from poly- (meaning "many") and -nomial (in this case meaning "term, so it says "many terms".



A polynomial can have:
constants (like 3, -20, or ½)
variables (like x and y)
exponents (like the 2 in y2), but only 0, 1, 2, 3, ... etc are allowed
that can be combined using addition, subtraction, multiplication and division ...
... except ...
... not division by a variable (so something like 2/x is right out).
So a polynomial can have constants, variables and exponents, but never division by a variable.

These are polynomials:

- 3x
- x - 2
- $-6y^2 - (^7/_9)x$

- $3xyz + 3xy^2z - 0.1xz - 200y + 0.5$
- $512v^5 + 99w^5$
- 5

(Yes, even "5" is a polynomial, one term is allowed, and it can even be just a constant!)

And these are not polynomials

- $3xy^{-2}$ is not, because the exponent is "-2" (exponents can only be 0,1,2,...)
- 2/(x+2) is not, because dividing by a variable is not allowed
- 1/x is not either
- √x is not, because the exponent is "½"

There are special names for polynomials with 1, 2 or 3 terms:

$3xy^2$	$5x - 1$	$3x + 5y^2 - 3$
Monomial (1 term)	Binomial (2 terms)	Trinomial (3 terms)

Adding and Subtracting Polynomials

To add polynomials we simply add any like terms together. Like Termsare terms whose variables (and their exponents such as the 2 in x^2) are the same.In other words, terms that are "like" each other. Note that the coefficients (the numbers you multiply by, such as "5" in 5x) can be different.
Example of like terms:

$$7x \qquad x \qquad -2x \qquad \pi x$$

are all like terms because the variables are all x

Similarly

$$(1/3)xy^2 \qquad -2xy^2 \qquad 6xy^2 \qquad xy^2/2$$

are all like terms because the variables are all xy^2.

Example of addition

(1) $2x^2 + 6x + 5 + 3x^2 – 2x – 1$

You may re arrange with like terms written together

$= 2x^2 + 3x^2 + 6x + –2x + 5 – 1$

$= 5x^2 – 4x – 4$

(2) (2x2 + 6y + 3xy) + (3x2 - 5xy - x)+ (6xy + 5)

$= 5x^2 + 6y + 4xy - x + 5$

(3) $(3x^2 - 6x + xy) +(2x^3 - 5x^2 -3y) + (7x + 8y)$

$= 2x^3 - 2x^2 + x + xy + 5y$

(4) $(2x^2 - 4y + 7xy - 6y^2) − (-3x^2 + 5y - 4xy + y^2)$

$= 2x^2 − 4y + 7xy − 6y^2 + 3x^2 − 5y + 4xy − y^2$

$= 5x^2 - 9y + 11xy - 7y^2$

(5) $(3x^2 - 5x + 8y + 7xy + 2y^2) − (4x^2 + 2x - 3y + 7xy - 3y^2)$

$= 3x^2 - 5x + 8y + 7xy + 2y^2 − 4x2 − 2x+3y − 7xy + 3y^2$

$= 3x^2 − 4x^2 - 5x − 2x + 8y + 3y + 7xy − 7xy + 2y^2 + 3y^2$

$= − x^2 − 7x + 11y + 5y^2$

(6) If $P = 5x^4 - 2x^2 + 4x - 3$ and $Q = 5x^4 + 3x^3 - 4x + 3$, what is P – Q

$= 5x^4 – 2x^2 + 4x – 3 – (5x^4 + 3x3 – 4x + 3)$

$= 5x^4 – 2x^2 + 4x – 3 – 5x^4 – 3x^3 + 4x – 3$

$= 5x^4 - 5x^4 - 3x^3 - 2x^2 + 4x + 4x – 3 – 3$

$= –3x^3 – 2x^2 + 8x – 6$

(7) If $P = 4x^4 - 3x^3 + x^2 - 5x + 11$ and $Q = -3x^4 + 6x^3 - 8x^2 + 4x - 3$

what is 2P + Q?

$2P = 8x^4 - 6x^3 + 2x^2 - 10x + 22$

2P + Q

$= 8x^4 - 6x^3 + 2x^2 - 10x + 22 + (-3x^4 + 6x^3 - 8x^2 + 4x - 3)$

$= 8x^4 - 3x^4 - 6x^3 + 6x^3 + 2x^2 - 8x^2 - 10x + 4x + 22 - 3$

$= 5x^4 - 6x^2 - 6x + 19$

(8) If $P = 4x^4 - 3x^3 + x^2 - 5x + 11$ and $Q = -3x^4 + 6x^3 - 8x^2 + 4x - 3$, what is P - 2Q

$2Q = -6x^4 + 12x^3 - 16x^2 + 8x – 6$

So $P - 2Q$ will be

$= 4x^4 - 3x^3 + x^2 - 5x + 11 - (-6x^4 + 12x^3 - 16x^2 + 8x - 6)$

$= 4x^4 - 3x^3 + x^2 - 5x + 11 + 6x^4 - 12x^3 + 16x^2 - 8x + 6$ (change signs correctly!)

$= 4x^4 + 6x^4 - 3x^3 - 12x^3 + x^2 + 16x^2 - 5x - 8x + 11 + 6$

$= 10x^4 - 15x^3 + 17x^2 - 13x + 17$

(9) $(-4x^3 + x^2y - xy^2) + (3x^3 - xy^2 + 5x^2y) + (7xy^2 + 3y^3)$

$= -x^3 + 5xy^2 + 6x^2y + 3y^3$

$= -x^3 + 3y^3 + 6x^2y + 5xy^2$

(10) If $P = 3x^2 + xy - 5y^2$, $Q = 2x^2 - xy + 3y^2$ and $R = -6x^2 + 4xy - 7y^2$,

find $P + Q - R$

$P + Q – R = (3x^2 + xy - 5y^2) + (2x^2 - xy + 3y^2) - (-6x^2 + 4xy - 7y^2)$

$= 3x^2 + xy - 5y^2 + 2x^2 - xy + 3y^2 + 6x^2 - 4xy + 7y^2$

$= 3x^2 + 2x^2 + 6x^2 + xy - xy - 4xy - 5y^2 + 3y^2 + 7y^2$

$= 11x^2 - 4xy + 5y^2$

(11) $P = 3x^2 + xy - 5y^2$,

$Q = 2x^2 - xy + 3y^2$ and

$R = -6x^2 + 4xy - 7y^2$

Find $P - (Q + R)$?

$Q + R$

$= (2x^2 - xy + 3y^2) + (-6x^2 + 4xy - 7y^2)$

$= 2x^2 - xy + 3y^2 - 6x^2 + 4xy - 7y^2$

$= -4x^2 + 3xy - 4y^2$

So $P - (Q + R)$

$= 3x^2 + xy - 5y^2 - (-4x^2 + 3xy - 4y^2)$

$= 3x^2 + xy - 5y^2 + 4x^2 - 3xy + 4y^2$

$= 7x^2 - 2xy - y^2$

Multiplying Polynomials by Monomials

The Distributive Property can be used to multiply polynomials by monomials.

Below are two examples of polynomials being multiplied by monomials:

Example 1

$$2(4x - 2w + 3)$$

$$8x - 4w + 6$$

Example 2

$$x^2y(2x^2 - 3)$$

$$2x^4y - 3x^2y$$

Multiplying Polynomials

Example 1: Simplify (x + 3) (x + 2)

Remember to multiply each term in the first bracket with each term in the second bracket.

(See the sketch shown with the next example)

$(x + 3)(x + 2)$

$= (x + 3)(x) + (x + 3)(2)$

$= x(x) + 3(x) + x(2) + 3(2)$

$= x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

Example 2

$(3g – 3) (2g^2 + 4g – 4)$

$$(3g - 3)(2g^2 + 4g - 4)$$

$$6g^3 + 12g^2 - 12g - 6g^2 - 12g + 12$$

$$6g^3 + 6g^2 - 24g + 12$$

Question 1: Simplify $(x + 2y)(3x - 4y + 5)$

$= 3x^2 - 4xy + 5x + 6xy - 8y^2 + 10y$

$= 3x^2 + 2xy + 5x - 8y^2 + 10y$

Question 2: Simplify $(x + 3) (x + 2)$

$= x^2 + 2x + 3x + 6$

$= x^2 + 5x + 6$

Question 3: Simplify $(x - 4)(x - 3)$

$= x^2 - 3x - 4x + 12$

$= x^2 - 7x + 12$

Question 4: Simplify $(x - 3y)(x + y)$

$= x^2 + xy - 3xy - 3y^2$

$= x^2 - 2xy - 3y^2$

Question 5: Simplify$(3x + 2)(4x - 5)$

$= 3x(4x - 5) + 2(4x - 5)$

$= 12x^2 - 15x + 8x - 10$

$= 12x^2 - 7x - 10$

Question 6: Simplify$(x - 4)(3x - y + 3)$

$= x(3x - y + 3) - 4(3x - y + 3)$

$= 3x^2 - xy + 3x - 12x + 4y - 12$

$= 3x^2 - xy - 9x + 4y - 12$

Question 7: Simplify$(3x - 4y)$ and $(-2x + 5y - 6)$

$= 3x(-2x + 5y - 6) - 4y(-2x + 5y - 6)$

$= -6x^2 + 15xy - 18x + 8xy - 20y^2 + 24y$

$= -6x^2 + 23xy - 18x - 20y^2 + 24y$

Question 8: Simplify $(2y - 1)(2y + 1)(4y2 + 1)$

Step1 Find the product of the first two terms:

$(2y - 1)(2y + 1)$

$= 2y(2y + 1) - 1(2y + 1)$

$= 4y^2 + 2y - 2y - 1$

$= 4y^2 - 1$

Step 2: Find the product of this with the third term:

$(4y^2 - 1)(4y^2 + 1)$

$= 4y^2(4y2 + 1) - 1(4y^2 + 1)$

$= 16y^4 + 4y^2 - 4y^2 - 1$

$= 16y^4 - 1$

Question 9: Simplify $(3x - 2)(2x + 3)(2x - 3)$

The product $(3x - 2)(2x + 3)(2x - 3)$ can be bracketed in two different ways:

$[(3x - 2)(2x + 3)](2x - 3)$ or

$(3x – 2)[(2x + 3)(2x – 3)]$

The answers will be equal because multiplication is associative, but the second is easier since we can use the difference of two squares: $(2x + 3)(2x – 3) = (2x)^2 – 3^2 = 4x^2 – 9$

So:

$(3x – 2)(2x + 3)(2x – 3)$

$= (3x – 2)[(2x + 3)(2x – 3)]$

$= (3x – 2)(4x^2 – 9)$

$= 12x^3 – 27x – 8x^2 + 18$

$= 12x^3 – 8x^2 – 27x + 18$

Question 10: Simplify $(3a – 4b + 1)(2a + 5b – 3)$

Multiply each term in one polynomial by each term in the other polynomial:

$(3a – 4b + 1)(2a + 5b – 3)$

$= 3a(2a + 5b - 3) – 4b(2a + 5b – 3) + 1(2a + 5b – 3)$

$= 6a^2 + 15ab – 9a – 8ab – 20b^2 + 12b + 2a + 5b – 3$

$= 6a^2 – 20b^2 + 7ab – 7a + 17b – 3$

FRACTIONS

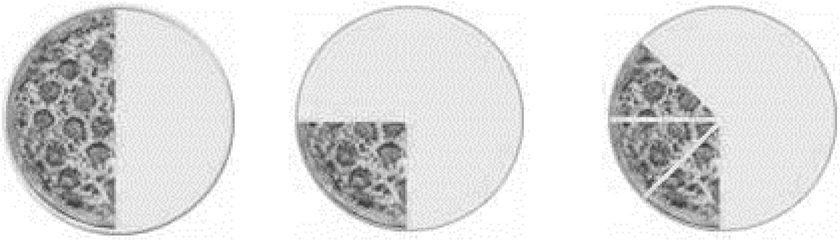
A fraction is a part of a whole. For instance the fraction $\frac{95}{100}$ is representing your mark in Maths exam. It says of the 100, you got 95. The bottom number in a fraction (the denominator) says how many parts the whole is divided into. The top number in a fraction (the numerator) says how many parts we have.

Thus a Fraction has two numbers:

$$\frac{Numerator}{Denominator}$$

The top number is the Numerator, it is the number of parts you have.
The bottom number is the Denominator, it is the number of parts the whole is divided into.

With the help of a pizza (or a pie diagram) we can easily explain the concept. Slice a pizza, and you will have fractions:

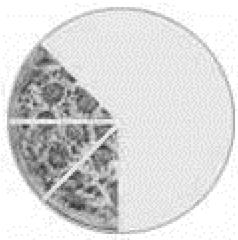


$^1/_2$ (One-Half) $^1/_4$ (One-Quarter) $^3/_8$ (Three-Eighths)

The top number tells how many slices you have
The bottom number tells how many slices the pizza was cut into.

Here are some other important fraction terms.

1. Proper fraction: A Proper Fraction has a top numberless than its bottom number, that is, numerator is less than the denominator. $Eg. \frac{95}{100}$



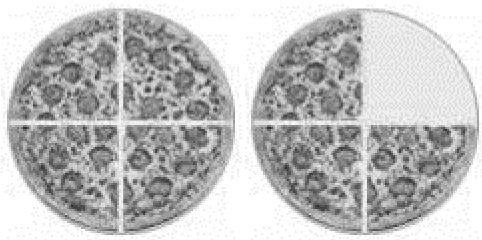
3/8

(Three-Eighths)

Examples

3/8 1/4 14/15 4/5

2. Improper fraction: numerator is greater than or equal to denominator. $Eg. \frac{10}{3}$



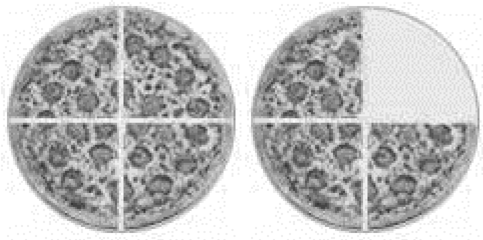
$$\frac{7}{4}$$

(seven-fourths or seven-quarters)

Examples

$$\frac{3}{2} \quad \frac{7}{3} \quad \frac{16}{15} \quad \frac{15}{15} \quad \frac{99}{5}$$

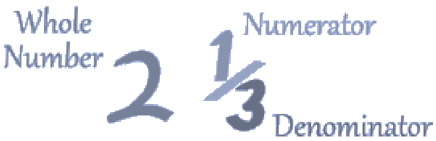
3. Mixed Fractions: (Also called "Mixed Numbers") :A Mixed Fraction is a whole number and a proper fraction combined. $Eg.\ 1\frac{3}{4}$



$$1\ \frac{3}{4}$$

(one and three-quarters)

We can give names to every part of a mixed fraction:



So we can summarise the three types of fractions like this:

Proper Fractions: The numerator is less than the denominator

Examples: $^1/_3$, $^3/_4$, $^2/_7$

Improper Fractions: The numerator is greater than (or equal to) the denominator

Examples: $^4/_3$, $^{11}/_4$, $^7/_7$

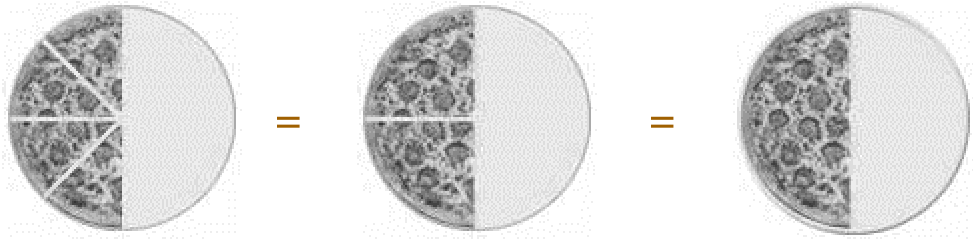
Mixed Fractions: A whole number and proper fraction together

Examples: $1\ ^1/_3$, $2\ ^1/_4$, $16\ ^2/_5$

Equivalent Fractions

Some fractions may look different, but are really the same, for example:

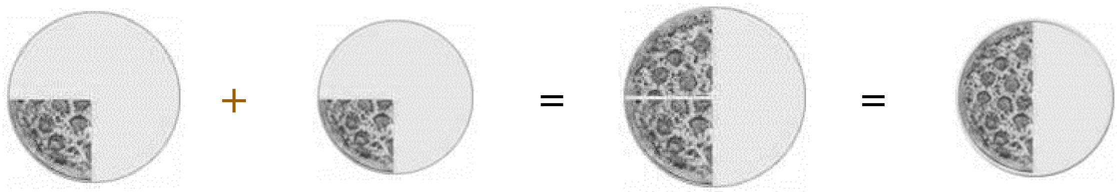
$^4/_8$	=	$^2/_4$	=	$^1/_2$
(Four-Eighths)		Two-Quarters)		(One-Half)



Adding Fractions

You can add fractions easily if the bottom number (the denominator) is the same:

$^1/_4$	+	$^1/_4$	=	$^2/_4$	=	$^1/_2$
(One-Quarter)		(One-Quarter)		(Two-Quarters)		(One-Half)



Another example:

$$\frac{5}{8}+\frac{1}{8}=\frac{6}{8}=\frac{3}{4}$$

Adding Fractions with Different Denominators

If we want to add $\frac{3}{8}+\frac{1}{4}$, the denominators are different. We must somehow make the denominators the same.

In this case it is easy, because we know that 1/4 is the same as 2/8 :

So we can write $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$

But it can be harder to make the denominators the same, so you may need to use one of these methods (a) Least Common Denominator (b) Common Denominator

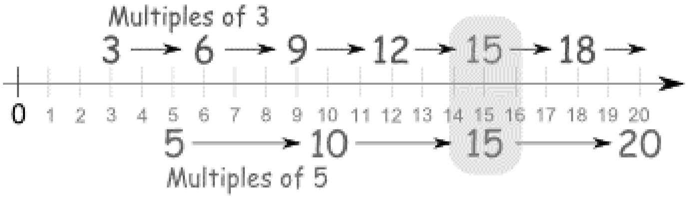
(a) Least Common Denominator

Least Common Denominatoris the Least Common Multiple of the denominators. Least Common Multiple is the smallest positive number that is a multiple of two or more numbers.

Let's start with an Example.

Least Common Multiple of 3 and 5.

First, list the Multiples of each number.



As you can see in the figure multiples of 3 are 3 (because 3×1=3), 6 (because 3×2=6), 9 (because 3×3=9), 12, 15, 18, 21, 24, 27 and so on.

Similarly, the multiples of 5are 5 (because 5×1=5), 10 (because 5×2=10), 15(because 5×3=15),20, 25, 30, 35 and so on.

Nowfrom the multiples of the two numbers find the first value that is the same for both the numbers. Here it is 15.

(*there may be other multiples which are the same. Here we consider only the lowest value which are the same for both. For example, consider the numbers* 4 and 5.The multiples of 4 are: 4,8,12,16,**20**,24,28,32,36,**40**,44,...The multiples of 5 are: 5,10,15,**20**,25,30,35,**40**,45,50,...Here we see that there are two common multiples, 20 and 40. The "Least Common Multiple" is simply the smallest of the common multiples, so here it is 20)

15 is a common multiple of 3 and 5, and is the smallest (least) common multiple. So we can say that the Least Common Multiple of 3 and 5 is 15.

So, coming back to our problem, we were discussing about adding fractions with different denominators and of the two methods for that we were dealing with the method called Least Common Denominator.

So to add $\frac{1}{3}$ and $\frac{1}{6}$ by the "Least Common Multiple" method, first list all the multiples of the two denominators, 3 and 6.

List the multiples of 3:→ 3, **6,** 9, 12, 15, 18, 21, ...

List the multiples 6: →**6**, 12, 18, 24, ...

Now find the common multiple with the lowest value. Here it is 6.

So 6 is the Least Common Denominator.

So now should think of ways to make the denominator the equal to 6 for both the fractions. For 1/6 there is no problem, as its denominator is already equal to 6. For 1/3, we have to convert the denominator to 6. We can do this if we multiply top and bottom of 1/3 by 2. So we get 2/6.

So let us rewrite the problem as $\frac{2}{6} + \frac{1}{6}$

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$$

We may do a simplification and write $\frac{3}{6}$ as $\frac{1}{2}$.

<u>Question 1</u>

Compute 1/6 + 7/15

The Denominators are 6 and 15:

multiples of 6:→ 6, 12, 18, 24, 30, 36, ...

multiples 15:→ 15, 30, 45, 60, ...

So the Least Common Multiple of 6 and 15 is 30.

Now let's try to make the denominators the same, that is equal to 30.

If we multiply 1/6 by 5 and 7/15 by 2, we can do this.

1/6 × 5 = 5/30

7/15 × 2 = 14/30

Now we can do the addition by adding the top numbers:

5/30 + 14/30 = 19/30

Question 2: Compute $\frac{3}{8} + \frac{5}{12}$

List the multiples of the denominators 8 and 12

multiples of 8:→ 8, 16, 24, 32, 40, ...

multiples 12:→ 12, 24, 36, 48, ...

The Least Common Multiple is 24

Let us try to make the denominators the same.When we multiply 8 × 3 you get 24, and when you multiply 12 × 2 you also get 24. So, let's use that:

$$\frac{9}{24}+\frac{10}{24}=\frac{19}{24}$$

Question 3: What is the least common denominator for the fractions $\frac{3}{5},\frac{4}{9}\ and\ \frac{2}{3}$

(a) 12 (b) 27 (c) 45 (d) 135

Correct answer is option (c), 45.

The least common denominator is equal to the least common multiple of the denominators: 5, 9 and 3

The multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...

The multiples of 9 are: 9, 18, 27, 36, 45, 54, ...

The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, ...

The least common multiple of 5, 9 and 3, therefore, is 45and the least common denominator of 3/5, 4/9 and 2/3 is also 45.

Question 4: What is the least common denominator for the fractions $\frac{5}{16},\frac{7}{12}\ and\ \frac{8}{9}$

(a) 72 (b) 144 (c) 288 (d) 1728

Correct answer is option (b), 144.

The least common denominator is equal to the least common multiple of the denominators: 16, 12 and 9

The multiples of 16 are: 16, 32, 48, 64, 80, 96, 112, 128, 144, 160, ...

The multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, ...

The multiples of 9 are: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117, 126, 135, 144, 153 ...

The least common multiple of 16, 12 and 9, therefore, is 144,and the least common denominator of $\frac{5}{16},\frac{7}{12}\ and\ \frac{8}{9}$ is also 144.

Question 5: What is the least common denominator for the fractions $\frac{2}{3},\frac{5}{9}\ and\ \frac{7}{12}$

(a) 24 (b) 36 (c) 48 (d) 72

Correct answer is option (b), 36.

The least common denominator is equal to the least common multiple of the denominators: 3, 9 and 12

The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, ...

The multiples of 9 are: 9, 18, 27, 36, 45,...

The multiples of 12 are: 12, 24, 36, 48, ...

The least common multiple of 3, 9 and 12, therefore, is 36,

So, the least common denominator of 2/3,5/9 and 7/12is also 36.

Question 6: What is the least common denominator for the fractions$\frac{7}{8}, \frac{11}{12}\, and\, \frac{13}{18}$

(a) 72 (b) 84 (c) 96 (d) 120

Correct answer is option (a), 76.

The least common denominator is equal to the least common multiple of the denominators: 8, 12 and 18

The multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, ...

The multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, ...

The multiples of 18 are: 18, 36, 54, 72, 90,...

The least common multiple of 8, 12 and 18, therefore, is 72,

So, the least common denominator of 7/8, 11/12 and 13/18 is also 72.

Question 7: What is$the\ value\ of\ \frac{1}{3}+\frac{5}{9}$

(a) $\frac{6}{12}$ (b) $\frac{1}{2}$ (c) $\frac{4}{9}$ (d) $\frac{8}{9}$

Correct answer is option (d), $\frac{8}{9}$

To add or subtract two fractions, you need to find the least common denominator.

The least common denominator is equal to the least common multiple of the denominators: 3 and 9

The multiples of 3 are: 3, 6, 9, 12, ...

The multiples of 9 are 9, 18, ...

The least common multiple of 3 and 9, therefore, is 9,

and the least common denominator of $\frac{1}{3}+\frac{5}{9}$is also9.

Since we already have 9 as denominator in $\frac{5}{9}$, we need not convertthis.

To convert the denominator of $\frac{1}{3}$ to 9, let us multiply $\frac{1}{3} \times 3 = \frac{3}{9}$

So we can write $\frac{5}{9} + \frac{3}{9} = \frac{8}{9}$

<u>Question 8:</u> What is $\frac{11}{12} - \frac{2}{9}$

(a) $\frac{25}{36}$ (b) $\frac{9}{3}$ (c) 3 (d) $1\frac{5}{36}$

Correct answer is option (a), $\frac{25}{36}$

To add or subtract two fractions, you need to find the least common denominator. The least common denominator is equal to the least common multiple of the denominators: 12 and 9
The multiples of 12 are: 12, 24, **36**, 48, ...
The multiples of 9 are: 9, 18, 27, **36**, 45, ...
The least common multiple of 12 and 9, therefore, is **36**.
So the least common denominator of $\frac{11}{12} - \frac{2}{9}$ is also 36.
Change the two fractions to equivalent fractions with denominator 36:

$$\frac{11}{12} = \frac{11 \times 3}{12 \times 3} = \frac{33}{36}$$

$$\frac{2}{9} = \frac{2 \times 4}{9 \times 4} = \frac{8}{36}$$

And we get

$$\frac{11}{12} - \frac{2}{9} = \frac{33}{36} - \frac{8}{36} = \frac{33 - 8}{36} = \frac{25}{36}$$

<u>Question 9:</u> What is $\frac{1}{6} + \frac{3}{4}$

(a) $\frac{4}{10}$ (b) $\frac{11}{12}$ (c) $\frac{2}{5}$ (d) $\frac{11}{24}$

Correct answer is option (b), $\frac{11}{12}$

The denominator (the bottom numbers) are different, so we need to make them the same before we can add or subtract.

Let us find the least common denominator, which is equal to the least common multiple of the denominators: 6 and 4

The multiples of 6 are: 6, **12**, 18, ...

The multiples of 4 are: 4, 8, **12**, 16, ...

The least common multiple of 6 and 4, therefore, is **12**,

NOW we can change the fractions so they have denominator 12 (remember to multiply the top and bottom):

$$\frac{1}{6} = \frac{1 \times 2}{6 \times 2} = \frac{2}{12}$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

So

$$\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{2+9}{12} = \frac{11}{12}$$

<u>Question 10:</u> What is $\frac{2}{15} + \frac{3}{10}$

(a) $\frac{13}{30}$ (b) $\frac{13}{60}$ (c) $\frac{5}{25}$ (d) $\frac{1}{5}$

Correct answer is option (a), $\frac{13}{30}$

To add or subtract two fractions, you need a common denominator.

The multiples of 15 are: 15, **30**, 45, ...

The multiples of 10 are: 10, 20, **30**, 40, ...

The least common multiple of 15 and 10 is **30**,

and the least common denominator of $\frac{2}{15} + \frac{3}{10}$ is also 30

Change the two fractions to equivalent fractions with denominator 30:

$$\frac{2}{15} = \frac{2 \times 2}{15 \times 2} = \frac{4}{30}$$

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$$

So

$$\frac{2}{15} + \frac{3}{10} = \frac{4}{30} + \frac{9}{30} = \frac{4+9}{30} = \frac{13}{30}$$

<u>Question 11:</u> What is $\frac{7}{8} - \frac{5}{6}$

(a) $\frac{2}{2}$ (b) 1 (c) $\frac{1}{24}$ (d) $1\frac{17}{24}$

Correct answer is option (c), $\frac{1}{24}$

The denominators are different, so we need to find the least common denominator.

The least common denominator is equal to the least common multiple of the denominators: 8

and 6

The multiples of 8 are: 8, 16, **24**, 32, ...

The multiples of 6 are: 6, 12, 18, **24**, 30, ...

The least common multiple of 8 and 6, therefore, is **24**,

and the least common denominator of $\frac{7}{8}-\frac{5}{6}$is also 24

Change the two fractions to equivalent fractions with denominator 24:

$$\frac{7}{8}=\frac{7\times 3}{8\times 3}=\frac{21}{24}$$

$$\frac{5}{6}=\frac{5\times 4}{6\times 4}=\frac{20}{24}$$

So

$$\frac{7}{8}-\frac{5}{6}=\frac{21}{24}-\frac{20}{24}=\frac{21-20}{24}=\frac{1}{24}$$

Question 12: What is$\frac{5}{6}-\frac{4}{9}$

(a) $\frac{1}{3}$ (b) $\frac{1}{15}$ (c) $1\frac{5}{18}$ (d) $\frac{7}{18}$

Correct answer is option (d), $\frac{7}{18}$

The denominators (bottom numbers) should be the same, so we need to find the least common denominator.
The least common denominator is equal to the least common multiple of the denominators: 6 and 9
The multiples of 6 are: 6, 12, **18**, 24, ...
The multiples of 9 are: 9, **18**, 27, ...
The least common multiple of 6 and 9, therefore, is **18**,
and the least common denominator of $\frac{5}{6}-\frac{4}{9}$is also 18.
Change the two fractions to equivalent fractions with denominator 18:

$$\frac{5}{6}=\frac{5\times 3}{6\times 3}=\frac{15}{18}$$

$$\frac{4}{9}=\frac{4\times 2}{9\times 2}=\frac{8}{18}$$

So

$$\frac{5}{6}-\frac{4}{9}=\frac{15}{18}-\frac{8}{18}=\frac{15-8}{18}=\frac{7}{18}$$

(b) Common Denominator

Common Denominatormethod to solve the problems is as follows.

To add $\frac{1}{3}$ and $\frac{1}{6}$, we need to make the denominators the same. For this multiply 1/3 with the denominator of 1/6, that is 6. Remember to multiply both numerator and denominator. So we get 6/18. Do the same for 1/6, that is multiply 1/6 by 3. So we get 3/18. Now we can write

$$\frac{6}{18}+\frac{3}{18}$$

Now we can easily add since the denominators are the same.

$$\frac{6}{18}+\frac{3}{18}=\frac{9}{18}=\frac{1}{2}$$

To present in general terms let us write the problem using alphabets.

$$\frac{a}{b}+\frac{c}{d}$$

1. Multiply each fraction by the denominator of the other.

$$\frac{a\times d}{b\times d}+\frac{c\times b}{d\times b}$$

Re arranging the denominator for better understanding we get

$$\frac{a\times d}{b\times d}+\frac{c\times b}{b\times d}$$

2. As two fractions now have the same denominator, we can add them

$$\frac{a\times d+c\times b}{b\times d}$$

So the formula is

$$\frac{a}{b}+\frac{c}{d}=\frac{a\times d+c\times b}{b\times d}$$

Question 1: Find $\frac{2}{3}+\frac{4}{5}$

The general form is $\frac{a}{b}+\frac{c}{d}$

So a = 2, b = 3, c = 4 and d = 5.)

Substituting in the formula $\frac{a\times d+c\times b}{b\times d}$ we get $\frac{2\times 5+3\times 4}{3\times 5}=\frac{10+12}{15}=\frac{22}{15}$

We may also solve the problem directly without using the formula as

$$\frac{2}{3}+\frac{4}{5}=\frac{2\times 5+3\times 4}{3\times 5}=\frac{10+12}{15}=\frac{22}{15}$$

Question 2: What is $\frac{1}{3}+\frac{2}{5}$

(a) $\frac{3}{8}$ (b) $\frac{11}{15}$ (c) $\frac{13}{15}$ (d) $\frac{16}{15}$

Correct answer is (b), $\frac{11}{15}$

$$\frac{1}{3}+\frac{2}{5}=\frac{1\times 5+3\times 2}{3\times 5}=\frac{5+6}{15}=\frac{11}{15}$$

If we use the formula to solve,

Substitute

a = 1, b = 3, c = 2 and d = 5

$$=\frac{1\times 5+3\times 2}{3\times 5}=\frac{5+6}{15}=\frac{11}{15}$$

Question 3: What is $\frac{1}{3}+\frac{2}{5}$

(a) $\frac{3}{8}$ (b) $\frac{11}{15}$ (c) $\frac{13}{15}$ (d) $\frac{16}{15}$

The correct answer is option (b), that is, $\frac{11}{15}$

This is how we workout.

The general form is $\frac{a}{b}+\frac{c}{d}$

So a = 1, b = 3, c = 2 and d = 5)

Substituting in the formula $\frac{a\times d+c\times b}{b\times d}$ we get

$$\frac{1}{3}+\frac{2}{5}=\frac{1\times 5+3\times 2}{3\times 5}=\frac{5+6}{15}=\frac{11}{15}$$

Question 4: What is $\frac{2}{7}+\frac{3}{5}$

(a) $\frac{31}{35}$ (b) $\frac{6}{7}$ (c) $\frac{29}{35}$ (d) $\frac{5}{12}$

The correct answer is option (a), that is, $\frac{31}{35}$

This is how we workout.

The general form is $\frac{a}{b}+\frac{c}{d}$

So a = 2, b = 7, c = 3 and d = 5)

Substituting in the formula $\frac{a\times d+c\times b}{b\times d}$ we get

$$\frac{2}{7}+\frac{3}{5}=\frac{2\times 5+7\times 3}{7\times 5}=\frac{10+21}{35}=\frac{31}{35}$$

Question 5: What is $\frac{5}{6}+\frac{2}{5}$

(a) $\frac{7}{11}$ (b) $\frac{29}{30}$ (c) $\frac{37}{30}$ (d) $\frac{4}{30}$

The correct answer is option (c), that is, $\frac{37}{30}$

This is how we workout.

The general form is $\frac{a}{b}+\frac{c}{d}$

So a = 6, b = 6, c = 2 and d = 5)

Substituting in the formula $\frac{a\times d+c\times b}{b\times d}$we get

$$\frac{5}{6}+\frac{2}{5}=\frac{5\times 5+6\times 2}{6\times 5}=\frac{25+12}{30}=\frac{37}{30}$$

Question 6: What is $\frac{3}{8}+\frac{2}{9}$

(a) $\frac{5}{17}$ (b) $\frac{41}{72}$ (c) $\frac{7}{12}$ (d) $\frac{43}{72}$

The correct answer is option (d), that is, $\frac{43}{72}$

This is how we workout.

The general form is $\frac{a}{b}+\frac{c}{d}$

So a = 3, b = 8, c = 2 and d = 9)

Substituting in the formula $\frac{a\times d+c\times b}{b\times d}$we get

$$\frac{3}{8}+\frac{2}{9}=\frac{3\times 9+8\times 2}{8\times 9}=\frac{27+16}{72}=\frac{43}{72}$$

Multiplying Fractions

There are 3 simple steps to multiply fractions

1. Multiply the top numbers (the numerators).
2. Multiply the bottom numbers (the denominators).
3. Simplify the fraction if needed.

Example:

$$\frac{1}{2}\times\frac{2}{5}$$

$$\frac{1}{2}\times\frac{2}{5}=\frac{1\times 2}{2\times 5}=\frac{2}{10}$$

Simplify the fraction:

$$\frac{2}{10} = \frac{1}{5}$$

Question 1: Multiply $\frac{1}{3}$ by $\frac{9}{16}$

$$\frac{1}{3} \times \frac{9}{16} = \frac{1 \times 9}{3 \times 16} = \frac{9}{48}$$

Simplify the fraction: $\frac{9}{48} = \frac{3}{16}$

Question 2: Multiply $\frac{2}{3}$ and 5

Here we are multiplying fractions and whole numbers. Make the whole number a fraction, by putting it over 1. So 5 may be written as $\frac{5}{1}$. Note that 5 and $\frac{5}{1}$ are the same.

So we can write the problem as

$$\frac{2}{3} \times \frac{5}{1}$$

$$\frac{2}{3} \times \frac{5}{1} = \frac{2 \times 5}{3 \times 1} = \frac{10}{3}$$

<u>Question 2</u>

$$3 \times \frac{2}{9}$$

Here also we are multiplying fractions and whole numbers. Instead of making the whole number a fraction as we did in the previous problem, we can just think of the whole number as being a "top" number and do the multiplication directly.

$$3 \times \frac{2}{9} = \frac{3 \times 2}{9} = \frac{6}{9}$$

We may simplify as

$$\frac{6}{9} = \frac{2}{3}$$

Multiplication and Division of Mixed Fractions

A Mixed Fraction is a whole number and a proper fraction combined.That is why it is called a "mixed" fraction (or mixed number)

Example of mixed fraction: $1\frac{3}{4}$. Here the whole number 1 is combined with a proper fraction.

Example

Find $1\frac{3}{8} \times 3$

First, convert the mixed fraction $1\frac{3}{8}$ to an improper fraction $\frac{1\times8+3}{8} = \frac{11}{8}$

Now multiply this by 3:

$$\frac{11}{8} \times 3 = \frac{33}{8}$$

You may convert the final answer back to mixed fraction just because the question was given as a mixed fraction. $\frac{33}{8} = 4\frac{1}{8}$

<u>Question 1:</u> Compute $1\frac{1}{2} \times 2\frac{1}{5}$

Convert to Improper Fractions

Multiply the Fractions

You may convert the result back to Mixed Fractions.

$$\frac{2 \times 1 + 1}{2} \times \frac{5 \times 2 + 1}{5} = \frac{3}{2} \times \frac{11}{5} = \frac{3 \times 11}{2 \times 5} = \frac{33}{10}$$

You may convert the result $\frac{33}{10}$ back to Mixed Fractions as $3\frac{3}{10}$

<u>Question 2:</u> Compute $3\frac{1}{4} \times 3\frac{1}{3}$

Convert to Improper Fractions

Multiply the Fractions

You may convert the result back to Mixed Fractions.

$$\frac{4 \times 3 + 1}{4} \times \frac{3 \times 3 + 1}{3} = \frac{13}{4} \times \frac{10}{3} = \frac{13 \times 10}{4 \times 3} = \frac{130}{12}$$

You may convert the result $\frac{130}{12}$ back to Mixed Fractions $10\frac{10}{2} = 10\frac{5}{6}$

<u>Question 3:</u> Compute $-1\frac{5}{9} \times -2\frac{1}{7}$

Convert to Improper Fractions, retaining the negative as such.

$$\frac{9 \times 1 + 5}{9} \times \frac{7 \times 2 + 1}{7} = \frac{-14}{9} \times \frac{-15}{7}$$

Multiply the Fractions (negative multiplied by negative gives positive)

$$= \frac{14 \times 15}{9 \times 7} = \frac{210}{63}$$

We may simplify further

$$\frac{210}{63} = \frac{30}{9} = \frac{10}{3} = 3\frac{1}{3}$$

Question 4: What is $2\frac{3}{4} \times 3$

(a) $6\frac{3}{4}$ (b) $6\frac{9}{12}$ (c) $7\frac{1}{4}$ (d) $8\frac{1}{4}$

The correct answer is (d), $8\frac{1}{4}$

First change $2\frac{3}{4}$ to an improper fraction.

$$2\frac{3}{4} = \frac{11}{4}$$

Rewriting we get

$$\frac{11}{4} \times 3 = \frac{33}{4}$$

Convert $\frac{33}{4}$ to a mixed fraction.

4 divides into 33, 8 times with remainder 1

Therefore $\frac{33}{4} = 8\frac{1}{4}$

Question 5: What is $2\frac{1}{2} \times 1\frac{3}{5}$

(a) $2\frac{3}{10}$ (b) $2\frac{4}{7}$ (c) 4 (d) $3\frac{3}{10}$

The correct answer is (c), 4.

This is how we worked out.

First change both mixed fractions to improper fractions:

$$2\frac{1}{2} = \frac{5}{2}$$

$$1\frac{3}{5} = \frac{8}{5}$$

Multiply the fractions and simplify

$$\frac{5}{2} \times \frac{8}{5} = \frac{40}{10} = 4$$

Question 6: What is $2\frac{1}{8} \times 1\frac{7}{9}$

(a) $1\frac{35}{72}$ (b) $3\frac{7}{9}$ (c) $5\frac{4}{6}$ (d) $8\frac{6}{10}$

The correct answer is (b), $3\frac{7}{9}$

This is how we worked out.

First change both mixed fractions to improper fractions:

$$2\frac{1}{8} = \frac{17}{8}$$

$$1\frac{7}{9} = \frac{16}{9}$$

Multiply the fractions and simplify

$$\frac{17}{8} \times \frac{16}{9} = \frac{272}{72}$$

272 and 72 have a common factor 8, so divide top and bottom by 8:

$$\frac{272}{72} = \frac{34}{9}$$

9 divides into 34, 3 times with remainder 7

$$\frac{34}{9} = 3\frac{7}{9}$$

Question 7: What is $3\frac{2}{3} \times 2\frac{7}{10}$

(a) $6\frac{7}{15}$ (b) $8\frac{9}{10}$ (c) $9\frac{7}{10}$ (d) $9\frac{9}{10}$

The correct answer is (d), $9\frac{9}{10}$

This is how we worked out.

First change both mixed fractions to improper fractions:

$$3\frac{2}{3} = \frac{11}{3}$$

$$2\frac{7}{10} = \frac{27}{10}$$

Multiply the fractions and simplify

$$\frac{11}{3} \times \frac{27}{10} = \frac{297}{30}$$

297 and 30 have a common factor 3, so divide top and bottom by 3

$$\frac{297}{30} = \frac{99}{10}$$

10 divides into 99, 9 times with remainder 9

$$\frac{99}{10} = 9\frac{9}{10}$$

Question 8: What is $-2\frac{2}{3} \times -1\frac{7}{12}$

(a) $2\frac{7}{18}$ (b) $-2\frac{7}{18}$ (c) $4\frac{2}{9}$ (d) $-4\frac{2}{9}$

The correct answer is (c), $4\frac{2}{9}$

This is how we worked out.

First change both mixed fractions to improper fractions, keeping negative as such:

$$-2\frac{2}{3} = -\frac{8}{3}$$

$$-1\frac{7}{12} = -\frac{19}{12}$$

Then multiply the Improper Fractions (Note: negative times negative gives positive)

$$-\frac{8}{3} \times -\frac{19}{12} = \frac{152}{36}$$

152 and 36 have a common factor 4, so divide top and bottom by 4:

$$\frac{152}{36} = \frac{38}{9}$$

9 divides into 38, 4 times with remainder 2

$$\frac{38}{9} = 4\frac{2}{9}$$

Question 9: What is $-2\frac{3}{5} \times 1\frac{1}{4}$

(a) $-2\frac{7}{18}$ (b) $-3\frac{1}{4}$ (c) $-4\frac{2}{9}$ (d) $3\frac{1}{4}$

The correct answer is (b), $-3\frac{1}{4}$

This is how we worked out.

First change $-2\frac{3}{5}$mixed fractions to improper fractions, keeping negative as such:

$$-2\frac{3}{5} = \frac{-13}{5}$$

Now change $1\frac{1}{4}$mixed fractions to improper fractions

$$1\frac{1}{4} = \frac{5}{4}$$

Then multiply the Improper Fractions and simplify (Note: negative times positive gives negative)

$$\frac{-13}{5} \times \frac{5}{4} = \frac{-65}{20} = \frac{-13}{4} = -3\frac{1}{4}$$

<u>Question 10: </u>What is $3\frac{2}{3} \div \frac{5}{9}$

(a) $6\frac{3}{5}$ (b) $-2\frac{7}{18}$ (c) $4\frac{2}{9}$ (d) $-4\frac{2}{9}$

The correct answer is (a), $6\frac{3}{5}$

This is how we worked out.

First change the mixed fraction to an improper fraction:

$$3\frac{2}{3} = \frac{11}{3}$$

Turn the second fraction upside down and multiply:

$$\frac{11}{3} \times \frac{9}{5} = \frac{99}{15}$$

99 and 15 have a common factor 3, so divide top and bottom by 3:

$$\frac{99}{15} = \frac{33}{5}$$

Convert to mixed fraction: 5 divides into 33, 6 times with remainder 3

$$\frac{33}{5} = 6\frac{3}{5}$$

<u>Question 11: </u>What is $4\frac{1}{2} \div 3\frac{3}{5}$

(a) $6\frac{3}{5}$ (b) $-4\frac{2}{9}$ (c) $4\frac{2}{9}$ (d) $1\frac{1}{4}$

The correct answer is (d), $1\frac{1}{4}$

This is how we worked out.

First change the mixed fraction to an improper fraction:

$$4\frac{1}{2} = \frac{9}{2}$$

$$3\frac{3}{5} = \frac{18}{5}$$

We can re write as.

$$4\frac{1}{2} \div 3\frac{3}{5} = \frac{9}{2} \div \frac{18}{5}$$

Turn the second fraction upside down and multiply:

$$\frac{9}{2} \div \frac{18}{5} = \frac{9}{2} \times \frac{5}{18} = \frac{45}{36}$$

45 and 36 have a common factor 9, so divide top and bottom by 9:

$$\frac{45}{36} = \frac{5}{4}$$

Convert to mixed fraction: 4 divides into 5, once with remainder 1

$$\frac{5}{4} = 1\frac{1}{4}$$

<u>Question 12:</u> What is $2\frac{1}{3} \div 1\frac{3}{4}$

(a) $7\frac{3}{5}$ (b) $4\frac{2}{9}$ (c) $1\frac{1}{3}$ (d) $1\frac{1}{4}$

The correct answer is (c), $1\frac{1}{3}$

This is how we worked out.

First change the mixed fraction to an improper fraction:

$$2\frac{1}{3} = \frac{7}{3}$$

$$1\frac{3}{4} = \frac{7}{4}$$

We can re write as.

$$2\frac{1}{3} \div 1\frac{3}{4} = \frac{7}{3} \div \frac{7}{4}$$

Turn the second fraction upside down and multiply:

$$\frac{7}{3} \div \frac{7}{4} = \frac{7}{3} \times \frac{4}{7} = \frac{28}{21}$$

28 and 21 have a common factor 7, so divide top and bottom by 7

$$\frac{28}{21} = \frac{4}{3}$$

Convert to mixed fraction: 3 divides into 4 once, with remainder 1

$$\frac{4}{3} = 1\frac{1}{3}$$

Question 13: What is $2\frac{7}{9} \div 3\frac{3}{4}$

(a) $\frac{20}{27}$ (b) $4\frac{2}{9}$ (c) $1\frac{1}{3}$ (d) $1\frac{1}{4}$

The correct answer is (a), $\frac{20}{27}$

This is how we worked out.

First change the mixed fraction to an improper fraction:

$$2\frac{7}{9} = \frac{25}{9}$$

$$3\frac{3}{4} = \frac{15}{4}$$

We can re write as.

$$2\frac{7}{9} \div 3\frac{3}{4} = \frac{25}{9} \div \frac{15}{4}$$

Turn the second fraction upside down and multiply:

$$\frac{25}{9} \times \frac{4}{15} = \frac{100}{135}$$

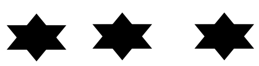
100 and 135 have a common factor 5, so divide top and bottom by 5:

$$\frac{100}{135} = \frac{20}{27}$$

Ratios

A ratio compares values.

A ratio says how much of one thing there is compared to another thing.



The above diagram may be described as 3:1, which implies, there are 3 filled squares to 1 empty square.

Ratios can be shown in different ways:

Using the ":" to separate the values: 3:1

Instead of the ":" we can use the word "to": 3 to 1

Or write it like a fraction: $\frac{3}{1}$

Question 1

A class of 32 students has 12 girls. What is the ratio of girls to boys.

(a) 3:5 (b) 5:3 (c) 3:8 (d) 8:3

The correct answer is (a), 3:5.

This is how we work out.

The number of boys in the class = 32 - 12 = 20

So the ratio of girls to boys is 12:20

Simplify by dividing by 4:

12 : 20 = 12/4 : 20/4 = 3 : 5

So the ratio of girls to boys is 3:5

Question 2

Raju can run a mile in 5 minutes and 50 seconds. Ramu can run a mile in 6 minutes and 40 seconds. What is the ratio of Raju's time to Ramu's time?

(a) 5.5:6.4 (b) 55:64 (c) 7:8 (d) 8:7

The correct answer is (c), 7:8.

This is how we workout.

You must first change both times into seconds:

Raju's time = 5 minutes 50 seconds = 350 seconds

Ramu's time = 6 minutes 40 seconds = 400 seconds

So the ratio of Raju's time to Ramu's time = 350 : 400

Which can be simplified to 350/50 : 400/50 = 7 : 8

Question 3

The distance between two towns on a map is 5 cm. If the real distance between the two towns is 25 km, what is the scale of the map

(a) 1 : 5,000,000 (b) 1 : 500,000 (c) 1 : 50,000 (d) 1 : 5,000

The correct answer is (b), 1 : 500,000.

This is how we work out.

You can only compare the two distances if they are measured in the same unit. So the first step is to convert the real distance to centimetres:

First convert to m: 25 km = 25 × 1,000 m = 25,000 m

Then to cm: 25,000 m = 25,000 × 100 cm = 2,500,000 cm

So the scale of the map is 5 cm : 2,500,000 cm

Which is 1 : 500,000

Question 4

A real horse is 1.8 m high. A statue of the horse is 3 m high. What is the ratio of the height of the horse to the height of the statue

(a) 3:5 (b) 2:3 (c) 5:3 (d) 6:1

The correct answer is (a), 3 : 5

This is how we work out.

Ratio = 1.8 m : 3 m = 18 : 30 = 3 : 5

Question 5

The tallest building in the world, the BurjKhalifa in Dubai, is 828 metres tall. A model of the building is made using the ratio 1 : 5,000. How high is the model in centimetres

(a) 1,656 cm (b) 165.6 cm (c) 16.56 cm (d) 1.656 cm

The correct answer is (c), 16.56 cm.

This is how we work out.

Height of the model = 828 m ÷ 5,000

= 82,800 cm ÷ 5,000

= 16.56 cm

Question 6

A recipe for pastry has flour, butter and water mixed in the ratio 24 : 8 : 3. If Raji follows the recipe and uses 3 cups of flour, how many cups of butter should she use

(a) $\frac{1}{3}$ Cup (b)) $\frac{3}{8}$ Cup (c)) $\frac{3}{4}$ Cup (d)) 1 Cup

The correct answer is (d), 1 cup.

This is how we work out.

Start with 24 : 8 : 3

Raji uses 3 cups of flour, not 24, and 24/3=8 so let's divide all values by 8:

$24 : 8 : 3 = 3 : 1 : \frac{3}{8}$

So 1 cup of butter should be used.

Question 7

A recipe for pastry has flour, butter and water mixed in the ratio 24 : 8 : 3. If Raji follows the recipe and uses 3 cups of flour, how many cups of butter should she use

(a) $\frac{1}{3}$ Cup (b)) $\frac{3}{8}$ Cup (c)) $\frac{3}{4}$ Cup (d)) 1 Cup

The correct answer is (d), 1 cup.

This is how we work out.

Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example 1:

Are the ratios 3 to 4 and 6:8 equal?

The ratios are equal if 3/4 = 6/8.

These are equal if their cross products are equal; that is, if 3 × 8 = 4 × 6. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!

A ratio of 1:7 is not the same as a ratio of 7:1.

Question1:

Are the ratios 7:1 and 4:81 equal? No!

7/1 > 1, but 4/81 < 1, so the ratios can't be equal.

Question 2:

Are 7:14 and 36:72 equal?

Notice that 7/14 and 36/72 are both equal to 1/2, so the two ratios are equal.

Proportion

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.

3/4 = 6/8 is an example of a proportion.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number.
Example:
Solve for n: 1/2 = n/4.

Using cross products we see that 2 × n = 1 × 4 =4, so 2 × n = 4. Dividing both sides by 2, n = 4 ÷ 2 so that n = 2.

Decimals, Fractions and Percentages

Decimals, Fractions and Percentages are just different ways of showing the same value:

A Half can be written...

As a fraction:	$^1/_2$
As a decimal:	0.5
As a percentage:	50%

A Quarter can be written...

As a fraction:	$^1/_4$
As a decimal:	0.25
As a percentage:	25%

Example Values

Here is a table of commonly used values shown in Percent, Decimal and Fraction form:

Percent	Decimal	Fraction
1%	0.01	$^1/_{100}$
5%	0.05	$^1/_{20}$
10%	0.1	$^1/_{10}$
12½%	0.125	$^1/_8$
20%	0.2	$^1/_5$
25%	0.25	$^1/_4$

$33\frac{1}{3}$%	0.333...	$^1/_3$
50%	0.5	$^1/_2$
75%	0.75	$^3/_4$
80%	0.8	$^4/_5$
90%	0.9	$^9/_{10}$
99%	0.99	$^{99}/_{100}$
100%	1	
125%	1.25	$^5/_4$
150%	1.5	$^3/_2$
200%	2	

Conversions

(a) From Percent to Decimal

To convert from percent to decimal: divide by 100, and remove the "%" sign.

(b) From Decimal to Percent

To convert from decimal to percent: multiply by 100, and add a "%" sign.

(c) From Fraction to Decimal

The easiest way to convert a fraction to a decimal is to divide the top number by the bottom number *(divide the numerator by the denominator in mathematical language).*

Adding Mixed Fractions

We may be asked to add mixed fractions:

These are the steps

1. convert them to Improper Fractions
2. then add them
3. thenyou may convert back to Mixed Fractions.

Example

Compute $2\frac{3}{4} + 3\frac{1}{2}$

Convert to Improper Fractions:

$$2\frac{3}{4} = \frac{11}{4}$$

$$3\frac{1}{2} = \frac{7}{2}$$

$$\frac{11}{4} + \frac{7}{2}$$

To add this, one way is to make the denominators common. One way is to retain 11/4 as such and then multiply 7/2 by 2 (don not forget to multiply both numerator and denominator). We get $\frac{7}{2} \times 2 = \frac{14}{4}$

Now add

$$\frac{11}{4} + \frac{14}{4} = \frac{11 + 14}{4} = \frac{25}{4}$$

Convert back to Mixed Fractions:

$$\frac{25}{4} = 6\frac{1}{4}$$

Question 1: Compute $3\frac{5}{8} + 1\frac{3}{4}$

Convert to improper fractions:

$$3\frac{5}{8} = \frac{29}{8}$$

$$1\frac{3}{4} = \frac{7}{4}$$

To add this, one way is to make the denominators common. One way is to retain 29/8 as such and then multiply 7/4 by 2 (don not forget to multiply both numerator and denominator). We get $\frac{7}{4} \times 2 = \frac{14}{8}$

Now add

$$\frac{29}{8} + \frac{14}{8} = \frac{29 + 14}{8} = \frac{43}{8} = 5\frac{3}{8}$$

Subtracting Mixed Fractions

For Subtracting Mixed Fractions, Just follow the same method as addition, but subtract instead of add:

Example

Compute $15\frac{3}{4} - 8\frac{5}{6}$

Convert to Improper Fractions:

$$15\frac{3}{4} = \frac{63}{4}$$

$$8\frac{5}{6} = \frac{53}{6}$$

Now we have to make the denominators equal. One way is to make both denominators equal to 12. For this we have to multiply 63/4 by 3 and 53/6 by 2.

$$\frac{63}{4} \times 3 = \frac{189}{12}$$

$$\frac{53}{6} \times 2 = \frac{106}{12}$$

Now we can write the problem as

$$15\frac{3}{4} - 8\frac{5}{6} = \frac{63}{4} - \frac{53}{6} = \frac{189}{12} - \frac{106}{12} = \frac{189 - 106}{12} = \frac{83}{12}$$

Convert back to Mixed Fractions:

$$\frac{83}{12} = 6\frac{11}{12}$$

<u>Question1</u>: Subtract $1\frac{9}{10}$ from $4\frac{3}{5}$

Remember that this means

$$4\frac{3}{5} - 1\frac{9}{10}$$

Convert to Improper Fractions:

$$4\frac{3}{5} = \frac{23}{5}$$

$$1\frac{9}{10} = \frac{19}{10}$$

Now we have to make the denominators equal. One way is to multiply 23/5 by 2

So $\frac{23}{5} \times 2 = \frac{46}{10}$

Therefore we can write the problem as

$$\frac{46}{10} - \frac{19}{10} = \frac{46 - 19}{10} = \frac{27}{10}$$

Convert 27/10 to a mixed fraction.

10 divides into 27, 2 times with remainder 7.

So we get

$$\frac{27}{10} = 2\frac{7}{10}$$

Question 2: Subtract $1\frac{5}{6}$ from $3\frac{1}{2}$

Remember that this means

$$3\frac{1}{2} - 1\frac{5}{6}$$

Convert to Improper Fractions:

$$3\frac{1}{2} = \frac{7}{2}$$

$$1\frac{5}{6} = \frac{11}{6}$$

Now we have to make the denominators equal. One way is to multiply 7/2 by 3.

$$\frac{7}{2} \times 3 = \frac{21}{6}$$

Now the denominators are the same, we just subtract the numerators.

$$\frac{21}{6} - \frac{11}{6} = \frac{21 - 11}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$$

Another method (least common denominator method) is as follows.

$$3\frac{1}{2} - 1\frac{5}{6} = \frac{7}{2} - \frac{11}{6}$$

Next find the least common denominator i.e. the least common multiple of 6 and 2

The multiples of 6 are 6, 12, ...

The multiples of 2 are 2, 4, 6, 8, ...

The least common multiple of 6 and 2, therefore, is 6

Convert $\frac{7}{2}$ to an equivalent fraction with denomenator 6

$$\frac{7}{2} = \frac{7 \times 3}{2 \times 3} = \frac{21}{6}$$

Now the denominators are the same, we just subtract the numerators. Remember to put the answer over the same denominator:

$$\frac{7}{2} - \frac{11}{6} = \frac{21}{6} - \frac{11}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$$

Question 3: Subtract $2\frac{5}{6}$ from $3\frac{3}{4}$

Remember that this means

$$3\frac{3}{4}-2\frac{5}{6}$$

Convert to Improper Fractions:

$$3\frac{3}{4}=\frac{15}{4}$$

$$2\frac{5}{6}=\frac{17}{6}$$

Next find the least common denominator i.e. the least common multiple of 6 and 4

The multiples of 6 are 6, 12, 18, ...

The multiples of 4 are 4, 8, 12, 16, ...

The least common multiple of 6 and 4, therefore, is 12

Convert both fractions to equivalent fractions with denominator 12:

$$\frac{15}{4}=\frac{15\times 3}{4\times 3}=\frac{45}{12}$$

$$\frac{17}{6}=\frac{17\times 2}{6\times 2}=\frac{34}{12}$$

Now the denominators are the same, we just subtract the numerators. Remember to put the answer over the same denominator:

$$\frac{15}{4}-\frac{17}{6}=\frac{45}{12}-\frac{34}{12}=\frac{11}{12}$$

Question 4

What is$2\frac{1}{2}+1\frac{1}{6}-1\frac{2}{3}$

(a) $5\frac{1}{3}$(b) $2\frac{11}{2}$ (c) 2 (d) $2\frac{1}{5}$

The correct answer is (c), 2.

This is how we worked out.

First change all the mixed fractions to improper fractions:

$$2\frac{1}{2}=\frac{5}{2}$$

$$1\frac{1}{6}=\frac{7}{6}$$

$$1\frac{2}{3}=\frac{5}{3}$$

Next find the least common denominator i.e. the least common multiple of 2, 6 and 3

The multiples of 2 are: 2, 4, 6, 8, 10, 12, ...

The multiples of 6 are: 6, 12, 18, ...

The multiples of 3 are: 3, 6, 9, 12, 15, ...

So the least common multiple of 2, 6 and 3 is 6

Convert all fractions to equivalent fractions with denominator 6:

$$\frac{5}{2} = \frac{5 \times 3}{2 \times 3} = \frac{15}{6}$$

$$\frac{7}{6} = \frac{7}{6}$$

$$\frac{5}{3} = \frac{5 \times 2}{3 \times 2} = \frac{10}{6}$$

Now the denominators are the same, we just add and subtract the numerators.

Remember to put the answer over the same denominator:

$$\frac{5}{2} + \frac{7}{6} - \frac{5}{3} = \frac{15}{6} + \frac{7}{6} - \frac{10}{6} = \frac{15 + 7 - 10}{12} = \frac{12}{6} = 2$$

Question 5

What is$4\frac{3}{4} + 1\frac{1}{12} - 2\frac{5}{6}$

(a) $5\frac{1}{3}$(b) $2\frac{11}{2}$ (c)$\frac{5}{6}$ (d) $2\frac{1}{5}$

The correct answer is (c), $\frac{5}{6}$.

This is how we worked out.

First change all the mixed fractions to improper fractions:

$$4\frac{3}{4} = \frac{19}{4}$$

$$1\frac{1}{12} = \frac{13}{12}$$

$$2\frac{5}{6} = \frac{17}{6}$$

Now we need the denominators to be the same:

Find the least common denominator i.e. the least common multiple of 4, 12 and 6

The multiples of 4 are: 4, 8, 12, 16, ...

The multiples of 12 are: 12, 24, ...

The multiples of 6 are: 6, 12, 18, ...

The least common multiple of 4, 12 and 6, therefore, is 12

Convert all fractions to equivalent fractions with denominator 12:

$$\frac{19}{4} = \frac{19 \times 3}{4 \times 3} = \frac{57}{12}$$

$$\frac{13}{12} = \frac{13}{12}$$

$$\frac{17}{6} = \frac{17 \times 2}{6 \times 2} = \frac{34}{12}$$

Now the denominators are the same, we just subtract the numerators. Remember to put the answer over the same denominator:

$$\frac{19}{4} - \frac{13}{12} - \frac{17}{6} = \frac{57}{12} - \frac{13}{12} - \frac{34}{12} = \frac{57 - 13 - 34}{12} = \frac{10}{12} = \frac{5}{6}$$

Question 6

What is$3\frac{9}{4} - 2\frac{3}{5} + 1\frac{1}{2}$

(a) $2\frac{4}{5}$(b) $2\frac{11}{2}$ (c)$\frac{5}{6}$ (d) $2\frac{1}{5}$

The correct answer is (a), $2\frac{4}{5}$.

This is how we worked out.

First change all the mixed fractions to improper fractions:

$$3\frac{9}{10} = \frac{39}{10}$$

$$2\frac{3}{5} = \frac{13}{5}$$

$$1\frac{1}{2} = \frac{3}{2}$$

Next find the least common denominator i.e. the least common multiple of 10, 5 and 2

The multiples of 10 are: 10, 20, ...

The multiples of 5 are: 5, 10, 15, ...

The multiples of 2 are: 2, 4, 6, 8 10, 12, ...

The least common multiple of 10, 5 and 2, therefore, is 10

Convert all fractions to equivalent fractions with denominator 10:

$$\frac{39}{10} = \frac{39}{10}$$

$$\frac{13}{5} = \frac{13 \times 2}{5 \times 2} = \frac{26}{10}$$
$$\frac{3}{2} = \frac{3 \times 5}{2 \times 5} = \frac{15}{10}$$

Now the denominators are the same, we just subtract and add the numerators. Remember to put the answer over the same denominator:

$$\frac{39}{10} - \frac{13}{5} + \frac{3}{2} = \frac{39}{10} - \frac{26}{10} + \frac{15}{10} = \frac{39 - 26 + 15}{10} = \frac{28}{10} = \frac{14}{5}$$

Convert $\frac{14}{5}$ to a mixed fraction:

5 divides into 14, 2 times with remainder 4:

So $\frac{14}{5} = 2\frac{4}{5}$

Comparing Fractions

Sometimes we need to compare two fractions to discover which is larger or smaller. There are two easy ways to compare fractions: using decimals, or using the same denominator.

(a) The Decimal Method of Comparing Fractions

Just convert each fraction to decimals, and then compare the decimals.

Example: which is bigger: $\frac{3}{8}$ or $\frac{5}{12}$

Convert each fraction to a decimal.

We can use a calculator (3÷8 and 5÷12), or the method on Converting Fractions to Decimals.

Anyway, these are the answers:

$$\frac{3}{8} = 0.375, and\ \frac{5}{12} = 0.4166$$

So $\frac{5}{12}$ is bigger.

(b) When the Denominators aresame

When two fractions have the same denominator they are easy to compare, just compare the numerators and decide.

Example:

Which is larger: $\frac{4}{9}$ or $\frac{5}{9}$

Since the denominators are the same, compare numerator. So we can easily say : $\frac{5}{9}$ is greater than $\frac{4}{9}$, simply because 5 is greater than 4.

(c) When denominators are different

But when the denominators are not the same we need to make them the same (using Equivalent Fractions).

Example:

Which is larger: $\frac{3}{8}$ or $\frac{5}{12}$

Since the denominators are different, let us make them same. If we multiply 8 by 3 and 12 by 2 both denominators will be equal to 24. Remember that both numerator and denominator has to be multiplied.

$$\frac{3}{8} \times 3 = \frac{9}{24}$$

$$\frac{5}{12} \times 2 = \frac{10}{24}$$

Now both fractions have the same denominator, so we can simply compare the numerators and decide.

$$We\ know\ that\ \frac{10}{24}\ is\ larger\ than \frac{9}{24}, because\ 10\ is\ larger\ than\ 9.$$

So $\frac{5}{12}$ is greater than $\frac{3}{12}$

Question 1

Which one of the following fractions is the largest?

(a) $\frac{29}{36}$ (b) $\frac{7}{9}$ (c) $\frac{5}{6}$ (d) $\frac{3}{4}$

To compare the fractions, we need to write each of them as an equivalent fraction with a common denominator i.e. we must find the least common multiple of 36, 9, 6 and 4.

Since 9, 6 and 4 are all factors of 36, the least common multiple is 36

So change them all to equivalent fractions with denominator 36:

$$\frac{29}{36} = \frac{29}{36}$$

$$\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36}$$

$$\frac{5}{6} = \frac{5 \times 6}{6 \times 6} = \frac{30}{36}$$

$$\frac{3}{4} = \frac{3 \times 9}{4 \times 9} = \frac{27}{36}$$

Now they all have the same denominator, we just need to find the one with the largest numerator, which is 30.

Therefore $\frac{5}{6}$ is the largest fraction.

MODULE II
Basic Mathematical Concepts:- Linear Equations

This chapter deals with the concept of simple linear equations, simultaneous linear equations, and the uses of these equations in economics (demand and supply analysis).

Equation

An equation is a statement of equality between two expressions. In other words, an equation sets two expressions, which involves one or more than one variable, equal to each other.

For example, (a) 2x=10, (b) 3x+2y=20, (c) x^2 -5x+6=0.

An equation consists of one or more unknown variables. In the above example first and second equation (a and c) contain only one unknown variable (x) and the second equation contains two unknowns(x and y)

Solutions of the Equation

An equation is true for some particular value or values of the unknown. The value of the unknown for which equation is true is called solutions of the equation. It is also known as root of the equation

For example, (a) 2x=10, so $x = \frac{10}{2} = 5$, Thus this equation is true for the value x=5

Linear and Non-linear Equations

The highest degree of the variables in an equation determines the nature of the equation. If the equation is first degree, then it is known as linear equation otherwise it is known as non-linear.

For example: $5x + y = 20$ is a linear equation. It is a linear equation because there is no term involving x^2, y^2 , $x \times y$, or any higher powers of *x* and *y*.

$x^2 - 7x + 12 = 0$ is a non-linear equation. It is non-linear because the highest degree of the equation is two.

Variables and Parameters

A variable is a symbol or letter used to denote a quantity whose value changes over a period of time. In other words, a variable is a quantity which can assume any one of the values from a range of possible values.

Example: income of the consumer is a variable, since it assumes different values at different time

Dependent and Independent Variable

If x and y are two variables such that $y = f(x)$,for any value of the x there is a corresponding y value, then x is independent variable and y is dependent variable. The value of y depends on the value of x.

Example: Consider the consumption function $c = f(y)$. Here consumption c depends on income. For each value of income there corresponds a value of consumption. Thus c is dependent variable and y is independent variable.

Parameters are similar to variables –that is, letters that stand for numbers– but have a different meaning. We use parameters to describe a set of similar things. Parameters can take on different values, with each value of the parameter specifying a member of this set of similar objects.

For example: $\int xdx = \frac{1}{3}x^3 + c$ where c is called the *constant of integration.* In this case, each value of c specifies a single antiderivative. We call c the parameter of the set of *all* the antiderivatives of x^2. Each value of the parameter c specifies a single antiderivative.

Solution of Simple Linear Equations

A simple linear equation is an equation which consists of only one unknown and its exponent is one.

Steps for Solving a Linear Equation in One Variable

1. Simplify both sides of the equation.
2. Use the addition or subtraction properties of equality to collect the variable terms on one side of the equation and the constant terms on the other.
3. Use the multiplication or division properties of equality to make the coefficient of the variable term equal to 1.

Note: In order to isolate the variable, perform operations on both sides of the equation.

1- Use of Inverse Operation

a) Use subtraction to undo addition.

If $a = b$

$a - c = b - c$

Example (1): Solve $x + 5 = 15$

Solution:

$x + 5 = 15$

Subtract 5 $\quad$ $\underline{\quad 5} = \underline{5}$

$$x = 10$$

OR

$x + 5 = 15 \qquad x = 15 - 5 = 10$

Example (2) $\quad y + 6 = 2y$

Solution:

$y + 6 = 2y$

Subtract y $\quad$ $\underline{y} = \underline{y}$

$$6 = y$$

OR

$y + 6 = 2y \quad 6 = 2y - y \quad y = 6$

b) Use Addition to Undo Subtraction

If $a = b$

then $a + c = b + c$

For example, solve $x - 4 = 6$

Solution:

$$x - 4 = 6$$

Add $\quad 4 = 4$

$$x = 10$$

OR

$$x - 4 = 6 \quad x = 6 + 4 \quad x = 10$$

c) Use Division to Undo Multiplications

If $a = b$

then $\frac{a}{c} = \frac{b}{c}$

Example $3x = 18$

Solution:

$$3x = 18$$

$$\frac{3x}{3} = \frac{18}{3}$$

Answer: $x = 6$

OR

$$x = \frac{18}{3} = 6$$

d) Use Multiplication to Undo Division

If $a = b$

then $ac = bc$

Example: $\frac{x}{4} = 6$

Solution:

$$4.\left(\frac{x}{4}\right) = 4.6$$

Answer $x = 24$

OR

$$x = 4.6 = 24$$

2. Equation having Fractional Coefficient

The coefficient of x also be a rational number. This section discusses how to solve the equation having only one fraction and equation having different fractions.

a) Equation having Only One Fraction

To clear fractions, multiply both sides of the equation by the denominator of the fractions or by the reciprocal of the fraction

Example (1) $\frac{1}{7}x = 5$

Solution

$$7.\frac{1}{7}x = 5.7$$

Answer: $x = 35$

Example (2) $\frac{2}{6}x = 15$

Solution:

$$\frac{6}{2}.\left(\frac{2}{6}x\right) = 15.\left(\frac{6}{2}\right), x = \frac{90}{2} = 45$$

Answer: $x = 45$

b) Equation Containing Fractions Having Different Denominator

To clear fractions, multiply both sides of the equation by the LCD of all the fractions. The Lowest Common Denominator (L.C.D) of two or more fractions is the smallest number divisible by their denominators without reminder

For example: solve $\frac{x}{3} + \frac{x}{4} = 14$

Solution: Here L.C.D is 12

$$12 \times \left(\frac{x}{3} + \frac{x}{4}\right) = 12 \times 14$$

$$4x + 3x = 168, \quad 7x = 168$$

Answer: $x = 24$

3. Equations Containing Parentheses

Follow the following steps to solve the equation which contains parenthesis

a) Remove the parenthesis
b) Solve the resulting equation

For example: solve $10 + 3(x - 6) = 16$

Solution: $10 + 3x - 18 = 16$

$$3x - 8 = 16$$

$$3x = 16 + 8$$

$$x = \frac{24}{3} = 8$$

SIMULTANEOUS EQUATIONS

Simultaneous equations are set of two or more equations, each containing two or more variables whose values can simultaneously satisfy both or all equations in the set. The number of variables will be equal to or less than the number of equations in the set. The simultaneous equation can be solved by the following methods.

a.) Elimination method
b.) Substitution method
c.) Cross multiplication method

(A) Elimination method

i. Multiply the equations with suitable non-zero constants, so that the coefficients of one variable in both equations become equal.
ii. Subtract one equation from another, to eliminate the variable with equal coefficients. Solve for the remaining variable.
iii. Substitute the obtained value of the variable in one of the equations and solve for the second variable.

Simultaneous Equation in Two Unknowns (First Degree)

Solve $2x+2y=40$

$3x+4y=65$

Solution:

$2x+2y=40$ .... (1)

$3x+4y=65$ .... (2)

Multiply the equation (1) by 2, we will get

$4x+4y=80$ .... (3)

Subtract equation (2) from equation (3)

$$\begin{array}{r} 4x+4y=80 \\ -\ 3x+4y=65 \\ \hline x=15 \end{array}$$

Substitute x=15 either in equation (1) or equation (2)

We substitute in equation (1), we will get

$$2(15)+2y=40$$

$$2y=40-30$$

$$y=\frac{10}{2}=5$$

Checking answers by substituting the obtained value into the original equation.

$$2(15)+2(5)=40$$

$30+10=40$ Both sides are equal (L.H.S=R.H.S)

So the answers x=15 and y=5

B. SUBSTITUTION METHOD

The substitution method is very useful when one of the equations can easily be solved for one variable. Here we reduce one equation in to the form of $y=f(x)$ or $x=f(y)$. That is expressing the equation either in terms of x or in terms of y. Then substitute this reduced equation in the non-reduced equation and find the values of both unknowns.

X	Y	1

Steps involved in Substitution Method

i. Choose one equation and isolate one variable; this equation will be considered the first equation.
ii. Substitute the transformed equation into the second equation and solve for the variable in the equation.
iii. Using the value obtained in step ii, substitute it into the first equation and solve for the second variable.
iv. Check the obtained values for both variables into both equations.

Solve $4x+2y=6$

$5x+y=6$

Solution:

$4x+2y=6$ .... (1)

$5x+y=6$ .... (2)

Express equation (2) in terms of x, we will get

$y=6-5x$ .... (3)

Substitute equation (3) in equation (2), we will get

$$4x+2(6-5x)=6$$
$$4x+12-10x=6$$
$$-6x=6-12$$
$$x=\frac{-6}{-6}=1$$

Substitute x=1 in equation (1)

$$4(1)+2y=6$$
$$2y=6-4$$
$$y=\frac{2}{2}=1$$

Checking answers by substituting the obtained value into the original equation.

$$4(1)+2(1)=6$$

$4+2=6$ Both sides are equal (L.H.S=R.H.S)

So the answers are x=1 and y=1

C. CROSS MULTIPLICATION

This method is very useful for solving the linear equation in two variables.Let us consider the general form of two linear equations $a_1x+b_1y+c_1=0$, and $a_2x+b_2y+c_2=0$. To solve this pair of equations for x and y using cross-multiplication, we will arrange the variables, coefficients, and the constants as follows.

coefficient of y	constant terms	constant terms	coefficient of x	coefficient of x	coefficient of y
b_1	c_1	c_1	a_1	a_1	b_1
b_2	c_2	c_2	a_2	a_2	b_2

That is

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Example: Solve $2x + 2y = 40$
$3x + 4y = 65$

Solution

On transposition, we get $2x + 2y - 40 = 0$
$3x + 4y - 65 = 0$

X		Y		1	
coefficient of y	constant terms	constant terms	coefficient of x	coefficient of x	coefficient of y
2	-40	-40	2	2	2
4	-65	-65	3	3	4

(2×-65) - (4×-40) = (-130) – (-160) = 30
(-40×3) – (-65×2) = (-120) - (-130) = 10
(2×4) – (3×2) = (8) – (6) = 2

S0 $x = \frac{30}{2} = 15$, and $y = \frac{10}{2} = 5$

The same answer that we got in the first problem

Simultaneous Equation in Three Unknowns (First Degree)

Steps

1. Take any two equation form the given equations and eliminate any one of the unknowns.
2. Take the remaining equation and eliminate the same unknown
3. Follow the rules of simultaneous equation in two unknowns

Solve $9x + 3y - 4z = 35$
$x + y - z = 4$
$2x - 5y - 4z = -48$

Solution:

$$9x + 3y - 4z = 35 .... (1)$$
$$x + y - z = 4 .... (2)$$
$$2x - 5y - 4z = -48 .... (3)$$

Take equation (1) and (2)
Multiply equation (2) by 4,we will get

$$4x + 4y - 4z = 16 .... (4)$$

Subtract it from equation (1), we will get

$$5x - y = 19 .... (5)$$

Take equation (2) and (3)
Multiply equation (2) by 4,we will get

$$4x + 4y - 4z = 16 .... (4)$$

Subtract equation (3) from (4), we will get

$$2x + 9y = 64 .... (6)$$

Take equation (5) and multiply it by 9

$$45x - 45y = 171 .... (7)$$

<u>Add equation (6) from equation (7)</u>

$$45x - 45y = 171 .... (7)$$
$$2x + 9y = 64 .... (6)$$

$$47x = 235 \qquad x = \frac{235}{47} = 5$$

Substitute *x=5* in equation (5), $5x - y = 19$

$$5(5) - y = 19$$

$$-y = 19 - 25 = -6$$

<u>So *y=6*</u>

Substitute *x=5 and y=6 in equation (1)* $9x + 3y - 4z = 35$

$$9(5) + 3(6) - 4z = 35$$
$$45 + 18 - 4z = 35$$
$$-4z = 35 - 63 = -28$$

$$z = \frac{-28}{-4} = 7$$

<u>**Answer**: *x=5, y=6, and z=7*</u>

DEMAND AND SUPPLY FOR A GOOD

Now we can apply simple linear equation and simultaneous linear equations in the analysis of demand and supply. Here we use both demand function and supply function. Demand function depicts the negative relationship between quantity demanded and price. The linear demand function can be written as $q = a - bp$ .where q denotes quantity demanded and p denotes price.

For example: $q = 80 - 2p$. This equation can be written as $p = 40 - \frac{1}{2}q$. This called inverse demand function.

Supply function depicts the positive relationship between quantity demanded and price. The linear supply function can be written as $q = a + bp$ .where q denotes quantity supplied and p denotes price.

For example: $q = 40 + 2p$. This equation can be written as $p = 20 + \frac{1}{2}q$. This called inverse supply function.

The equilibrium quantity and equilibrium price is determined by the interaction of both demand supply curve. At equilibrium point the demand will be equal to supply. The price that equates demand and supply is called equilibrium price. If current price exceeds the equilibrium price, there will be an excess supply. This situation will compel the producer to reduce the price of the product so that they can sell unsold goods. The reduction in the price will continue until it reaches equilibrium point ($q^d = q^s$) . On the other hand, if current price is below the equilibrium price there is an excess demand for the product. This shortage leads buyers to bid the price up. The increase in the price will continue until it reaches the equilibrium point ($q^d = q^s$).

Now we are able to find the equilibrium price and quantity by using the system of two linear equations; demand function and supply function. Consider the following equations.

$$p = 20 + \frac{1}{2}q$$

$$p = 40 - \frac{1}{2}q$$

This set of equation is system of two linear equation in the variable p and q. We have to find the values of both p and q that satisfy both equations simultaneously.

Example: Find the equilibrium price of the following demand and supply function

$$q^s = 20 + 3p$$
$$q^d = 160 - 2p$$

Solution:

At equilibrium demand is equal to supply

$$q^s = 20 + 3p = q^d = 160 - 2p$$

Collect all *p* values on left side and the constants on right side

$$3p + 2p = 160 - 20$$
$$5p = 140$$
$$p = \frac{140}{5} = 28$$

Now substitute p=28 in either q^d or q^s

$$q^s = 20 + 3p$$
$$q^s = 20 + 3(28)$$
$$q^s = 104$$

Check the answer with the q^d equation, $q^d = 160 - 2p$

$$q^d = 160 - 2(28)$$
$$q^d = 160 - 56 = 104$$

Thus, $q^d = q^s$. Here equilibrium price is Rupees 28 and the equilibrium quantity is 104.

QUESTIONS

Multiple Choice Questions

1. Unknown values in an equation are called

   (a) Constants (b) parameters (c) Variables (d) all the above

2. Given or known values in an equations are called

   (a) Constants (b) Parameters (c) Coefficients (d) all of the above

3. A variable which is free to take any value we choose to assign to it is called

   (a) Dependent variable (b) independent variable
   (c) Continuous variable (d) Discrete variable

4. The variable that stands alone on the left-hand side of the equation such as y = 5x + 2 is known as

   (a) Dependent variable (b) independent variable
   (c) Endogenous variable (d) explained variable

5. Given y=2x-6=2, then x= ………….

   (a) 8 (b) 4 (c) 6 (d) -4

6. Given $\frac{2}{3}x = 4, \textit{then } x = \ldots\ldots\ldots\ldots\ldots$

(a) 2.5 (b) 3 (c) $\frac{10}{3}$ (d) 6

7. If $4x+2=3x+12$, *then x is*

(a) 10 (b) 2 (c) *12* (d) 8

8. Given the demand and supply functions qD = -8p + 2000 and qS =12p – 200 respectively, the equilibrium price is

(a) p = 100 (b) p = 110 (c) p = 120 (d) p = 140

9. The equation $-7z+1=5-3z$ will be satisfied for z equal to

(a) 3 (b) 1 (c) -1 (d) 2

10. Pick up the correct value of x for $\frac{x}{30}=\frac{3}{15}$

(a) 7 (b) 6 (c) 5 (d) 4

11. 8 is the solution of the equation

(a) $\frac{x+4}{4}+\frac{x-5}{3}=12$ (b) $\frac{x+4}{2}=\frac{x+10}{9}$

(c) $\frac{x+6}{2}=\frac{x-5}{3}$ (d) $\frac{x+4}{2}+\frac{x+10}{9}=8$

12. $\frac{x+24}{5}=4+\frac{x}{4}$, then x=

(a) 8 (b) 10 (c) 16 (d) -8

13. The fourth part of a number exceeds the sixth part by four. The number is

(a) 46 (b) 64 (c) 58 (d) 48

14. The solution of the simultaneous equations $3x+4y=7, 4x-7=3$ is

(a) (2,2) (b) (1,1) (c) (1,2) (d) (-1,2)

15. The values of x and y satisfying the equations $\frac{x}{2}+\frac{y}{3}=2, x+2y=8$ are given by the pair

(a) (4,-3) (b) (4,6) (c) (2,3) (d) (-2,-6)

16. The pair satisfying the equations $x+6y=32$, $\frac{x+y}{x-y}=\frac{11}{9}$ is given by

(a) (18, 3) (b) (20,2) (c) (22,4) (d) None of these

17. The pair satisfying the equations $x-4y=0$, $x+3y=28$ is

(a) (3, 4) (b) (4,3) (c) (4, 16) (d) (16,4)

18. $\frac{x}{2}+\frac{x}{3}=10$, Here the value of x is

(a) 10 (b) -10 (c) -12 (d) 12

19. The value of x in the equation $4x+3=2x+5$

(a) 1 (b) 2 (c) -2 (d) 8

20. The values of x satisfying the equations $3x+2=x+6$ is given by

(a) 4 (b) 8 (c) 2 (d) -2

21. The curve of a linear equation is................

(a) parabola (b) a liner (c) hyper-parabola (d) none of the above

22. Two intersecting lines of simultaneous equation give.............

(a) An unique solution (b) infinite solution

(c) no solution (d) none of the above

23. Consider the equations, $6x-3y=-9$, $2x-y+9=0$ represents two lines which are,

(a) Intersecting at exactly two points (b) Intersecting at exactly one point

(c) parallel (d) none of the above

24. The pair satisfying the equations $x-4y-14=0$, $5x-y-13=0$ is

(a) (-3, 2) (b) (2,-3) (c) (1, 2) (d) (2,4)

25. Thrice a number decreased by 22

(a) x-22 (b) 3 + x (c) 3x-22 (d) 3(x-22)

26. $2(5x-2)=8x$

(a) 2 (b) -2 (c) 1 (d) 3

27. The equation x=12 can be written in the two variable case as

(a) 1.x+0.y=12 (b) 0.x+1.y=12 (c) 1.x+1.y=12 (d) All of the above

28. A linear equation in two variables is of the form $ax + by + c = 0$, where

(a) a=0, b=0 (b) a $\neq$0, b$\neq$0 (c) both a and b (d) none of the above

29. The linear equation 3x-4y=18 has

(a) Infinitely many solution (b) unique solution

(c) Two solution (d) no solution

30. $\frac{x}{3}-7=-11$, x =............

(a) 9 (b) -9 (c) 12 (d) -12

Answers

1. (c) Variables

2. (d) all of the above

3. (b) independent variable

4. (a) Dependent variable

5. (b) 4

6. (d) 6

7. (a) 10

8. (b) p = 110

9. (c) -1

10. (b) 6

11. (d) $\frac{x+4}{2}+\frac{x+10}{9}=8$

12. (c) 16

13. (d) 48

14. (b) (1,1)

15. (c) (2,3)

16. (b) (20,2)

17. (d) (16,4)

18. (d) 12

19. (a) 1

20. (c) 2

21. (b) a line

22. (a) An unique solution

23. (c) parallel

24. (b) (2,-3)

25. (c) 3x-22

26. (a) 2

27. (c) 1.x+1.y=12

28. (b) a ≠0, b≠o

29. (a) Infinitely many solution

30. (d) -12

Very Short Answer Type Questions

1. What do you mean by a linear equation
2. Distinguish between variable and constant
3. Distinguish between dependent variable and independent variable
4. *Solve (3x-1) + 2 = 12x + 6*

5. Solve $\frac{x}{2}+\frac{x}{3}=10$

6. Solve $\frac{4x+1}{7}=\frac{10x+25}{25}$

7. Solve $\frac{x}{3}+\frac{x}{2}+x=11$

8. $7(x-2)+8(x-3)-22=x+10$

9. The denominator of a fraction exceeds the numerator by 6 and if 4 be added to both the fraction becomes $\frac{4}{5}$ .find the fraction

10. If the numerator of a fraction is increased by 3 and the denominator by 2 it becomes 2. Again if the numerator is decreased by 5 and the denominator by 3 it becomes 2 Find x and y.

11. A number consist of two digits. The digit in the ten's place is twice the digit in the unit's Place. If 27 be subtracted from the number the digits are reversed. Find the number.

12. For a certain commodity the demand function is given as $q^d = 100(10 - p)$ and the Supply function is given as $q^s = 75(p-3)$ .Find the equilibrium quantity and price.

13. Anil's age is six times fiend's age. Four years hence, the age of Anil will be four times his friend's age. Find the present age of Anil and his friend.

14. Sha has only Rupees one and Rupees 2 coins with him. In total he has 50 coins and total amount with him is Rupees 75. Then find the numbers of Rs 1 and Rs 2 coins he has

15. Solve the following linear equations

a) $6(12x - 8) = 44$

b) $2(x+5) = 3(x-8)$

Answers

1. An equation is a statement in which one expression equals to another expression. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers such that $a \neq b$ and $b \neq 0$, is called a linear equation in two variables.. (The "two variables are the x and y.).The numbers a and b are called the coefficients of the equation $ax + by + c = 0$.The number c is called the constant of the equation $ax + by + c = 0$

2. A variable is a symbol or letter used to denote a quantity whose value changes over a period of time. In other words, a variable is a quantity which can assume any one of the values from a range of possible values.

Example: income of the consumer is a variable, since it assumes different values at different time

3. If x and y are two variables such that $y = f(x)$,for any value of the x there is a corresponding y value, then x is independent variable and y is dependent variable. The value of y depends on the value of x.

Example: Consider the consumption function $c = f(y)$. Here consumption c depends on income. For each value of income there corresponds a value of consumption. Thus c is dependent variable and y is independent variable.

4 ***5(3x-1) + 2 = 12x + 6***

$15x - 5 + 2 = 12x + 6$

$15x - 3 = 12x + 6$

$15x - 12x = 6 + 3$

$3x = 9$

$X = \frac{9}{3} = 3$

Answer x=3

5. $\frac{x}{2} + \frac{x}{3} = 10$

L.C.M=6

$6 \times \left(\frac{x}{2}\right) + 6 \times \left(\frac{x}{3}\right) = 6 \times 10$

$3x + 2x = 60$

$5x = 60$

$x = \frac{60}{5} = 12$

Answer x=12

6. $\frac{4x+1}{7} = \frac{10x+25}{25}$

Cross multiply equation 1 and equation 2

$25(4x+1) = 7(10x+25)$

$100x + 25 = 70x + 175$

$100x - 70x = 175 - 25$

$30x = 150$

$x = \frac{150}{30} = 5$ Answer x=5

7. $\frac{x}{3} + \frac{x}{2} + x = 11$

L.C.M=6

$6 \times \left(\frac{x}{3}\right) + 6 \times \left(\frac{x}{2}\right) + 6 \times x = 6 \times 11$

$2x + 3x + 6x = 66$

$11x = 66$

$x = \frac{66}{11} = 6$ Answer x=6

8. $7(x-2) + 8(x-3) - 22 = x + 10$

$7x - 14 + 8x - 24 - 22 = x + 10$

$15x - x = 10 + 14 + 24 + 22$

$14x = 70$

$x = \frac{70}{14} = 5$ *Answer x=5*

9. Let x be the numerator and the fraction be $x = \frac{x}{x+6}$

By the question $\frac{x+4}{x+6+4} = \frac{4}{5}$

$\Rightarrow 5x + 20 = 4x + 24 + 16$

$5x - 4x = 40 - 20$

$x = 20$, So the required fraction is $\frac{20}{20+6}$

10. Let $\frac{x}{y}$ be the required fraction

By the question

$$\frac{x+3}{y+2} = 2, \frac{x-5}{y-3} = 3$$

Thus, $(x+3) = 2y + 4$

$\Rightarrow x - 2y = 4 - 3 = 1$.............(1)

$(x-5) = 3y - 9$

$\Rightarrow x - 3y = -9 + 5 = -4$.........(2)

Subtract (2) from (1) $\Rightarrow y = 5$

From (1) $x - 2y = 1 \Rightarrow x - 2(5) = 1$

So x= 11

Thus the required fraction is $\frac{11}{5}$

11. Let x be the digit in the unit's place. So the digit in the ten's place is 2x. It is in the ten's place, so it becomes 10.(2x)

Thus the number becomes $10(2x) + x$

By the question $20x + x - 27 = 10x + 2x$,

$21x - 27 = 12x$

$9x = 27$

$x = \frac{27}{9} = 3$

So the required number is $10(2 \times 3) + 3 = 63$

12. At equilibrium demand= supply. Thus we have

$100(10 - p) = 75(p - 3)$

$1000 - 100p = 75p - 225$

$-175p = -1225$

$p = \frac{-1225}{-175} = 7$

Substitute *p=7* in $75(p-3) = 75(7-3)$

$75 \times 4 = 300$

Thus equilibrium quantity is 300 kg (if it measures in kg)

Equilibrium price is 7

13. Let $x = age$ of Anil
y=age of Anil's friend

We are given that

$$x = 6y............(1)$$

And $x + 4 = 4(y + 4)..........(2)$

Substitute equation (1) in (2), we get

$$6y + 4 = 4(y + 4)$$
$$\Rightarrow 6y + 4 = 4y + 16$$
$$6y - 4y = 16 - 4$$
$$2y = 12$$
$$y = \frac{12}{2} = 6$$

Put y=6 in equation (1)

$$x = 6(6) = 36$$

The present age of Anil is 36 and the present age of his friend is 6

14. Let x= number of 1 Rupee coin
y= number of 2 Rupee coin

Then $x + y = 50...........(1)$

$$x + 2y = 75.........(2)$$

Subtract equation (1) from equation (2). Then, we have

$$y = 25$$

Substitute y=25 in equation (1).

$$x + 25 = 50$$
$$x = 50 - 25 = 25$$

Thus, Sha has 25 One Rupee coins and 25 Two Rupee coins

15. a) $6(12x - 14) = 44x$

$$72x - 84 = 44x$$
$$72x - 44x = 84$$
$$28x = 84$$
$$x = \frac{84}{28} = 3$$

Answer: x=3

b) $2(x + 5) = 3(x - 8)$

$$\Rightarrow 2x + 10 = 3x - 24$$
$$\Rightarrow 2x - 3x = -24 - 10$$
$$-x = -34$$
$$\therefore x = 34$$

Answer: x=34

Short Answer Type Questions

1. Solve $2x+3y+10=0$
   $5x-3y+1=0$

2. Solve $x+2y=8$
   $x-y=-1$

3. Solve $12x-13y=-1$
   $8x+7y=15$

4. Solve $y=3(x+1)$
   $14x=y+1$

5. Solve $3x+7y=27$
   $5x+2y=16$

6. Solve $\frac{3a-4b}{5}=a-10$
   $\frac{a-b}{2}=\frac{b-1}{3}$

7. The sum of two digits is 30. The larger exceeds twice the smallest by 12.Find the numbers

8. A father is twice old as his son 20 years ago the father was 4 times as old as his son. Find their ages.

9. Ten years ago the age of a father was four times of his son. Ten years hence the age of the father will be twice that of his son. Find present ages of the father and the son.

10. 3 dozen books and 6 dozen pens cost comes to be Rupees 36 and 4 dozen books and 5 dozens cost is 43.5. Find the price of books and pens per dozen

11. Solve $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$
    $\frac{1}{x}-\frac{1}{y}=\frac{1}{6}$

12. 6 kg of wheat and 8 kg of rice cost is Rs 144 and 4 kg of wheat and 10 kg of rice cost is Rs 152. Find the price of wheat and rice

13. The demand for a commodity is given by $Q^d=40-8p$ and the supply is given by $Q^s=20p+18$
    A-Find the equilibrium price and quantity

Answers

1. $2x+3y+10=0$

   $5x-3y+1=0$

   Rearrange equations, we will get

   $2x+3y=-10$................................(1)

   $5x-3y=-1$..................................(2)

   Add equation (1) and (2), then we have

$$7x=-11$$

$$x=\frac{-11}{7}$$

Put $x=\frac{-11}{7}$ *in equation (1),* $2x+3y=-10$

$$2\times\left(\frac{-11}{7}\right)+3y=-10$$

$$\frac{-22}{7}+3y=-10$$

$$3y=-10+\frac{22}{7}$$

$$=\frac{-70+22}{7}$$

$$y=\frac{-48}{7}\times\frac{1}{3}$$

$$y=\frac{-48}{21}=\frac{-16}{7}$$

Answer: $x=\frac{-11}{7}$ and $y=\frac{-16}{7}$

2. $x+2y=8$ …………………..*(1)*

   $x-y=-1$ ………………….*(2)*

   From equation (2) $x=-1+y$ ………………….*(3)*

   Substitute equation (3) in equation (1)

$$-1+y+2y=8$$

$$3y=9$$

$$y=\frac{9}{3}=3$$

Put y=3 in equation (2), $x-y=-1$

$$x-3=-1$$

$$x=-1+3=2$$

Answer x=2 and y=3

3. $12x-13y=-1$ …………………..*(1)*

$8x + 7y = 15$ ……………….....*(2)*

Multiply equation (1)by 8and equation (2)by 12

$96x - 104y = -8$ ………………….. *(3)*

$96x + 84y = 180$ ………………….. *(4)*

Subtract equation (4) from equation (3)

$-188y = -188$

$$y = \frac{-188}{-188} = 1$$

Put y=1 in equation (1) $12x - 13y = -1$

$12x - 13(1) = -1$

$12x - 13 = -1$

$12x = -1 + 13$

$12x = 12$

$$x = \frac{12}{12} = 1$$

Answer: x=1 and y=1

4. $y = 3(x+1)$ ………………… *(1)*

$4x = y + 1$………………….. *(2)*

Substitute equation (1) in equation (2)

$4x = 3x + 3 + 1$

$4x - 3x = 4$

$x = 4$

Substitute x=4 in (1)

$y = 3(4+1)$

$y = 15$

Answer: x=4 and y=15

5. $3x + 7y = 27$

$5x + 2y = 16$

On transposition, we get $3x + 7y - 27 = 0$

$5x + 2y - 16 = 0$

X		Y		1	
coefficient of y	constant terms	constant terms	coefficient of x	coefficient of x	coefficient of y
7	-27	-27	3	3	7
2	-16	-16	5	5	2

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

(7×-16) - (2×-27) = (-112) – (-54) = -58
(-27×5) – (-16×3) = (-135) - (-48) = 87
(3×2) – (5×7) = (6) – (35) = -29

$$x = \frac{-58}{-29} = 2 \qquad y = \frac{-87}{-29} = 3$$

Answer x=2 and 3

6. $\frac{3a-4b}{5} = a - 10$

$\frac{a-b}{2} = \frac{b-1}{3}$ cross multiply each other by the denominators

$$3(a-b) = 2(b-1)$$
$$3a - 3b = 2b - 2$$
$$3a - 5b = -2 \quad \text{.................................. (1)}$$

$\frac{3a-4b}{5} = a - 10$ Again cross multiply each other by the denominators

$$3a - 4b = 5(a-10)$$
$$3a - 4b = 5a - 50$$
$$-2a - 4b = -50 \quad \text{................................. (2)}$$

$3a - 5b = -2$ .... (1)

$-2a - 4b = -50$ .... (2)

$(1) \times 2 \Rightarrow 6a - 10b = -4$ .... (3)
$(2) \times 3 \Rightarrow -6a - 12b = -150$ .... (4)
$(3) + (4) \Rightarrow -22b = -154$

$$b = \frac{-154}{-22} = 7$$

Substitute b=7 in equation (1) $3a - 5(7) = -2$

$$3a - 35 = -2$$
$$3a = 33$$
$$a = \frac{33}{3} = 11$$

Checking answers by substituting the obtained value into the original equation.

$$3(11) - 5(7) = -2$$

$33 - 35 = -2$ Both sides are equal (L.H.S=R.H.S)

So the answers a=11 and b=7

7. Let x be the larger digit and y be smaller digit. We are given that the sum of two digits is 30. So,

$x + y = 30$ …………………….(1)

The larger exceeds the smaller by 12

$x = 2y + 12$ ……………………(2)

Substitute (2) in (1).We will get,

$2y + 12 + y = 30$

$3y = 30 - 12$

$y = \frac{18}{3} = 6$

Substitute y=6 in equation (1) $x + y = 30$

$x + 6 = 30$

$x = 30 - 6 = 24$

So first digit is 6 and the second digit is 24

8. Let $x = age\ \ of\ son$

y=age of son

We are given that

$$y = 2x......................(1)$$

$$y - 20 = 4(x - 20)......................(2)$$

$$4x - 80$$

$$-20 + 80 = 4x - y$$

$$60 = 4x - y$$

$4x - y = 60$ …………………..(3)

Substitute (1) in (3)

$4x - 2x = 60$

$2x = 60$

$x = \frac{60}{2} = 30$

$y = 2x$

$2 \times 30 = 60$

$Age\ of\ \ son\ \ \ x = 30$

$Age\ of\ \ father\ \ \ y = 60$

9. *Let* $x = age\ of\ son$

y=age of son

We are given that

$$y - 10 = 4(x - 10)$$

$$\Rightarrow \ y - 10 = 4x - 40$$

$$y - 4x = -30..............(1)$$

And $y + 10 = 2(x + 10)$

$$\Rightarrow \qquad y + 10 = 2x + 20.$$

$$y - 2x = 10..................(2)$$

Subtract equation (2) from (1) $\Rightarrow \ -2x = -40$

$$x = \frac{-40}{-2} = 20$$

Substitute *x*=20 in equation (2), $y - 2x = 10.$

$$\Rightarrow \ y - 2(20) = 10.$$

$$y = 10 + 40 = 50$$

The present age of son is 20 and the present age of father is 30

10. *Let* $x = books$

y= pens

We are given that

$$3x + 6y = 36...........(1)$$

$$4x + 5y = 43.5.......(2)$$

$$(1) \times 4 \Rightarrow 12x + 24y = 144...........(3)$$

$$(2) \times 3 \Rightarrow 12x + 15y = 130.50........(4)$$

$$(3) - (4) \Rightarrow 9y = 13.50$$

$$y = \frac{13.50}{9} = 1.50$$

Put $y = 1.50$ in (1)

$$3x + 6y = 36$$

$$3x + 6(1.5) = 36$$

$$3x + 9 = 36$$

$$3x = 36 - 9$$

$$x = \frac{27}{3} = 9$$

Answer:

The price of books/dozen=Rupees 9

The price of pens/dozen=Rupees 1.5

11. $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$...............(1)

$\frac{1}{x}-\frac{1}{y}=\frac{1}{6}$..............(2)

Put $\frac{1}{x}=a$, and $\frac{1}{y}=b$

$\therefore (1)\Rightarrow a+b=\frac{1}{2}$..............(3)

$\therefore (2)\Rightarrow a-b=\frac{1}{6}$..............(4)

$(1)+(2)\Rightarrow 2a=\frac{4}{6}=\frac{2}{3}$

$a=\frac{2}{3}\times\frac{1}{2}=\frac{2}{6}=\frac{1}{3}$

Put $a=\frac{1}{3}$ in (3)

$\Rightarrow \frac{1}{3}+b=\frac{1}{2}$

$b=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

$a=\frac{1}{x}\therefore x=\frac{1}{a}=\frac{1}{1/3}$

$=1\times\frac{3}{1}=3$

$b=\frac{1}{y}\therefore y=\frac{1}{b}=\frac{1}{1/6}$

$=1\times\frac{6}{1}=6$

Answer x = 3, and y = 6

12. 6 kg of wheat and 8 kg of rice cost is Rs 144 and 4 kg of wheat and 10 kg of rice cost is Rs 152. Find the price of wheat and rice

We denote the price of wheat by x and the price of rice by y. Thus, we get simultaneous equation as,

$6x+8y=144$...........(1)

$4x + 10y = 152$...........(2)

$(1) \times 4 \Rightarrow 24x + 32y = 576$...........(3)
$(2) \times 6 \Rightarrow 24x + 60y = 912$...........(4)

Subtract equation (3) from equation (4). Then, we have

$$28y = 336$$

$$\therefore y = \frac{336}{28} = 12$$

Substitute y=12 in equation (1)

$$6x + 8y = 144$$
$$6x + 8(12) = 144$$
$$6x + 96 = 144$$
$$6x = 144 - 96$$
$$6x = 48$$

Answer:

$$\therefore x = \frac{48}{6} = 8$$

The price of wheat per kg=8
The price of rice per kg=12

13. $Q^d = 54 - 8p$

$Q^s = 10p + 18$

A equilibrium quantity demanded is equal quantity supplied. Thus, we have,

$$54 - 8p = 10p + 18$$
$$-8p - 10p = 18 - 54$$
$$-18p = -36$$
$$\therefore p = \frac{-36}{-18} = 2$$

Put p=2 in the demand equation

$$Q^d = 54 - 8p$$
$$Q^d = 54 - 8(2)$$
$$Q^d = 54 - 16$$
$$\therefore Q = 38$$

Thus, equilibrium price is Rs 2 and equilibrium quantity is 38 kg.

Essay Questions

1. *Solve* $3x + y + 2z = 11$
   $x + y + z = 6$
   $2x + y - z = 1$

2. Solve $x + y + z = 53$

$x+2y+3z=105$
$x+3y+4z=134$

3. Solve $2x-y+z=3$
$x+3y-2z=11$
$3x-2y+4z=1$

4. Solve $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=11$
$\frac{3}{x}+\frac{4}{y}-\frac{1}{z}=12$
$\frac{2}{x}-\frac{3}{y}-\frac{4}{z}=-29$

5. Solve $\frac{x}{2}-\frac{y}{3}=0$
$\frac{x}{4}+\frac{y}{2}=4$

6. Solve $2x+y-3z=7$
$3x-y-z=2$
$4x+3y+2z=8$

7. Solve $2x+y+z=3$
$x-3y-2z=8$
$3x-y-z=8$

8. Solve $x+2y+3z=26$
$x-3y+2z=1$
$3x+y-z=8$

ANSWERS

1. $3x+y+2z=11$...............(1)
$x+y+z=6$......................(2)
$2x+y-z=1$......................(3)

Take equation (1) and (2) and subtract equation (2) from equation (1)

$3x+y+2z=11$
$x+y+z=6$
$2x+z=5$...............(4)

Take equation (2) and (3) and subtract equation (3) from equation (2)

$x+y+z=6$
$2x+y-z=1$

$$-x+2z=5\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Multiply equation (5) by 2

$$-2x+4z=10\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Take equation (4) and (6) and add it.

$$2x+z=5$$
$$-2x+4z=10$$
$$5z=15$$

$$z=\frac{15}{5}=3.$$

Put z=3 in (4)

$$2x+z=5$$
$$2x+3=5$$
$$2x=5-3=2$$
$$x=\frac{2}{2}=1$$

Put z=3 and x=1 in (1)

$$3x+y+2z=11$$
$$3(1)+y+2(3)=11$$
$$y=11-9=2$$

<u>Answer: x = 1, y = 2, and z = 3</u>

2. $x+y+z=53\ldots\ldots\ldots\ldots\ldots(1)$

$$x+2y+3z=105\ldots\ldots\ldots\ldots(2)$$
$$x+3y+4z=134\ldots\ldots\ldots\ldots\ldots(3)$$

Take equation (1) and (2) and subtract equation (1) from (2)

$$x+2y+3z=105$$
$$x+y+z=53$$
$$y+2z=52\ldots\ldots\ldots\ldots\ldots(4)$$

Take equation (2) and (3) and subtract equation (2) from (3)

$$x+3y+4z=134$$
$$x+2y+3z=105$$
$$y+z=29\ldots\ldots\ldots\ldots\ldots(5)$$

Now,take equation (4) and (5) and subtract equation (5) from (4)

$$y+2z=52$$
$$y+z=29$$
$$z=23$$

Substitute Z = 23 in equation (5), $y+z=29$

$$y+23=29$$
$$y=29-23=6$$

Now substitute z=23 and y=6 in equation (1), $x+y+z=53$

$$x+6+23=53$$
$$x=53-29=24$$

Answer: x = 24, y = 6, and z = 23

3. $2x-y+z=3$...............(1)

$x+3y-2z=11$............(2)

$3x-2y+4z=1$...............(3)

Take equation (1) and multiply it by (2). Thus we have,

$$4x-2y+2z=6 \text{...............}(4)$$

Take equation (2) and (4) , add it

$$4x-2y+2z=6$$
$$x+3y-2z=11$$

$$5x+y=17 \text{...........}(5)$$

Take equation (2) and multiply it by (2). Thus we have,

$$2x+6y-4z=22 \text{............}(6)$$

Take equation (3) and (6) , add it

$$2x+6y-4z=22$$
$$3x-2y+4z=1$$

$$5x+4y=23 \text{.................}(7)$$

Now take equation (5) and (7). Then subtract equation (5) from (7)

$$5x+4y=23$$
$$5x+y=17.$$

$$3y=6, y=\frac{6}{3}=2$$

Substitute *y=2* in equation (5) $5x+2=17$

$$5x=15, x=\frac{15}{5}=3$$

Now, substitute x=3 and y=2 in equation (1)

$$2(3)-2+z=3$$
$$6-2+z=3$$
$$z=-1$$

Answer: ***x = 3, y = 2 , and z =- 1***

4.
$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=11$$
$$\frac{3}{x}+\frac{4}{y}-\frac{1}{z}=12$$
$$\frac{2}{x}-\frac{3}{y}-\frac{4}{z}=-29$$

Let $a=\frac{1}{x}$, $b=\frac{1}{y}$, and $c=\frac{1}{z}$

Thus the equation becomes;

$$a+b+c=11..............(1)$$
$$3a+4b-c=12..........(2)$$
$$2a-3b-4c=-29..........(3)$$

Take equation (1), (2) and add it

$$a+b+c=11.$$
$$3a+4b-c=12$$

$$4a+5b=23.............(4)$$

Multiply equation (1) by 4. Thus we have

$$4a+4b+4c=44..............(5)$$

Add equation (3) and (5)

$$4a+4b+4c=44$$
$$2a-3b-4c=-29$$

$$6a+b=15..........(6)$$

Multiply equation (6) by 5

$$30a+5b=75..........(7)$$

Subtract equation (4) from (7)

$$30a + 5b = 75$$

$$4a + 5b = 23$$

$$26a = 52$$

$$a = \frac{52}{26} = 2$$

Substitute in equation (6) , $6(2) + b = 15$

$$b = 15 - 12 = 3$$

Now substitute a=2, and b=3 in equation (1)

$$2 + 3 + c = 11$$

$$c = 11 - 5 = 6$$

$x = \frac{1}{a} = \frac{1}{2}$, $y = \frac{1}{b} = \frac{1}{3}$, and $z = \frac{1}{c} = \frac{1}{6}$

5. $\frac{x}{2} - \frac{y}{3} = 0.........(1)$

$\frac{x}{4} + \frac{y}{2} = 4..........(2)$

We can rewrite the first equation as

$$\frac{3x - 2y}{6} = 0$$

Do cross multiplication. Then, we have

$$3x - 2y = 0...............(3)$$

Next take equation (2). We get,

$$\frac{2x + 4y}{8} = 4$$

$$2x + 4y = 32............(4)$$

$$3x - 2y = 0...............(3)$$

$$2x + 4y = 32............(4)$$

$$(3)\times 4 \Rightarrow 12x - 8y = 0..............(5)$$

$$(4)\times 2 \Rightarrow 4x + 8y = 64..............(6)$$

Add equation (5) and (6), we have

$$16x = 64$$

$$x = \frac{64}{16} = 4$$

Put x=4 in equation (3) or (4)

$$3x - 2y = 0$$

$$3(4) - 2y = 0$$

$$12 - 2y = 0$$

$$2y = 12$$

$$y = \frac{12}{2} = 6$$

S0 x = 4, and y = 6

Check the answer

$$\frac{x}{2} - \frac{y}{3} = 0.......(1)$$

$$\frac{4}{2} - \frac{6}{3} = 0$$

$$2 - 2 = 0$$

$$\frac{x}{4} + \frac{y}{2} = 4$$

$$\frac{4}{4} + \frac{6}{2} = 4$$

$$1 + 3 = 4$$

6. $2x + y - 3z = 7.........(1)$

$3x - y - z = 2.........(2)$

$4x + 3y + 2z = 8.........(3)$

Add equation (1) and (2).

$$5x - 4z = 9.........(4)$$

Multiply equation (2) by 3

$$9x - 3y - 3z = 6.........(5)$$

Take equation (5) and (3) and add it

$$9x - 3y - 3z = 6$$

$$4x + 3y + 2z = 8$$

$$(3) + (5) \Rightarrow 13x - z = 14.........(6)$$

Multiply equation (6) by 4

$$52x - 4z = 56.........(7)$$

Take equation (4) and (7) and subtract equation (4) from (7)

$$52x - 4z = 56.........(7)$$

$$5x - 4z = 9.........(4)$$

$$(7) - (4) \Rightarrow 47x = 47$$

$$\therefore x = \frac{47}{47} = 1$$

Put x=1 in (4) $\quad 5x - 4z = 9$

$$5(1) - 4z = 9.........(4)$$

$$4z = -4$$

$$\therefore z = \frac{-4}{4} = -1$$

Put x=1, z=-1 in (1)

$$2x + y - 3z = 7.........(1)$$

$$2(1) + y - 3(-1\_) = 7.$$

$$2+y+3=7.$$

$$\therefore y=7-5=2$$

Solution: ***x=1, y=2, and z=-1***

7. $2x+y+z=3..........(1)$
 $x-3y-2z=8.......(2)$
 $3x-y-z=6...........(3)$

Add equation (1) and (3)

$$2x+y+z=3$$

$$\underline{3x-y-z=6}$$

$$5x=9$$

$$\therefore x=\frac{9}{5}$$

Multiply equation (1) by 2

$$(1)\times 2 \Rightarrow 4x+2y+2z=6..........(4)$$

Add equation (4) and (2)

$$4x+2y+2z=6$$

$$\underline{x-3y-2z=8}$$

$$5x-y=14.....(5)$$

Substitute $x=\frac{9}{5}$ in equation (5)

$$5\times\frac{9}{5}-y=14.$$

$$\frac{45}{5}-y=14$$

$$9-y=14$$

$$-y=14-9=5$$

$$\therefore y=-5$$

Substitute $x = \frac{9}{5}$ and $y = -5$ in equation (1)

$$2 \times \frac{9}{5} + (-5) + z = 3.$$

$$\frac{18}{5} - 5 + z = 3$$

$$\frac{18 - 25}{5} + z = 3$$

$$\frac{-7}{5} + z = 3$$

$$z = 3 + \frac{7}{5}$$

$$z = \frac{15 + 7}{5}$$

$$\therefore z = \frac{22}{5}$$

Thus, $x = \frac{9}{5}$, $y = -5$ and $z = \frac{22}{5}$

8. $x + 2y + 3z = 26 ....(1)$
   $x - 3y + 2z = 1 ....(2)$
   $3x + y - z = 8 ....(3)$

Subtract equation (2) from equation (1)

$$x + 2y + 3z = 26$$
$$\underline{x - 3y + 2z = 1}$$
$$5y + z = 25 ....(4)$$

Multiply equation (1) by 3

$$(1) \times 3 \Rightarrow 3x + 6y + 9z = 78 ....(5)$$

Subtract equation (3) from (4)

$$3x + 6y + 9z = 78$$

$$\underline{3x + y - z = 8}$$

$$5y + 10z = 70 \ldots\ldots\ldots\ldots(6)$$

Subtract equation (4) from equation (6)

$$5y + 10z = 70$$

$$\underline{5y + z = 25}$$

$$9z = 45$$

$$\therefore z = \frac{45}{9} = 5$$

Substitute z=5 in equation (4)

$$5y + z = 25$$

$$5y + 5 = 25$$

$$5y = 25 - 5$$

$$\therefore y = \frac{20}{5} = 4$$

Substitute y=4 and z=5 in equation (1)

$$x + 2(4) + 3(5) = 26$$

$$x + 8 + 15 = 26$$

$$x = 26 - 23$$

$\therefore x = 3$ **Solution:** ***x=3, y=4, and z=5***

MODULE III

Basic Mathematical Concepts – Quadratic Equations

In the previous chapter we learned how to solve simple linear equations and linear simultaneous equations. This chapter discusses how to solve quadratic equation. After reading this chapter, you will be able to:

- Find the roots of quadratic equation
- Various methods to solve quadratic equations
- The uses of quadratic equation in Economics

Quadratic Function

A quadratic function is one which involves at most the second power of the independent variable in the equation $ax^2 + bx + c$ where a and b are coefficients and c is constant. The graph of a quadratic function is parabola.

Quadratic Equation

Equation of degree two is known as quadratic equation. This is one of the non-linear equations. The general format of this equation can be written as $ax^2 + bx + c = 0$. Where *a*, *b* and *c* are real numbers and *a* is not equal to zero. The numbers *b* and *c* can also be zero .The number *a* is the coefficient of x^2, *b* is the coefficient of *x*, and *c* is the constant term. These numbers can be positive or negative.

For example: $x^2 + 5x + 6 = 0$. Solving this equation we get two values for x. These two values are known as the roots of the quadratic equation.

Methods to Find the Roots of the Quadratic Equation:

The general quadratic equation $ax^2 + bx + c = 0$ can be solved by one of the following methods

1) By factorization method
2) By quadratic formula
3) By completing the square method

1. By Factorization Method

The factorization is an inverse process of multiplication. When an algebraic expression is the product of two or more quantities, each these quantities is called factor. Consider this example, if (x+3) be multiplied by (x+2) the product is $x^2 + 5x + 6$ .The two expressions

A. Procedures to Factorise the Quadratic Equation $x^2 + bx + c$

1. Factor the first term (x^2 is the product of x and x)
2. Find two numbers that their sum becomes equal to b (the coefficient of x) and the product becomes equal to c (the constant term)
3. Equate these two expressions with zer0.
4. Apply Zero Property: if we have two expressions multiplied together resulting in zero, then one or both of these must be zero. In other words, if m and n are complex numbers, then m × n= 0, iff m=0 or n=0

Example: Find the roots of $x^2 - 5x + 6 = 0$

Factors of x^2 are x and x. Next find two numbers whose sum is -5 and the product is six. The numbers are -2 and -3

$(x-3)\ (x-2) = 0$

Thus either $(x-3)$ or $(x-2)$ should be equal to zero

$(x-3) = 0$, *x=3*

$(x-2) = 0$ *x=2*

OR

$x^2 - 5x + 6 = 0$

This equation can rewrite as

$x^2 - 3x - 2x + 6 = 0$ -5 broken into two numbers

$x(x-3) - 2(x-3) = 0$ by factorising the first two terms and last two terms

$(x-3)(x-2) = 0$ by noting the common factor of $x + 3$

$(x-3) = 0$ or $(x-2) = 0$

So x=3 or x=2

2. Quadratic Formula

The roots of a quadratic equation $ax^2 + bx + c = 0$ can be solved by the following quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We can split this formula into two parts as

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad , \quad \text{and}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Accordingly, sum of roots: $\alpha + \beta = -\frac{b}{a}$ *and*

Product of roots $\alpha \times \beta = \frac{c}{a}$

Example: Find the roots of $6x^2 - 10x + 4 = 0$

Here a=6, b= -10, and c=4

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) + \sqrt{(-10)^2 - 4 \times 6 \times 4}}{2 \times 6}$$

$$= \frac{10 + \sqrt{100 - 96}}{2 \times 6}$$

$$= \frac{10 + \sqrt{4}}{2 \times 6} \qquad = \frac{10 + 2}{12} = 1$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) - \sqrt{(-10)^2 - 4 \times 6 \times 4}}{2 \times 6}$$

$$= \frac{10 - \sqrt{100 - 96}}{2 \times 6}$$

$$= \frac{10 - \sqrt{4}}{2 \times 6} \qquad = \frac{10 - 2}{12} = \frac{8}{12} = \frac{2}{3}$$

Answer: x=1 or x= $= \frac{2}{3}$

3. Completing the Square

This is based on the idea that a perfect square trinomial is the square of a binomial. Consider the following examples:

$x^2 + 10x + 25$ is a perfect trinomial because this can be written in the square of a binomial as $(x+5)^2$. Consider $x^2 - 6x + 9$, this equation can be written as $(x-3)^2$

Now look at the constant terms of the above two equations, it is the square of half of the coefficient of x equals the constant term;

$\left(\frac{1}{2}\times 10\right)^2 = 25$, and $\left(\frac{1}{2}\times(-6)\right)^2 = 9$. Thus we use this idea in the completing the square method.

Steps under Completing the Square Method

1) Rewrite the equation $x^2 + bx - c$ in to $x^2 + bx = c$
2) Add to each side of the equation $\left(\frac{1}{2}b\right)^2$
3) Factor the perfect-square trinomial
4) Take the square root of both sides of the equation
5) Solve for x

Example: Solve $x^2 + 6x - 4 = 0$ by completing the square method.

Solution: First rewrite the equation as $x^2 + bx = c$

$$x^2 + 6x = 4$$

Add $\left(\frac{1}{2}b\right)^2$ on both sides. Here b=6 and $\left(\frac{1}{2}b\right)^2 = 3^2 = 9$

$$x^2 + 6x + 9 = 4 + 9$$

$$(x+3)^2 = 13$$

Now take the square root of both sides

$$\sqrt{(x+3)^2} = \sqrt{13}$$

$$(x+3) = \pm\sqrt{13}$$

$$(x = -3 \pm \sqrt{13}$$

So x= $-3+\sqrt{13}$ **or** $-3-\sqrt{13}$

OR

Rewrite the equation so that it becomes complete square. To rewrite the equation take the half of the coefficient of x, add or subtract (depends on the sign of coefficient of x) with the x and square it. Here, $\left(\frac{1}{2}b\right) = 3$

$$(x+3)^2 = x^2 + 6x + 9$$

$$\Rightarrow x^2 + 6x - 4 = (x+3)^2 - 9 - 4 \quad \text{Deduct 9 from the expression}$$

$$= (x+3)^2 - 13 = 0$$

Take 13 to right side and put square root on both sides

$$\Rightarrow \sqrt{(x+3)^2} = \sqrt{13}$$

$$(x+3) = \pm\sqrt{13}$$

$$(x = -3 \pm \sqrt{13}$$

So x= $\underline{-3+\sqrt{13}}$ **or** $\underline{-3-\sqrt{13}}$

SIMULTANEOUS QUADRATIC EQUATIONS

In the second module you have learned simultaneous equations where both equations are linear. In this section we would learn how to solve simultaneous quadratic equation. We start with simultaneous equations where one equation is linear and other is quadratic. This will give you a quadratic equation to solve.

Example: solve simultaneous equations

$$y = x^2 - 1$$

$$y = 5 - x$$

Solution:

$$y = x^2 - 1 ....(1)$$

$$y = 5 - x ....(2)$$

Subtract equation (2) from (1)

$(y = x^2 - 1)$ - $(y = 5 - x)$ = $x^2 - 1 - 5 + x$ y will be cancelled

$$x^2 + x - 6 = 0$$

Now solve this quadratic equation either by factorisation method or by quadratic formula.

By factorisation $(x+3)\,(x-2) = 0$

So $x + 3 = 0$ or $x - 2 = 0$ Therefore, *x=-3 or x= 2*

OR

Substitute equation (2) in equation (1)

$$\Rightarrow x^2 - 1 = 5 - x$$

$$x^2 + 1 - 5 + x = x^2 + x - 6 = 0$$

By factorisation $(x+3)\,(x-2) = 0$

So $x + 3 = 0$ or $x - 2 = 0$ Therefore, *x=-3 or x= 2*

Now we can move to simultaneous quadratic equations

Solve simultaneous quadratic equations

$$y = 2x^2 + 3x + 2$$

$$y = x^2 + 2x + 8$$

Solution:

$$y = 2x^2 + 3x + 2 \text{.............}(1)$$

$$y = x^2 + 2x + 8 \text{..............}(2)$$

Now equate equation (1) and equation (2)

$$2x^2 + 3x + 2 = x^2 + 2x + 8$$

$$2x^2 + 3x + 2 - x^2 - 2x - 8 = 0$$

$$x^2 + x - 6 = 0$$

By factorisation $(x+3)\ (x-2) = 0$

So $x + 3 = 0$ or $x - 2 = 0$ Therefore, *x=-3 or x= 2*

ECONOMIC APPLICATION

The quadratic equation has application in the field of economics. Here we discuss two important Economics application of quadratic equation.

Supply and Demand

The quadratic equation can be used to represent supply and demand function. Market equilibrium occurs when the quantity demanded equals the quantity supplied. If we solve the system of quadratic equations for quantity and price we get equilibrium quantity and price. For example: The supply function for a commodity is given by $p = q^2 + 50$ and the demand function is given by $p = -10q + 650$ find the point of equilibrium.

Solution:

At the equilibrium demand is equal to supply

$$q^2 + 50 = -10q + 650$$

$$q^2 + 50 + 10q - 650 = 0$$

$$q^2 + 10q - 600 = 0$$

By factorization $(q+30)(q-20) = 0$

So q=-30 or 20

Since negative quantity is not possible we take positive value as quantity. Thus the equilibrium quantity is 20. Put q=20 in either demand function or supply function.

Supply function $p = q^2 + 50$

$$p = (20)^2 + 50$$

P=450

Cost and Revenue

The cost and revenue function can be represented by the quadratic equation. The total cost is composed of two parts, fixed cost and variable cost. The fixed cost remains the same regardless of the number of units produced. It does not depend on the quantity produced. Rent on building and machinery is an example for the fixed cost. The variable cost is directly related to the number of unit produced. Cost on raw material is an example for the variable cost. Thus,

TC=FC+VC

The revenue of the firm depends on the number of unit sold and its price.

TR= P×Q. Where TR denotes total revenue, P shows price, and Q denotes quantity.

BREAK-EVEN POINT

Firm's break-even point occurs when total revenue is equal to total cost.

Steps: 1-Find the profit function

2- Equate profit function with zero and solve for q.

If we deduct total cost function from total revenue function we get profit function.

Example: A firm has the total cost function $TC = 10.75q^2 + 5q + 125$ and demand function $p = 180 - 0.5q$ Find revenue function, profit function, and break-even point .

Solution:

Total revenue function= price × quantity (TR=p × q)

$$p \times q = (180 - 0.5q)q$$

$$= 175q - 0.5q^2$$

Profit function= Total revenue- total cost $(\pi = TR - TC)$

$$= 175q - 0.5q^2 - 10.75q^2 + 5q + 125$$

$$= 180q - 11.25q^2 - 125$$

$$11.25q^2 - 180q - 125$$

Break –even point

$$11.25q^2 - 180q - 125 = 0$$

Use quadratic formula

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here a=11.25, b=-180, and c= -125

$$q = \frac{-(-180) \pm \sqrt{(-180)^2 - 4 \times 11.25 \times -125}}{2 \times 11.25}$$

$$= \frac{180 \pm \sqrt{32400 + 5625}}{22.5}$$

$$= \frac{180 \pm \sqrt{38025}}{22.5}$$

$$= \frac{180 \pm 195}{22.5}$$

$$= \frac{180+195}{22.5} = 16.66 \approx 17$$

$$= \frac{180-195}{22.5} = -0.67$$

Since negative quantity is not possible we take positive value as quantity. Thus the break-even point is 17.

QUESTIONS

Multiple Choice Questions

1. The roots of the equation $x^2 + 7x + 12 = 0$ are
   a) 2, 5　　b) -5, 2　　c) 3, 4　　d) -3, -4

2. The product of two numbers is 143 and the sum is 24, then the numbers are.

   a) 18, 16　　b) 11, 13　　c) 14, 10　　d) 15, 9

3. $9x^2 + kx + 81 = 0$ has equal roots when k = …….
   a) ±54　　b) ±34　　c) ±64　　d) ±44

4. The sum of two consecutive numbers is 13 and the sum of its square is 85, then the numbers are.
   a) 8, 5　　b) 9, 4　　c) 6,7　　d) None of the above

5. The roots of the equation $x^2 - 19x + 90 = 0$ are

   a) -9, 10　　b) 10, -9　　c) -9, -10　　d) 9, 10

6. $x^2 + 4x + 6 = 0$, has………………… roots

   a) Imaginary　　b) Equal　　c) Rational　　d) None-of the above

7. $x^2 + 6x + 9 = 0$, roots are……………
   a) Rational and unequal　　b) Imaginary
   c) Rational but equal　　d) Equal to each other
8. $x^2 - 9x + 18 = 0$, roots are……………
   a) Equal to each other　　b) Imaginary
   c) Rational but unequal　　d) None of the above

9. The discriminant of the equation $3x^2 - 8x + 4 = 0$ is
   a) 18　　b) 16　　c) -16　　d) -18

10. The discriminant of the equation $x^2 + 4x + 4 = 0$ is
   a) 0 b) 4 c) 1 d) None of the above

11. Sum of the two roots of a quadratic equation is 7 and their product is 12, equations is

   a) $x^2 + 12x - 7 = 0$ b) $x^2 + 12x + 7 = 0$
   c) $x^2 - 7x + 12 = 0$ d) $x^2 + 7x + 12 = 0$

12. The roots of the equation $x^2 - 16 = 0$ are
   a) 8, 2 b) 4, -4 c) 4 d) -4

13. The roots of the equation $12x^2 - 30x + 18 = 0$ are

   a) 1, 2 b) 2, 3 c) 3, 4 d) None of the above

14. X and Y are two numbers such that their sum is 15 and the sum of their reciprocal is $\frac{3}{10}$, then the value of X and Y are…………..
   a) 6, 9 b) 5, 10 c) 12, 3 d) 8, 7
15. Find two consecutive numbers whose square has the sum 41
   a) 7, 8 b) 6, 7 c) 4, 5 d) None of the above
16. The value of k for which the quadratic equation $4x^2 - 2(k+1)x + (k+1) = 0$
   a) 5, -3 b) 5, 3 c) -2, 3 d) 1, 2
17. The value of k for which the quadratic equation $(k-5)x^2 + 6(k-5)x + 9 = 0$ has equal roots.
   a) 5 b) -5 c) 6 d) -6
18. If one root of the equation is $3 + \sqrt{5}$, then equation is
   a) $x^2 + 3 + \sqrt{5}x + 3 = 0$ b) $x^2 - 6x + 4 = 0$
   c) $x^2 + 3 - \sqrt{5}x + 3 = 0$ d) $x^2 - 4x + 6 = 0$
19. If the roots of the equation $2x^2 + 8x - y^3 = 0$ are equal then the value of y is
   a) 2 b) 3 c) -3 d) -2

20. $x^2 + 5x - 14 = 0$, the roots of the equation are
   a) (2, 5) b) (-2, 5) c) (7, 2) d) (-7, 2)

21. $x^2 + 9x + 18 = 0$, the factors of the equation are
   a) (x+2)(x+9) b) (x+3)(x+6)
   c) (x-2)(x-9) d) none of the above

22. $x^2 - 6x + 9 = 0$, roots are

a) (3, 3) b) (-3, -3) c) (3, -3) d) (2, 9)

23. $2x^2 - 3 = 0$, roots are

a) (0, 3/2) b) (0, 3) c) (2/3, 0) d) none of the above

24. $(x-2)^2 = ...$

a) $x^2 + 4x + 2 = 0$ b) $x^2 - 4x + 2 = 0$

c) $x^2 - 4x + 4 = 0$ d) $x^2 + 4x + 4 = 0$

25) $x^2 - 4x + 3 = 0$, $(\alpha - \beta)^2 = ...$

a) 4 b) 6 c) 2 d) 1

ANSWERS

1. d) -3, -4
2. b) 11, 13
3. a) ±54
4. c) 6,7
5. d) 9, 10
6. a) Imaginary
7. d) Equal to each other
8. c) Rational but unequal
9. c) 6
10. b) $x^2 - 6x + 4 = 0$
11. d) -2
12. d) (-7, 2)
13. b) (x+3)(x+6)
14. b) (-3, -3)
15. a) (0, 3/2)
16. $x^2 - 4x + 4 = 0$
17. a) 4

Very Short Answer Type Questions

1. Solve the equation by factorization method $x^2 - 8x + 16 = 0$
2. Examine the nature of the roots of $3x^2 - 8x + 4 = 0$
3. Examine the nature of the roots of $x^2 + 4x + 4 = 0$
4. Examine the nature of the roots of $x^2 - 5x + 6 = 0$
5. If the sum of a number and its reciprocal adds to $\frac{5}{2}$, then what is the number?
6. Construct a quadratic equation for the roots 6 and 4.
7. Find two consecutive numbers whose sum of squares is 85.

8. Solve the equation by factorization method $x^2 - x - 6 = 0$
9. Solve the equation by factorization method $x^2 - 4x + 3 = 0$
10. Solve the equation by factorization method $3x^2 - 14x + 8 = 0$
11. Solve x+y=5
    xy=6
12. Examine the nature of the roots of $x^2 - 15 + 56 = 0$
13. Write a short note on the importance of discriminant in the quadratic equation.
14. Solve the equation by factorization method $14x^2 + 45x + 9 = 0$
15. Solve $6x^2 - 6x - 36 = 0$

Answers

1. $x^2 - 8x + 16$

This equation can rewrite as

$x^2 - 4x - 4x + 16 = 0$ -8 broken into two numbers

$x(x-4) - 4(x-4) = 0$ By factorising the first two terms and last two terms

$(x-4)(x-4) = 0$ by noting the common factor of x -4

$(x-4) = 0$

So x=4, Both roots are same

2. $3x^2 - 8x + 4 = 0$

The discriminant $D = b^2 - 4ac$

Here, a=3, b=-8, and c=4

$D = b^2 - 4ac = (-8)^2 - 4 \times 3 \times 4$

=64 - 48=16

Since D >0 and perfect square. The roots are real, rational and unequal

3. $x^2 + 4x + 4 = 0$

Here, a=1, b=4, and c=4

$D = b^2 - 4ac = (4)^2 - 4 \times 1 \times 4$

=16- 16=0

Since D =0. The roots are real and equal

4. $x^2 - 5x + 6 = 0$

Here, a=1, b=-5, and c=6

$D = b^2 - 4ac = (-5)^2 - 4 \times 1 \times 6$

=25 - 24=0

Since D >0. The roots are rational and unequal

5. Let be the number x.

Then $x+\left(\frac{1}{x}\right)=\frac{5}{2}$

$$\frac{x^2+1}{x}=\frac{5}{2}$$

$$\Rightarrow 2x^2-5x+2=0$$

$$\Rightarrow 2x^2-4x-x+2=0$$

$$\Rightarrow 2x(x-2)-1(x-2)=0$$

$$\Rightarrow (2x-1)(x-2)=0$$

So, x= ½ or 2

6. The equation can be constructed as

$$x^2-(\text{Sum of roots})\times x+(\text{product of roots})=0$$

Here, $\alpha=6$ and $\beta=4$

$\alpha+\beta=10$, and $\alpha\times\beta=24$

Thus the equation is $x^2-10x+24=0$

7. Let the first number is x and the second number is x+1

From the question $x^2+(x+1)^2=85$

$$x^2+x^2+2x+1=85$$

$$2x^2+2x-84=0$$

$$x^2+x-42=0$$

By factorization $(x+7)(x-6)=0$

X= - 7 , x=6

From the question the two numbers are consecutive numbers. Thus numbers are 6 and 7

8. $x^2-x-6=0$

Find two numbers whose sum is equal to the value of the coefficient of x (-1) and the product is equal to the value of constant (-6)

The numbers are -3 and 2. Thus the factors are

$$(x-3)(x+2)=0$$

i) $(x-3)=0$, $x=3$ or ii) $(x+2)=0$, $x=-2$

9. $x^2-4x+3=0$

Find two numbers whose sum is equal to the value of the coefficient of x (-4) and the product is equal to the value of constant (-3)

The numbers are -3 and -1. Thus the factors are

$$(x-3)(x-2)=0$$

i) $(x-3)=0$ or ii) $(x-1)=0$

$x=3$ $x=1$

10. $3x^2-14x+8=0$

Here, the coefficient of x^2 is not unity. So multiply the coefficient of x^2 by Constant. 3×8=24

Now, find two numbers whose sum is -14 and the product is 24

The numbers are -12 and -2

This equation can rewrite as

$$3x^2-14x+8=3x^2-12x-2x+8=0$$

$$3x(x-4)-2(x-4)=0$$

$$(3x-2)(x-4)=0$$

i) $(3x-2)=0$ or ii) $(x-4)=0$

$3x=2$ $x=4$

$x=\frac{2}{3}$

11. x+y=5………….(1)

xy=6…………..(2)

From the equation (1) y=5-x…….(3)

Put equation (3) in (1). Thus, we have,

$$x(5-x)=6$$

$$5x-x^2=6 \quad =-x^2+5x-6=0$$

$$\div -1 \Rightarrow x^2-5x+6=0$$

By factorization $(x-3)(x-2)=0$

So, x=3 or x=2

When x=3, y=5-3=2

When x= 2, y=5-2=3

***Answer:* x = 3, y = 2**

X = 2, y = 3

12. $x^2 - 15 + 56 = 0$

Here, a=1, b=-15, and c=56

$D = b^2 - 4ac = (-15)^2 - 4 \times 1 \times 56$

=225 - 224=1

D>0, there are two distinct real solutions

13.The value of the expression $b^2 - 4ac$ is called the discriminant. The nature of roots of the quadratic equation can be predicted by the value of the discriminant. Generally it is denoted by D.

If $D < 0 \Rightarrow$ Roots are imaginary and of the form p+iq and p-iq

$D = 0 \Rightarrow$ Roots are equal to each other

$D > 0$ and D is a perfect square $\Rightarrow$ the roots are rational but unequal

$D > 0$ and D is not aperfect square $\Rightarrow$ the roots are irrational and of the form $p + \sqrt{q}$ and $p - \sqrt{q}$

14. $14x^2 + 45x + 9 = 0$

Here, the coefficient of x^2 is not unity. So multiply the coefficient of x^2 by constant. 14×9=126

Now, find two numbers whose sum is 45 and the product is 126

The numbers are 42 and 3

This equation can rewrite as

$$14x^2 + 45x + 9 = 14x^2 + 42x + 3x + 9 = 0$$

$$14x(x+3) + 3(x+3) = 0$$

$$(14x+3)(x+3) = 0$$

i) $(14x+3) = 0$ or ii) $(x+3) = 0$

$14x = -3$ $\quad x = -3$

$x = -\frac{3}{14}$

15. $6x^2 - 6x - 36 = 0$

$\div 6 \Rightarrow x^2 - x - 6 = 0$

Now, find two numbers whose sum is -1 and the product is -6

The numbers are -3 and 2

$$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$$

$(x-3) = 0$ $\quad\quad$ $(x+2) = 0$

X=3 $\quad\quad$ x=-2

Short Answer Type Questions

1. Find the roots of the equation by quadratic formula $3x^2 - 8x + 4 = 0$
2. *Solve* $4x^2 - 4x + 2 = 0$
3. *Solve* $2x^2 - 6x - 3 = 0$
4. The product of two numbers is 3200 and when the larger number is divided by the smaller number the result is 2. Find the numbers.
5. *Solve* $y = x^2 - 2x + 2$
   $y = 2x - 2$
6. *Solve* $y = x^2 + 5x + 5$
   $y = 2x + 15$
7. Divide 75 in to two parts so that the sum of their reciprocal is 1/18.
8. The sum of two consecutive numbers is 13 and the sum of its square is 85. Find numbers.
9. Find the roots of the equation $3x^2 - 14x + 8 = 0$
10. $Q^s = p^2 + 30p - 180$, $Q^d = 130 - 10p$ are supply and demand function of a firm respectively. Find equilibrium quantity and price.
11. Demand for goods of a firm is given by the equation 2pq=200 and supply is given by the equation 40+6p=2q, where p is the price and q is the quantity. Find equilibrium quantity and price.
12. Solve $4x^4 - 10x^2 + 36 = 0$
13. Solve $x^8 - 82x^4 + 81 = 0$
14. *Solve* $2x - 2y - 24 = 0$
    $2x^2 + 2y^2 - 148 = 0$
15. Solve $x + y = 3$
    $\frac{x}{y} + \frac{y}{x} = \frac{5}{2}$

ANSWERS

1. $3x^2 - 8x + 4 = 0$

Here a=3, b= -8, and c=4

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-8)+\sqrt{(-8)^2-4\times 3\times 4}}{2\times 3}$$

$$= \frac{8+\sqrt{64-48}}{6}$$

$$= \frac{8+\sqrt{16}}{6} \qquad = \frac{8+4}{6} = 2$$

$$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{8-\sqrt{64-48}}{6}$$

$$= \frac{8-\sqrt{16}}{6} \qquad = \frac{8-4}{6} = \frac{4}{6} = \frac{2}{3}$$

Answer: x = 2 or x $= \frac{2}{3}$

2. $4x^2 - 4x + 2 = 0$

Here a=4, b= -4, and c=2

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-4)+\sqrt{(-4)^2-4\times 4\times 2}}{2\times 4}$$

$$= \frac{4+\sqrt{16-32}}{8}$$

$$= \frac{4+\sqrt{-16}}{8} \qquad = \frac{4+i\sqrt{16}}{8}$$

$$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{4-\sqrt{16-32}}{8}$$

$$= \frac{4-\sqrt{-16}}{8} \qquad = \frac{4-i\sqrt{16}}{8}$$

Answer: x $= \frac{4+i\sqrt{16}}{8}$ ***or x*** $= \frac{4-i\sqrt{16}}{8}$

3. $2x^2 - 6x - 3 = 0$

Here *a*=2, *b*= -6, and *c*=-3

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-6)+\sqrt{(-6)^2-4\times 2\times -3}}{2\times 2}$$

$$= \frac{6+\sqrt{36+24}}{4}$$

$$= \frac{6+\sqrt{60}}{4} \qquad = \frac{6+7.75}{4} = 3.44$$

$$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{6-\sqrt{36+24}}{4}$$

$$= \frac{6-\sqrt{60}}{4} \qquad = \frac{6+7.75}{4} = -0.4375$$

Answer: x = 3.44 or x = -0.4375

4. Let x be the larger number and y be the small number

By the question $x \times y = 3200............(1)$ and

$$\frac{x}{y} = 2$$

$$\Rightarrow x = 2y..................(2)$$

Substitute equation (2) in (1) . Thus we have,

$$2y \times y = 3200$$

$$2y^2 = 3200$$

$$\Rightarrow y^2 = \sqrt{1200} = 40$$

Substitute *y=40 equation in (2)* $x = 2y,$

So x=80

Answer x = 80 and y = 40

5. $y = x^2 - 2x + 2$................(1)

$y = 2x - 2$.........................(2)

Equate equation (1) and (2)

$x^2 - 2x + 2 = 2x - 2$

$x^2 - 2x + 2 - 2x + 2 = 0$

$x^2 - 4x + 4 = 0$

By factorisation

$(x-2)(x-2) = 0$

Therefore, x=2,

6. $y = x^2 + 5x + 5$.....................(1)

$y = 2x + 15$.......................(2)

Equate equation (1) and (2)

$x^2 + 5x + 5 = 2x + 15$

$x^2 + 5x + 5 - 2x - 15 = 0$

$x^2 + 3x - 10 = 0$

By factorisation

$(x-2)(x+5) = 0$

Therefore, x= 2 or x= -5

7. Let the parts be x and 75-x

By the question $\frac{1}{x} + \frac{1}{75-x} = \frac{1}{18}$

$$\frac{75 - x + x}{x(75-x)} = \frac{1}{18}$$

$$1350 = 75x - x^2$$

$$x^2 - 75x + 1350 = 0$$

$$x^2 - 30x - 45x + 1350 = 0$$

$$x(x-30) - 45(x-30) = 0$$

$$(x-45)(x-30) = 0$$

Thus x = 45, 30

8. Let X and Y are two numbers

From the question $(x+y) = 13 \quad \Rightarrow y = 13 - x$

$$(x^2 + y^2) = 85$$

Put $y = 13 - x$ in equation

$$(x^2 + (13-x)^2 = 85$$

$$x^2 + 169 - 26x + x^2 = 85$$

$$2x^2 - 26x + 84 = 0$$

$$x^2 - 13x + 42$$

By factorization $(x-6)(x-7) = 0$

i) $x-6=0$ or $x-7=0$

$x=6$ $x=7$

Thus, the two consecutive numbers are 6 and 7

9. $3x^2-14x+8=0$

Here a=3, b= -14, and c=8

$$\alpha=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-14)+\sqrt{(-14)^2-4\times3\times8}}{2\times3}$$

$$=\frac{14+\sqrt{196-96}}{6}$$

$$=\frac{14+\sqrt{100}}{6}$$

$$=\frac{14+10}{6}\quad=\frac{24}{6}=4$$

$$\beta=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-14)-\sqrt{(-14)^2-4\times3\times8}}{2\times3}$$

$$=\frac{14-\sqrt{196-96}}{6}$$

$$=\frac{14-\sqrt{100}}{6}$$

$$=\frac{14-10}{6}\quad=\frac{4}{6}=\frac{2}{3}$$

Answer: x = 4 or x = 2/3

10. $Q^s=p^2+30p+80$, $Q^d=420-10p$

At equilibrium $Q^s = Q^d$

$$p^2+30p+80=420-10p$$

$$p^2+30p+80-420+10p=0$$

$$p^2+40p-420=0$$

$$p=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

a=1, b=40, c=-420

$$= \frac{-(40) \pm \sqrt{(40)^2 - 4 \times 1 \times -340}}{2 \times 1}$$

$$= \frac{-40 \pm \sqrt{1600 + 1360}}{2}$$

$$= \frac{-40 + \sqrt{2960}}{2}$$

$$= \frac{-40 \pm 54.40588}{2}$$

$$= \frac{-40 + 54.40}{2} = 7.20$$

$$= \frac{-40 - 54.40}{2} = -47.2$$

Take positive value as price. Thus equilibrium price is 7.20

$$Q^d = 420 - 10p$$

To get equilibrium quantity put p=7.20 in demand function

$$Q^d = 420 - 10(7.2)$$

$$420 - 72 = 348$$

Answer: Equilibrium quantity is348 and equilibrium price is 7.20

11. $2pq = 200............(1)$

$40 + 6p = 2q............(2)$

From the equation (2)

$$q = \frac{(40 + 6p)}{2}$$

$$\Rightarrow q = 20 + 3p.............(3)$$

Substitute equation (3) in equation (1)

$$2p(20 + 3p) = 200$$

$$40p + 6p^2 = 200$$

$$\Rightarrow 6p^2 + 40p - 200 = 0$$

$$\div 2 \Rightarrow 3p^2 + 20p - 100 = 0$$

Apply quadratic formula. Here a=3, b=20, and c= -100

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(20) \pm \sqrt{(20)^2 - 4 \times 3 \times -100}}{2 \times 3}$$

$$= \frac{-20 \pm \sqrt{400 + 1200}}{6}$$

$$= \frac{-(20) \pm \sqrt{1600}}{6}$$

$$= \frac{-20 \pm 40}{6}$$

$p = \frac{-20+40}{6} = \frac{20}{6} = \frac{10}{3}$ or $p = \frac{-20-40}{6} = \frac{-60}{6} = -10$

P can't be negative $\therefore p = \frac{10}{3}$

Put obtained p value in equation (3), we get

$$q = 20 + 3(\frac{10}{3}) = 30$$

Answer: $p = \frac{10}{3}, q = 30$

12. $4x^4 - 40x^2 + 36 = 0$

Put y=x^2,.Then equation becomes,

$$4y^2 - 40y + 36 = 0$$

$$\Rightarrow y^2 - 10y + 9 = 0$$

Use factorization method, find two numbers whose sum is -10 and its product is 9.
The numbers are -1 and -9

$$y^2 - 10y + 9 = 0 \Rightarrow (y-9)(y-1) = 0$$

$(y-9) = 0 \quad \therefore y = 9$

$(y-1) = 0 \quad \therefore y = 1$

Now, we are able to find the value of x. *As* $x^2 = y, \therefore x = \sqrt{y}$

When y=1, $x = \sqrt{1} = \pm 1$

When y=9 $x = \sqrt{9} = \pm 3$

Answer: x = 1, x = -1, x = 3, and x = -3

13. $x^8 - 82x^4 + 81 = 0$

Put y=x^4,.Then equation becomes,

$$y^2 - 55y + 54 = 0$$

Use factorization method, find two numbers whose sum is -82 and its product is 81.
The numbers are -1 and -81

$$y^2 - 82y + 81 = 0 \Rightarrow (y-81)(y-1) = 0$$

$(y-81) = 0 \quad \therefore y = 81$

$(y-1) = 0 \quad \therefore y = 1$

Now, we are able to find the value of x. *As* $x^4 = y, \therefore x = \sqrt[4]{y}$

When y=81, $x = \sqrt[4]{81} = 3$

When y=1 $x = \sqrt[4]{1} = 1$

Answer: x = 3, and x = 1

14. $2x + 2y - 24 = 0$..................(1)

$2x^2 + 2y^2 - 148 = 0$...........(2)

From equation (1)

$2x = 24 - 2y \Rightarrow x = 12 - y$...........(3)

Put equation (3) in (2)

$$2(12-y)^2 + 2y^2 - 148 = 0$$
$$2(144 - 24y + y^2) + 2y^2 - 148 = 0$$
$$288 - 48y + 2y^2 + 2y^2 - 148 = 0$$
$$\Rightarrow 4y^2 - 48y + 140 = 0$$
$$= y^2 - 12y + 35 = 0$$

Use factorization method, find two numbers whose sum is -12 and its product is 35. The numbers are -7 and -5

$$y^2 - 12y + 35 = 0 \Rightarrow (y-7)(y-5) = 0$$
$$(y-7) = 0 \qquad \therefore y = 7$$
$$(y-5) = 0 \qquad \therefore y = 5$$

When y=7 x=12-7=5

When y-5 x=12-5=7

Thus, the solutions are:

Y = 7 x = 5

Y = 5 x = 7

15. $x + y = 3$...............(1)

$\frac{x}{y} + \frac{y}{x} = \frac{5}{2}$................(2)

Take equation (2) and rewrite it as

$$\frac{x^2 + y^2}{yx} = \frac{5}{2}$$

Do cross multiplication

$2x^2 + 2y^2 = 5yx$..........(3)

From equation (1) y=3-x……(4). Apply equation (4) in (3)

$$2x^2 + 2(3-x)^2 = 5(3-x)x$$
$$2x^2 + 2(9 - 6x + x^2) = (15 - 5x)x$$
$$2x^2 + 18 - 12x + 2x^2) = 15x - 5x^2$$

$$2x^2 + 18 - 12x + 2x^2 - 15x + 5x^2 = 0$$
$$9x^2 - 27x + 18 = 0$$
$$\div 9 \Rightarrow x^2 - 3x + 2 = 0$$

By factorisation (x-2)(x-1)=0

X=2, or x=1

Put x values in equation (4)

When x=2, y=3-2=1

When x=1 y=3-1=2

Solutions: x = 2 y = 1

x = 1 y = 2

ESSAY QUESTIONS

1. Find the roots of the equation $(x^2 + (13 - x)^2 = 85$

   a). By factorization method

   b).By quadratic formula

   Show that sum of roots is equal to $-\frac{b}{a}$ and the product is equal to $\frac{c}{a}$

2. Consider the equation $18x^2 + 12x - 30 = 0$.Find the solution by completing the square method.

3. Solve the following pair of simultaneous equations

   $$2x^2 + 6y^2 = 114$$
   $$6x - 2y = 10$$

4. Solve the following simultaneous equation

   $$12x^2 + 8xy + 32 = 0$$
   $$16x - 4y = 52$$

5. A firm has the total cost function $TC = 16q^2 + 12q + 250$ and demand function $p = 180 + 2q$

   Find, a). TR function

   b). Profit function

   c). Profit when q=25

   d). Breakeven point

6. A firm has the total cost function $TC = 61.25q^2 - 428q + 240$ and demand function $Q = 600 - 0.5p$

   Find, a). TR function

   b). Profit function

   c). Profit when q=12

   d). Breakeven point

7. Solve the following simultaneous equation

$$2x^2 + 3y^2 = 147$$
$$3x - y = 13$$

8. Solve the equation by completing the Square

$$4x^2 + 4x = 3$$

ANSWERS

1. $(x^2 + (13 - x)^2 = 85$

$x^2 + 169 - 26x + x^2 = 85$

$$2x^2 - 26x + 84 = 0$$
$$x^2 - 13x + 42$$

a) By factorization $(x-6)(x-7) = 0$

i) $x - 6 = 0$ or $x - 7 = 0$

$x = 6$ $x = 7$

b) By quadratic formula $x^2 + 169 - 26x + x^2 = 85$

$$2x^2 - 26x + 84 = 0$$
$$x^2 - 13x + 42$$

Here *a*=1, *b*= -13, and *c*=42

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) + \sqrt{(-13)^2 - 4 \times 1 \times 42}}{2 \times 1}$$

$$= \frac{13 + \sqrt{169 - 168}}{2}$$

$$= \frac{13 + \sqrt{1}}{2} \qquad = \frac{13 + 1}{2} = 7$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) - \sqrt{(-13)^2 - 4 \times 1 \times 42}}{2 \times 1}$$

$$= \frac{13 - \sqrt{169 - 168}}{2}$$

$$= \frac{13 - \sqrt{1}}{2} \qquad = \frac{13 - 1}{2} = 6$$

$\alpha = 6$, and $\beta = 7$ Thus, x=6 or x=7

Sum of roots: $\alpha + \beta = -\frac{b}{a}$

$$= -\frac{-13}{1} = 13$$

$$\alpha + \beta = 7 + 6 = 13$$

Product of roots: $\alpha \times \beta = \frac{c}{a}$

$$= \frac{42}{1} = 42$$

$$\alpha \times \beta = 7 \times 6 = 42$$

2. $18x^2 + 12x - 30 = 0$

Divide the whole equation by 18

$$\frac{18x^2}{18} + \frac{12x}{18} - \frac{30}{18} = 0$$

$$x^2 + \frac{2x}{3} - \frac{15}{9} = 0$$

$$\Rightarrow x^2 + \frac{2x}{3} = \frac{15}{9}$$

Add the square of half of the coefficient of x to both sides. Here, the coefficient of x is $\frac{2}{3}$. Hence, the half of the coefficient is $\frac{1}{3}$

$$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$x^2 + \frac{2x}{3} + \frac{1}{9} = \frac{15}{9} + \frac{1}{9}$$

Express the left-hand side as perfect square

$$\left(x + \frac{1}{3}\right)^2 = \frac{15}{9} + \frac{1}{9}$$

$$\left(x + \frac{1}{3}\right)^2 = \frac{16}{9}$$

Take square root on both sides,

$$\sqrt{\left(x + \frac{1}{3}\right)^2} = \sqrt{\frac{16}{9}}$$

$$x + \frac{1}{3} = \pm\frac{4}{3}$$

So,

$$x+\frac{1}{3}=\frac{4}{3}$$

$$x=\frac{4}{3}-\frac{1}{3}=1$$

$$x+\frac{1}{3}=-\frac{4}{3}$$

$$x=-\frac{4}{3}-\frac{1}{3}=-\frac{5}{3}$$

Answer: x=1 or x= -5/3

3.

$$2x^2+6y^2=114\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$
$$6x-2y=10\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$
$$6x-2y=10$$
$$-2y=10+6x$$
$$y=-5-3x$$

Substitute the value of y in the equation (1)

$$2x^2+6(-5-3x)^2=114$$
$$2x^2+6(25+30x+9x^2=114$$
$$2x^2+150+180x+54x^2-114=0$$
$$56x^2+180x+36=0$$
$$28x^2+90x+18=0$$
$$14x^2+45x+9=0$$
$$14x^2+45x+9=0$$

Here, a=14, b=45, and c=9

$$\alpha=\frac{-b+\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(45)+\sqrt{(45)^2-4\times14\times9}}{2\times14}$$
$$=\frac{-45+\sqrt{2025-504}}{28}$$
$$=\frac{-45+\sqrt{1521}}{28} \qquad =\frac{-45+39}{28}=-\frac{6}{28}=-\frac{3}{14}$$

$$\beta=\frac{-b-\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(45)-\sqrt{(45)^2-4\times14\times9}}{2\times14}$$

$$= \frac{-45-\sqrt{2025-504}}{28}$$

$$= \frac{-45-\sqrt{1521}}{28} \qquad = \frac{-45-39}{28} = -\frac{84}{28} = -3$$

$\alpha = -\frac{3}{14}$, and $\beta = -3$ Thus, $x = -\frac{3}{14}$ or $x = -3$

4. $12x^2 + 8xy + 32 = 0$..................(1)

$16x - 4y = 52$..................(2)

Rewrite the equation (2)

$$-4y = 52 - 16x$$

$$y = -13 + 4x \text{..........(3)}$$

Substitute equation (3) in (1)

$$12x^2 + 8x(-13 + 4x) + 32 = 0$$

$$\Rightarrow 12x^2 - 104x + 32x^2 + 32 = 0$$

$$44x^2 - 104x + 32 = 0$$

$$\div 4 \Rightarrow 11x^2 - 26x + 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, a=11, b=-26, and c=8

$$= \frac{-(-26) \pm \sqrt{(-26)^2 - 4 \times 11 \times 8}}{2 \times 11}$$

$$= \frac{26 \pm \sqrt{676-352}}{22}$$

$$= \frac{26 \pm \sqrt{324}}{22} \qquad = \frac{26 \pm 18}{22}$$

$$\frac{26+18}{22} = \frac{44}{22} = 2$$

Or

$$\frac{26-18}{22} = \frac{8}{22} = \frac{4}{11}$$

Substitute the value of x (x=2) in the equation (2). It gives the y value

$$16(2) - 4y = 52$$

$$32 - 4y = 52$$

$$-4y = 20 \Rightarrow y = \frac{20}{-4} = -5$$

Next, substitute $x = \frac{4}{11}$ in equation (2)

$$16(\frac{4}{11}) - 4y = 52$$

$$\frac{64}{11} - 4y = 52$$

$$-4y = 52 - \frac{64}{11} \Rightarrow -4y = \frac{572 - 64}{11}$$

$$-4y = \frac{508}{11} \Rightarrow y = \frac{508}{11} \times -\frac{1}{4}$$

$$y = -\frac{508}{1441} = -\frac{127}{11}$$

The solutions are

X = 2 y = -5

X = 4/11 y = -127/11

5. $TC = 16q^2 + 12q + 250$

$p = 180 + 2q$

a). Total Revenue function

$$TR = p \times q \Rightarrow (180 + 2q)q$$

$$180q + 2q^2$$

b). Profit Function

$$\pi = TR - TC \Rightarrow (180q + 2q^2) - (16q^2 + 12q + 250)$$

$$168q - 10q^2 - 250$$

$$\Rightarrow -10q^2 + 168q - 250$$

$$-1 \Rightarrow 10q^2 - 168q + 250$$

c). Profit when q=25

$$10(25)^2 - 168(25) + 250$$
$$10(625) - 4200 + 250$$
$$6250 - 4200 + 250$$
=2300

d). Breakeven point

$$10q^2 - 168q + 250$$

Use quadratic formula

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here a = 10, b =- 168, and c = 250

$$q = \frac{-(-168) \pm \sqrt{(-168)^2 - 4 \times 10 \times 250}}{2 \times 10}$$

$$= \frac{168 \pm \sqrt{28224 - 10000}}{20}$$

$$= \frac{168 \pm \sqrt{18224}}{20}$$

$$= \frac{168 \pm 134.9963}{20}$$

$$= \frac{168 + 134.9963}{20} = 15.14 \approx 15$$

$$= \frac{168 - 134.9963}{20} = 1.65 \approx 1$$

The break-even points are 1 and 15.

6. $TC = 61.25q^2 - 428q + 240$................(1)

$Q = 600 - 0.5p$.......................(2)

a). Total Revenue Function

$TR = p \times q$

From equation (2)

$0.5p = 600 - q \Rightarrow p = 300 - 05q$

$TR = (300 - 05q)$

$$300q - 0.5q^2$$

b). Profit Function

$$\pi = TR - TC \Rightarrow (300 - 05q^2) - (61.25q^2 - 428q + 240)$$

$$300 - 05q^2 - 61.25q^2 + 428q - 240$$

$$-61.75q^2 + 428.q + 60$$

$$\div -1 \Rightarrow 61.75q^2 - 428.q - 60$$

C) Profit when q=12

$$61.75(12)^2 - 428(12) - 60$$

$$8892 - 5136 - 60 = 3696$$

d) Break –even point

$$61.75q^2 - 428.q - 60 = 0$$

Use quadratic formula

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here a=61.75, b=-428, and c= -60

$$q = \frac{-(-428) \pm \sqrt{(-428)^2 - 4 \times 61.75 \times -60}}{2 \times 61.75}$$

$$= \frac{428 \pm \sqrt{183184 + 14820}}{123.5}$$

$$= \frac{428 \pm \sqrt{198004}}{123.5}$$

$$= \frac{428 \pm 444.9764}{123.5}$$

$$= \frac{428 + 444.9764}{123.5} = 7.06 \approx 7$$

$$= \frac{428 - 444.9764}{123.5} = -0.1375$$

Since negative quantity is not possible we take positive value as quantity. Thus the break-even point is 7.

7. $3x^2 + 3y^2 = 183$.......(1)

$3x - y = 13$............(2)

Rewrite the equation (2)

$$-y = 13 - 3x$$

$$y = -13 + 3x\text{..........(3)}$$

Substitute equation (3) in (1)

$$3x^2 + 3(-13 + 3x)^2 = 183$$

$$\Rightarrow 3x^2 + 3(169 - 78x + 9x^2) = 183$$

$$3x^2 + 507 - 234x + 27x^2 = 183$$

$$30x^2 - 234x + 324 = 0$$

$$\Rightarrow 10x^2 - 78x + 108 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, a=10, b=-78, and c=108

$$= \frac{-(-78) \pm \sqrt{(-78)^2 - 4 \times 10 \times 108}}{2 \times 10}$$

$$= \frac{78 \pm \sqrt{6084 - 4320}}{20}$$

$$= \frac{78 \pm \sqrt{1764}}{22} \qquad = \frac{78 \pm 42}{20}$$

$$\frac{78 + 42}{20} = \frac{120}{20} = 6$$

Or

$$\frac{78 - 42}{20} = \frac{36}{20} = \frac{9}{5}$$

Substitute the value of x (x=6) in the equation (2). It gives the y value

$$3(6) - y = 13$$

$$18 - y = 13$$

$$-y = 13 - 18 = -5$$

$$\therefore y = 5$$

Next, substitute $x = \frac{9}{5}$ in equation (2)

$$3 \times \frac{9}{5} - y = 13$$

$$\frac{27}{5} - y = 13$$

$$-y = 13 - \frac{27}{5}$$

$$-y = \frac{65 - 27}{5} = \frac{38}{5}$$

$$y = -\frac{38}{5}$$

Thus, solutions are ***x=6, y=5***

x=9/5 ***y=-38/5***

8. $4x^2 + 4x = 3$

Rewrite the equation as

$$4x^2 + 4x - 3 = 0$$

Divide the equation by 4 and reduce it

$$x^2 + x - \frac{3}{4} = 0$$

$$x^2 + x = \frac{3}{4}$$

Add square of half of the coefficient of x to both sides. The coefficient of x is 1. Therefore half of the coefficient of x is ½

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$x^2 + x + \frac{1}{4} = \frac{3}{4} + \frac{1}{4}$$

Express left hand side as perfect square. Then, we have

$$\left(x + \frac{1}{2}\right)^2 = 1$$

Take square root on both sides

$$\sqrt{\left(x + \frac{1}{2}\right)^2} = \sqrt{1}$$

$$x + \frac{1}{2} = \pm 1$$

Thus, $x + \frac{1}{2} = 1$ or $x + \frac{1}{2} = -1$

$$x = 1 - \frac{1}{2} \quad \text{or} \quad x = -1 - \frac{1}{2}$$

$$\therefore x = \frac{1}{2} \qquad \therefore x = -\frac{3}{2}$$

Solutions: $x = \frac{1}{2}$, $x = -\frac{3}{2}$

MODULE IV

Graphs and Economic Applications

Linear Equations

A linear equation looks like any other equation. It is made up of two expressions set equal to each other. A linear equation is special because:

1. It has one or two variables.
2. No variable in a linear equation is raised to a power greater than 1 or used as the denominator of a fraction.
3. When you find pairs of values that make the linear equation true and plot those pairs on a coordinate grid, all of the points for any one equation lie on the same line. Linear equations graph as straight lines.

A linear equation in two variables describes a relationship in which the value of one of the variables depends on the value of the other variable. In a linear equation in *x* and *y*, x is called the independent variable and *y* depends on it. We call *y* the dependent variable. The independent variable is plotted along the horizontal axis. Most linear equations are functions (that is, for every value of *x*, there is only one corresponding value of *y*). When you assign a value to the independent variable, *x*, you can compute the value of the dependent variable, *y*. You can then plot the points named by each (*x*,*y*) pair on a coordinate grid.

A linear equation is an equation for a straight line.

There are many ways of writing linear equations, but they usually have constants (like "2" or "c") and must have simple variables (like "x" or "y").

These are linear equations:

$$y = 3x - 6$$

$$y - 2 = 3(x + 1)$$

$$y + 2x - 2 = 0$$

$$5x = 6$$

$$y/2 = 3$$

But the variables (like ‘x’ or ‘y’) in Linear Equations do NOT have:

Exponents (like the 2 in x^2)

Square roots, cube roots, etc

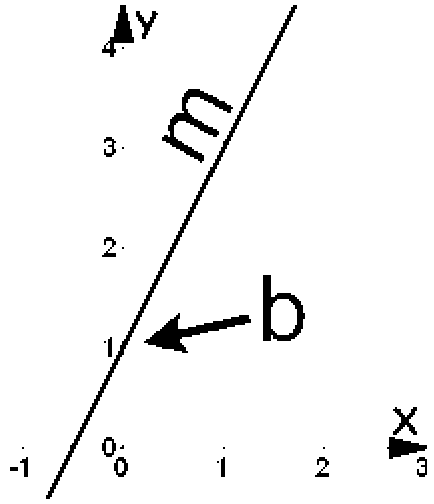
Forms of linear Equations

(a) Slope-Intercept Form

The most common form is the slope-intercept equation of a straight line

Y = mx + b

Where 'y' is slope, and 'b' is intercept.



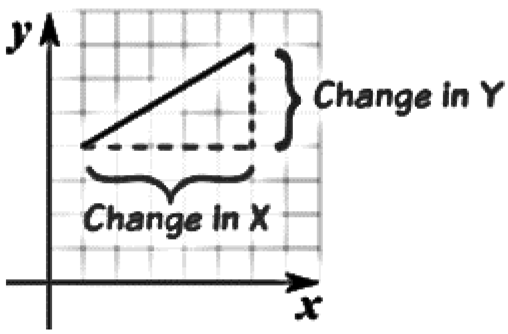
Before we move further, we will make two concepts clear (a) What is a slope and how is it measured. (b) What is intercept and how it is measured.

Slope

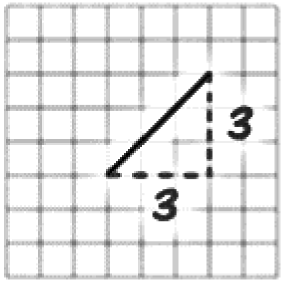
The Slope (also called Gradient) of a straight line shows how steep a straight line is.

The method to calculate the slope is:Divide the change in height by the change in horizontal distance. (see figure)

$$Slope = \frac{Change\ in\ Y}{Change\ in\ X}$$



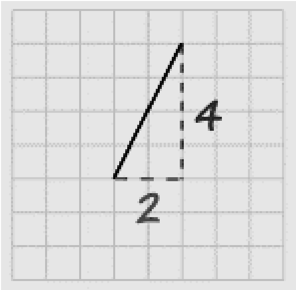
Example1



The slope of this line is$\frac{3}{3} = 1$

So the slope is equal to 1

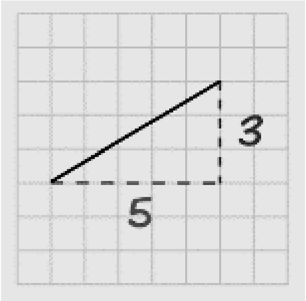
Example 2



The slope of this line is$\frac{4}{2} = 2$

So the slope is equal to 2.The line is steeper, and so the slope is larger.

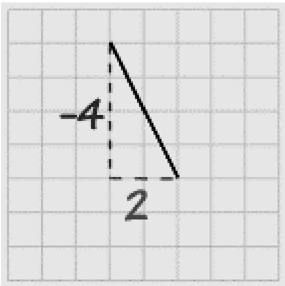
Example 3



The slope of this line is $\frac{3}{5} = 0.6$

So the slope is equal to 0.6.The line is less steep, and so the slope is smaller.

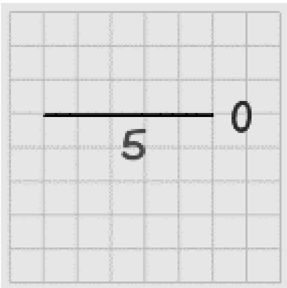
Example 4



The slope of this line is $\frac{-4}{2} = -2$

That line goes down as you move along, so it has a negative slope.

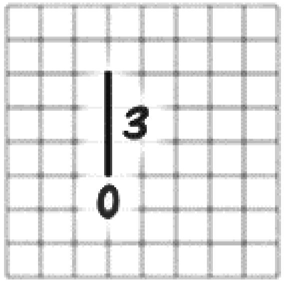
Example 5



The slope of this line is $\frac{0}{5} = 0$

A line that goes straight across has a slope of zero.

Example 6



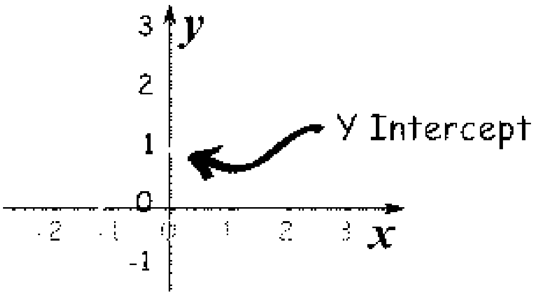
The slope of this line is $\frac{3}{0} = undefined$

(b) Intercept of a Straight Line

The intercept of a straight line is the point where it crosses the axis.

The Y intercept of a straight line is simply where the line crosses the Y axis.

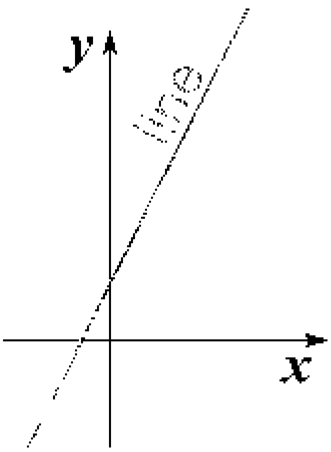
Example:



In the above diagram the line crosses the Y axis at 1.So the Y intercept is equal to 1.

Graph of a linear equation

y = 2x+1 is a linear equation. When graphed, we get the following.



We see that The graph of y = 2x+1 is a straight line. In this equation 2x means when x increases, y increases twice as fast. We can also see from the diagram that when x is 0, y is already 1. Hence +1 is also needed in the equation of the line. That is how we get the equation y = 2x +1.

Different forms of equation of a straight line

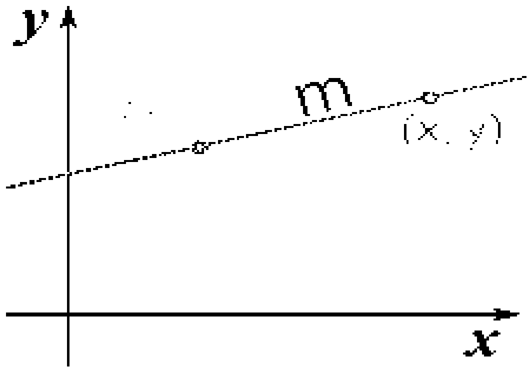
There are different forms by which a straight line is expressed. The most popular one is the slope – intercept for we have already seen. Here is a list of all the forms.

1. Slope-Intercept Form; y = mx + b*(not discussing in detail, as we have already done that)*

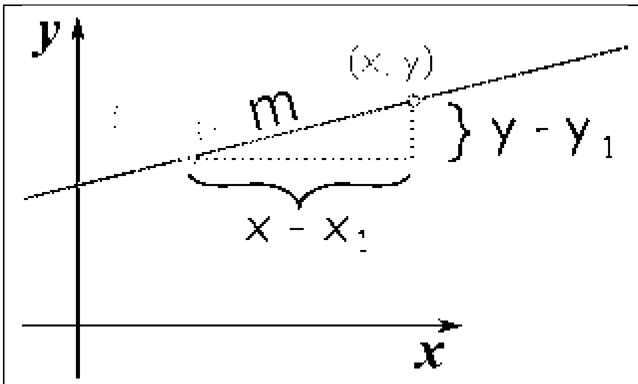
2. Point-Slope Form:

The "point-slope" form of the equation of a straight line is:$y - y_1 = m(x - x_1)$

Using this formula, when we know:one point on the lineand the slope of the line, then we can find other points on the line.

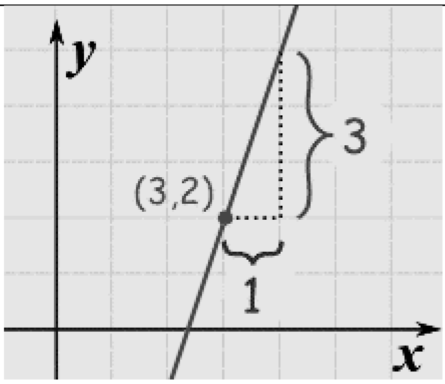
	(x_1, y_1) is a **known** point m is the **slope** of the line (x, y) is any other point on the line

We can see the Point-Slope Form is a different form of the Slope-Intercept Form.

	$Slope\ m = \frac{change\ in\ y}{change\ in x}$ $= \frac{y - y_1}{x - x_1}$ So the slope is $\frac{y - y_1}{x - x_1} = m$ Cross multiplying we get

	$y - y_1 = m\ (x - x_1)$ So, it is just the slope formula in a different way.

Example 1

	Here slope is $$m = \frac{3}{1} = 3$$ $y - y_1 = m(x - x_1)$ We know the value of m from the above calculation. We can also read from the figure that the values of x_1 is 3 and y_1 is 2. That is $(x_1,y_1) = (3,2)$. Substituting in the form $y - y_1 = m(x - x_1)$ we get equation of this straight line as $$y - 2 = 3(x - 3)$$

We can further simplify the equation that we just got

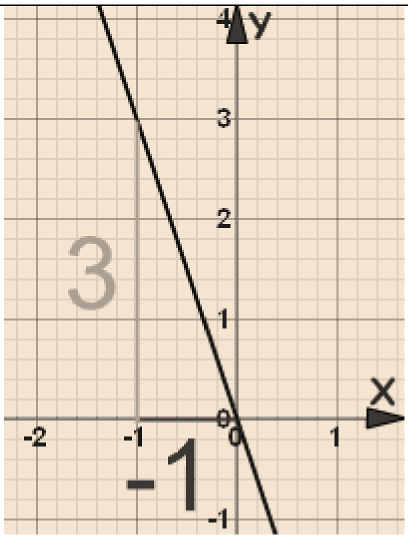
$y – 2 = 3(x – 3)$

$y – 2 = 3x–9$

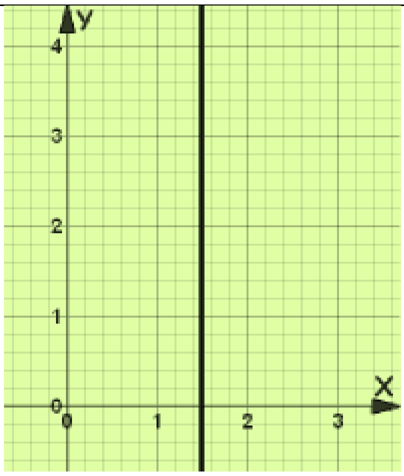
$y = 3x – 9 +2$

$y = 3x – 7$ (which is of the slope – intercept form)

Example 2

	Here slope is $$m = \frac{3}{-1} = -3$$ $y - y_1 = m(x - x_1)$ We know the value of m from the above calculation. We can also read from the figure that the values of x_1 is 0 and y_1 is 0. That is $(x_1,y_1) = (0,0)$. Substituting in the form $y - y_1 = m(x - x_1)$ we get equation of this straight line as $y - 0 = -3(x - 0)$ $y = -3x$

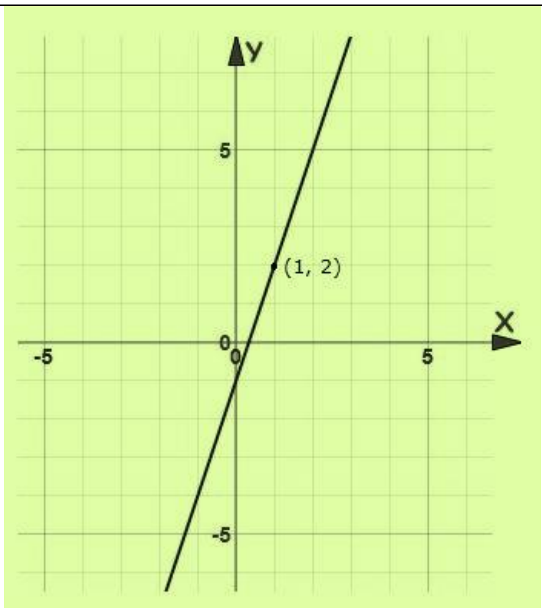
Example 3: Vertical Line

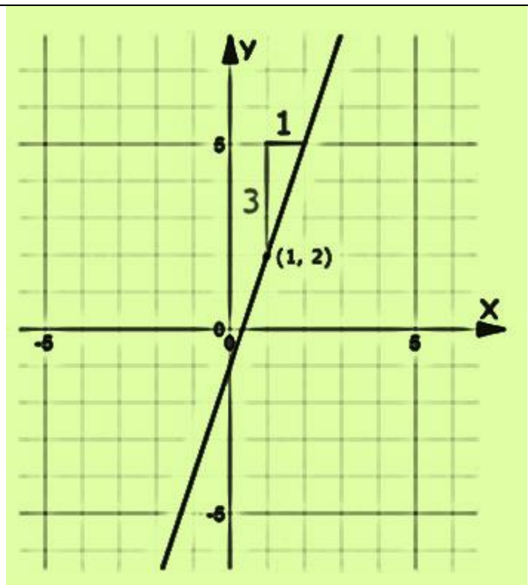
	What is the equation for a vertical line? Here the slope is undefined. In fact, this is a special case, and we use a different equation, like this: X = 1.5 Every point on the line has x coordinate 1.5, that's why its equation is x = 1.5

Question 1

Using the given point(in Fig 1), find the equation of this straight line in Point-Slope Form

Fig 1	Fig 2





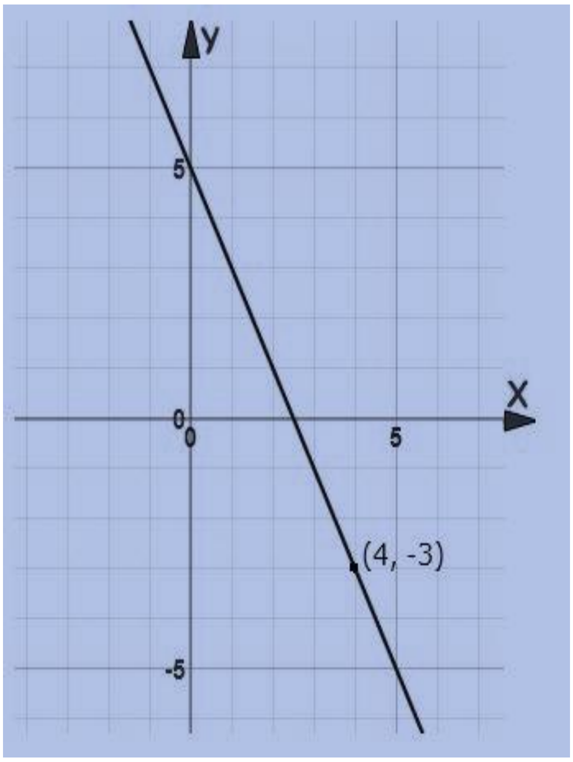
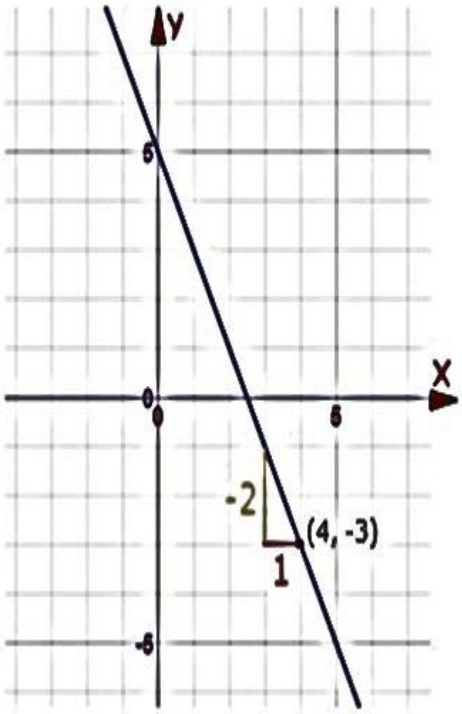
The equation of this straight line in Point-Slope Form is $y - 2 = 3(x - 1)$

This is how we work out it. (refer to fig. 2)

First find the slope:
$m = = 3/1 = 3$
Next use the formula $y - y_1 = m(x - x_1)$
Substitute $x_1 = 1$, $y_1 = 2$ and $m = 3$
Therefore $\mathbf{y - 2 = 3(x - 1)}$

Example 2

Using the given point (in Fig1), what is the Point-slope equation of this line

Fig 1	Fig 2
	

the Point-slope equation of this line is $y + 3 = -2(x - 4)$

This is how we workout

Fist find the slope, $m = -2/1 = -2$

Now use $y - y_1 = m(x - x_1)$
Substitute $x_1 = 4$, $y_1 = -3$ and $m = -2$
$\Rightarrow y - (-3) = -2(x - 4)$
$\Rightarrow y + 3 = -2(x - 4)$.

Graphical solution of linear equations

Example 1: Graph the function $3y + 15x = 30$

To solve this first set the equation in the slope intercept form $y = mx + b$. Then solve it for y in terms of x.

$3y + 15x = 30$

$3y = -15x + 30$

$y = -5x + 10$

From this we can find the value of slope, which is the value of m, that is, −5.

Slope m = −5

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

$y = -5(0) + 10$

$y = 10$

So the y intercept is (0,10)

Similarly find the x intercept by equating y = 0.

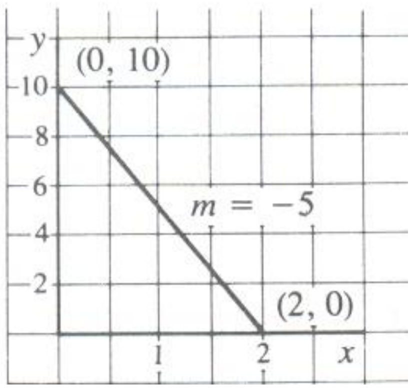
$0 = -5x + 10$

$5x = 10$

$x = 2.$

So the x intercept is (2,0)

By plotting the x and y intercepts we can graph the equation as given in the following figure.



Question 1:

Graph the function $2y - 6x = 12$

$2y = 6x + 12$

$y = 3x + 6$

From this we can find the value of slope, which is the value of m, that is, m = 3.

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

$y = 3(0) + 6$

y = 6

So the y intercept is (0,6)

Similarly find the x intercept by equating y = 0.

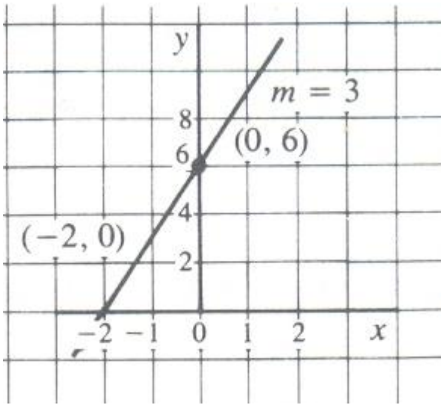
0 = 3x + 6

3x = −6

x = −2.

So the x intercept is (−2,0)

By plotting the x and y intercepts we can graph the equation as given in the following figure.



Question 2:

Graph the function 8y − 2x + 16 = 0

8y = 2x − 16

$$y = \frac{1}{4}x - 2$$

From this we can find the value of slope, which is the value of m,

$$m = \frac{1}{4}$$

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

$$y = \frac{1}{4}(0) - 2$$

y = −2

So the y intercept is (0,−2)

Similarly find the x intercept by equating y = 0.

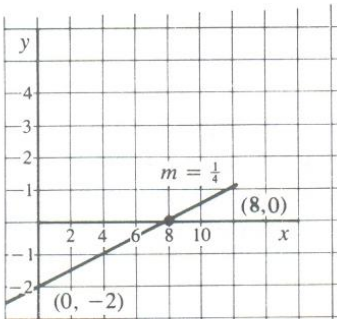
$$0 = \frac{1}{4}x - 2$$

$$\frac{1}{4}x = 2$$

$$x = 2 \times \frac{4}{1} = 8$$

So the x intercept is (8,0)

By plotting the x and y intercepts we can graph the equation as given in the following figure.



Question 3:

Graph the function 6y + 3x − 16 = 0

6y = −3x + 16

$$y = -\frac{1}{2}x + 3$$

From this we can find the value of slope, which is the value of m,

$$m = -\frac{1}{2}$$

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

$$y = -\frac{1}{2}(0) + 3$$

y = 3

So the y intercept is (0, 3)

Similarly find the x intercept by equating y = 0.

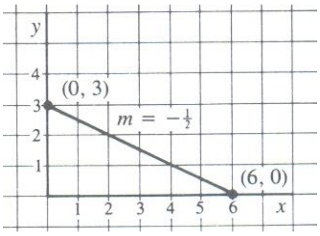
$$0 = -\frac{1}{2}x + 3$$

$$-3 = -\frac{1}{2}x$$

$$x = 3 \times \frac{2}{1} = 6$$

So the x intercept is (6,0)

By plotting the x and y intercepts we can graph the equation as given in the following figure.



Simultaneous linear equations

Simultaneous equations can be solved in several ways. Here we see how they are solved graphically. We shall explain it with the help of the following example.

Example

Solve simultaneously for x and y:

x + y =10

x − y =2

The two equations in the problem are linear equations and let us plot them.

Equation 1: x + y=10

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

0 + y = 10

y = 10

So the y intercept is (0, 10)

Similarly find the x intercept by equating y = 0.

$x + 0 = 10$

$x = -10$

So the x intercept is $(-10,0)$

By plotting the x and y intercepts we can graph the equation $x + y = 10$ as given in the figure below. But before that we have to find the intercepts for the second equation also.

Equation 2: $x - y = 2$

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

$0 - y = 2$

$y = -2$

So the y intercept is $(0, -2)$

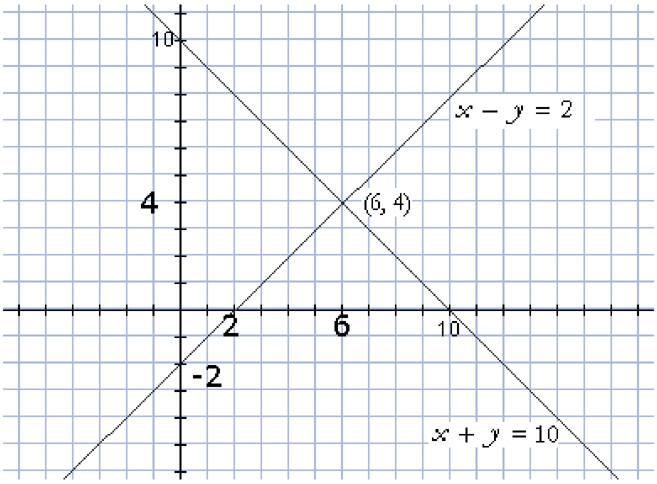
Similarly find the x intercept by equating y = 0.

$x - 0 = 2$

$x = 2$

So the x intercept is $(2,0)$

Let us now plot the two equations. The intersection of the graphs of the two equations will be the values of x and y that will solve both equations. See the following figure.



From the intersection of the graphs of the two curves we know that the solution to the problem is y = 4 and x = 6.

Question 1

Solve simultaneously for x and y:

$x - y = 2$

$2x + y = 10$

The two equations in the problem are linear equations and let us plot them.

Equation 1: $x - y = 2$

We know that at y intercept, $x = 0$. So to find y intercept, equate $x = 0$ in the equation.

$0 - y = 2$

$y = -2$

So the y intercept is $(0, -2)$

Similarly find the x intercept by equating $y = 0$.

$x - 0 = 2$

$x = 2$

So the x intercept is $(2,0)$

By plotting the x and y intercepts we can graph the equation $x - y = 2$ as given in the figure below. But before that we have to find the intercepts for the second equation also.

Equation 2: $2x + y = 10$

We know that at y intercept, $x = 0$. So to find y intercept, equate $x = 0$ in the equation.

$2(0) + y = 10$

$y = 10$

So the y intercept is $(0, 10)$

Similarly find the x intercept by equating $y = 0$.

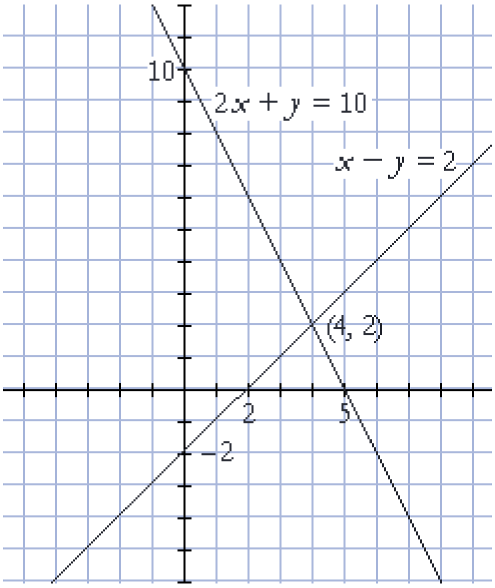
$2x + 0 = 10$

$2x = 10$

$x = 5$

So the x intercept is $(5,0)$

Let us now plot the two equations. The intersection of the graphs of the two equations will be the values of x and y that will solve both equations. See the following figure.



From the intersection of the graphs of the two curves we know that the solution to the problem is y = 2 and x = 4.

<u>Question 2</u>

Solve simultaneously for x and y:

2x + 3y = 7

4x + y = 9

The two equations in the problem are linear equations and let us plot them.

Equation 1: 2x + 3y = 7

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

2(0) + 3y = 7

3y = 7

y = 2.33

So the y intercept is (0, 2.33)

Similarly find the x intercept by equating y = 0.

2x + 3(0) = 7

2x = 7

x = 3.5

So the x intercept is (3.5,0)

By plotting the x and y intercepts we can graph the equation x – y = 2 as given in the figure below. But before that we have to find the intercepts for the second equation also.

Equation 2: 4x + y = 9

We know that at y intercept, x = 0. So to find y intercept, equate x = 0 in the equation.

4(0) + y = 9

y = 9

So the y intercept is (0, 9)

Similarly find the x intercept by equating y = 0.

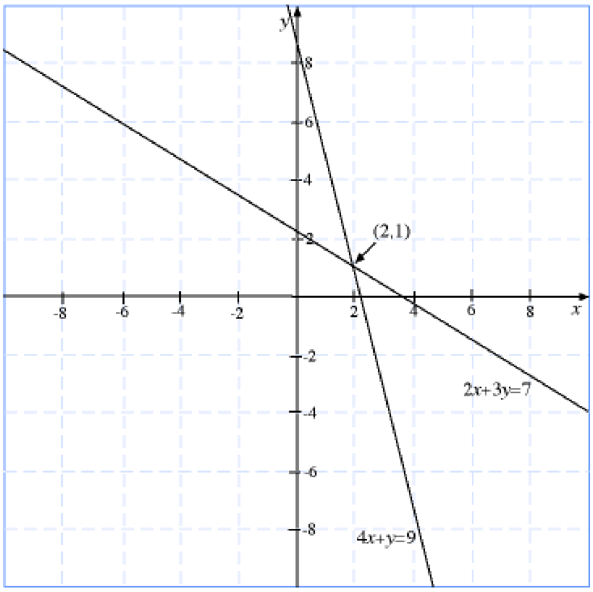
4x + 0 = 9

4x =9

x =2.25

So the x intercept is (2.25,0)

Let us now plot the two equations. The intersection of the graphs of the two equations will be the values of x and y that will solve both equations. See the following figure.



From the graph we see that the point of intersection of the two lines is (2, 1)

Hence, the solution of the simultaneous equations is x = 2, y =1.

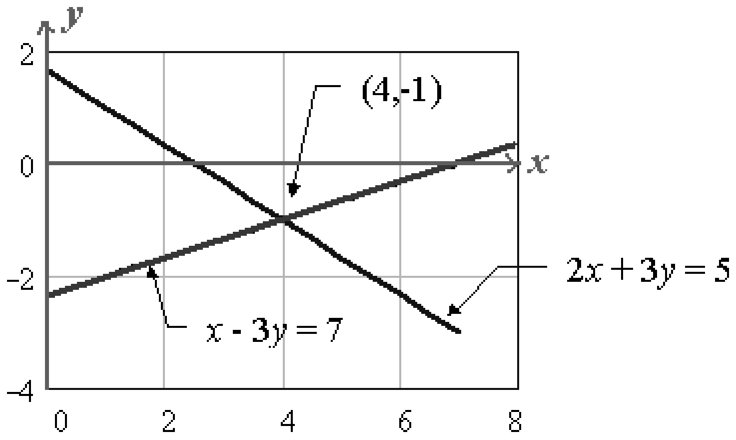
<u>Question 3</u>

Solve simultaneously for x and y:

2x + 3y = 5

x − 3y = 7

The two equations in the problem are linear equations and let us plot them.



From the graph we see that the point of intersection of the two lines is (4, −1)

Hence, the solution of the simultaneous equations is x = 4, y =−1.

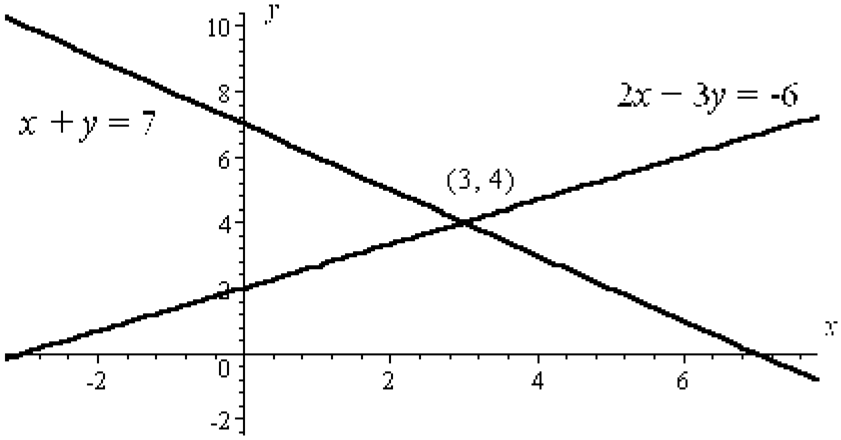
<u>Question 4</u>

Solve simultaneously for x and y:

2x − 3y = −6

x + y = 7

The two equations in the problem are linear equations and let us plot them.



From the graph we see that the point of intersection of the two lines is (3, 4)

Hence, the solution of the simultaneous equations is x = 3, y = 4.

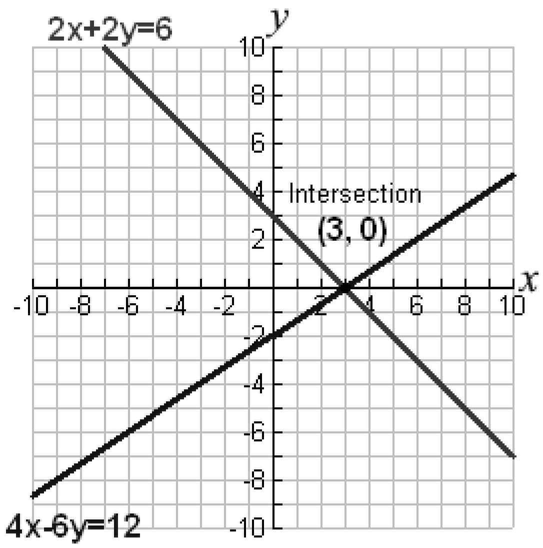
<u>Question 5</u>

Solve simultaneously for x and y:

4x - 6y = 12

2x + 2y = 6

The two equations in the problem are linear equations and let us plot them.



From the graph we see that the point of intersection of the two lines is (3, 0)

Hence, the solution of the simultaneous equations is x = 3, y =0 .

Note:

Please note the following points.

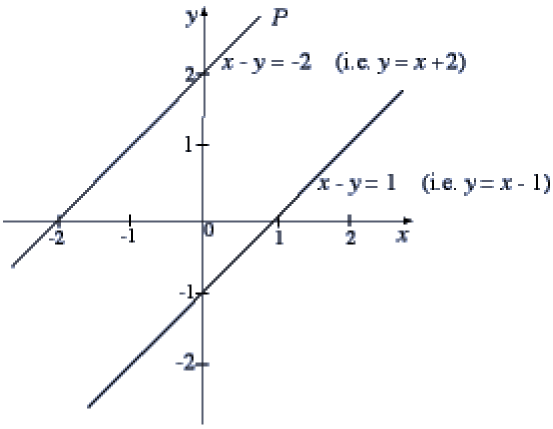
1. When we plot the two equations, if the lines are parallel, they do not intersect, and in such cases there is no solution.

Consider the following system of linear equations:

$x - y = -2$

$x - y = 1.$

When we plot, we get two parallel lines.



In this cases there is no solution.

2. When we plot the two equations, if the lines are concurrent (both equations give same line) then there are infinite solutions.

Consider the following system of linear equations:

$x - y = 1$

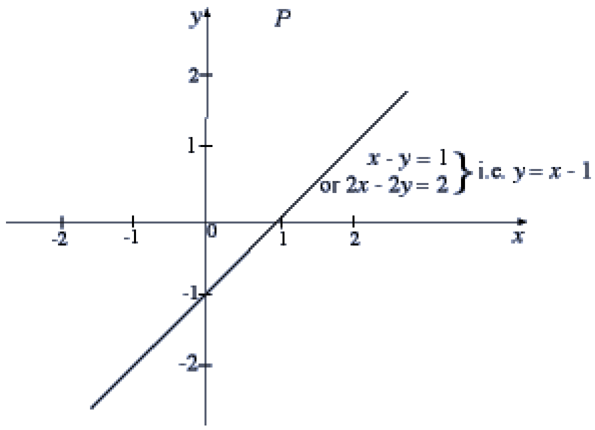
$2x - 2y = 2$

Multiply the first equation by 2 to put the equations in the form

$2x - 2y = 2$

$2x - 2y = 2$

When we plot we get the same (concurrent) line for both equations.



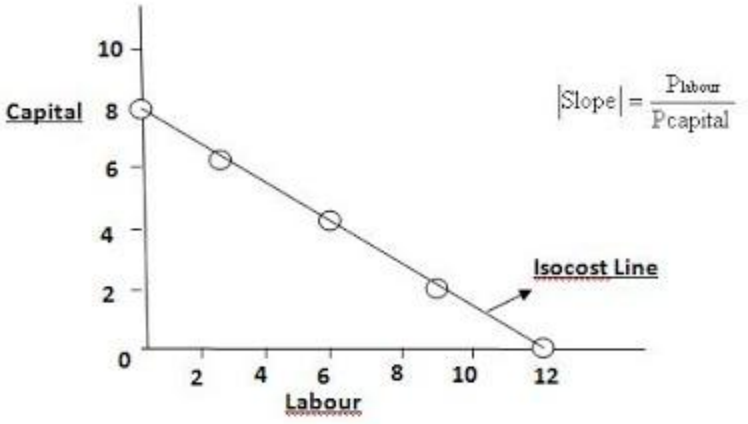
Since any point on this line has coordinates which will satisfy both equations, so there are an infinite number of solutions.

Applications in Economics - Isocost Lines

The isocost line is an important component when analysing producer's behaviour. The isocost line illustrates all the possible combinations of two factors that can be used at given costs and for a given producer's budget. In simple words, an isocost line represents a combination of inputs which all cost the same amount.

suppose that a producer has a total budget of Rs 120 and and for producing a certain level of output, he has to spend this amount on 2 factors A and B. Price of factors A and B are Rs 15 and Rs. 10 respectively.

Combinations	Units of Capital Price = 150Rs	Units of Labour Price = Rs 100	Total expenditure (in Rupees)
A	8	0	120
B	6	3	120
C	4	6	120
D	2	9	120
E	0	12	120



The general formula of an isocost is $P_kK + P_LL = T$, where K and L are capital and labour, P_k and P_L are the prices of capital and labour respectively, and T is total outlay.

The budget line or price line of the indifference curve analysis is a similar concept.

Example 1

In a factorytwo inputs coal (C) or gas (G) is used in the production of steel. The cost of coal used is 100 and the cost of gas used is 500. Draw an isocost curve showing the different combinations of gas and coal that can be purchased (a) with an initial expenditure (E) of 10000, (b) if expenditure increases by 50 percent, (c) if the price of gas is reduced by 20 percent. (d) if the price of coal rises by 25 percent.

(a) Expenditure of coal + expenditure on gas = total expenditure

$PcC + PgG= E$

$100C + 500G = 10000$

$C = 100 – 5G$

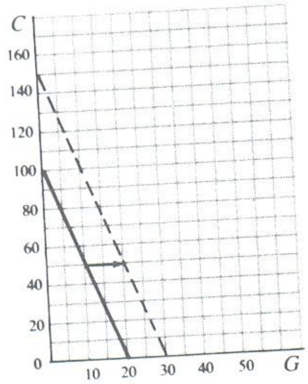
The graph of this is the solid line in figure (a)

(b) A 50 percent increase in expenditure makes the new outlay 15000. So the new equation will be

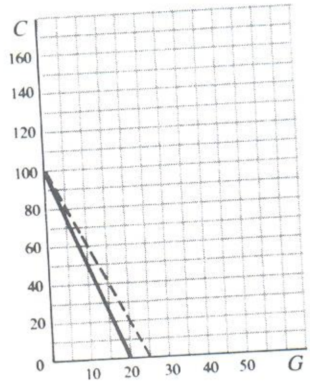
$100C + 500G = 15000$

$C = 150 – 5G$

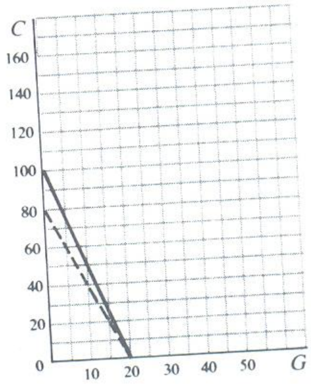
The graph of this is the dashed line in figure (a)



(a) Increase in budget



(b) Reduction in P_G



(c) Increase in P_C

(c) If the price of gas is reduced by 20 percent, the new price will be 400. So the new equation is

100C + 400G = 10000

C = 100 – 4G

The graph is the dashed line in figure (b)

(d)) If the price of coal is reduced by 25 percent, the new price will be 125. So the new equation is

125C + 500G = 10000

C = 80 – 4G

The graph is the dashed line in figure (c)

<u>Problem 1</u>

A person has Rs. 120 to spend on two goods (X and Y) whose respective prices are Rs. 3 and Rs.5. (a) draw a budget line (b) what happens to the original budget line if the budget falls by 25 percent. (c) what happens to the original budget line if the price of X doubles (d) what happens to the original budget line if the price of Y falls to 4.

(Note that a budget line is similar to an isoquant).

(a) The general function of a budget line is PxX + PyY = B

If Px = 3, Py = 5and B = 120, we can write the budget line as

3X + 5Y = 120

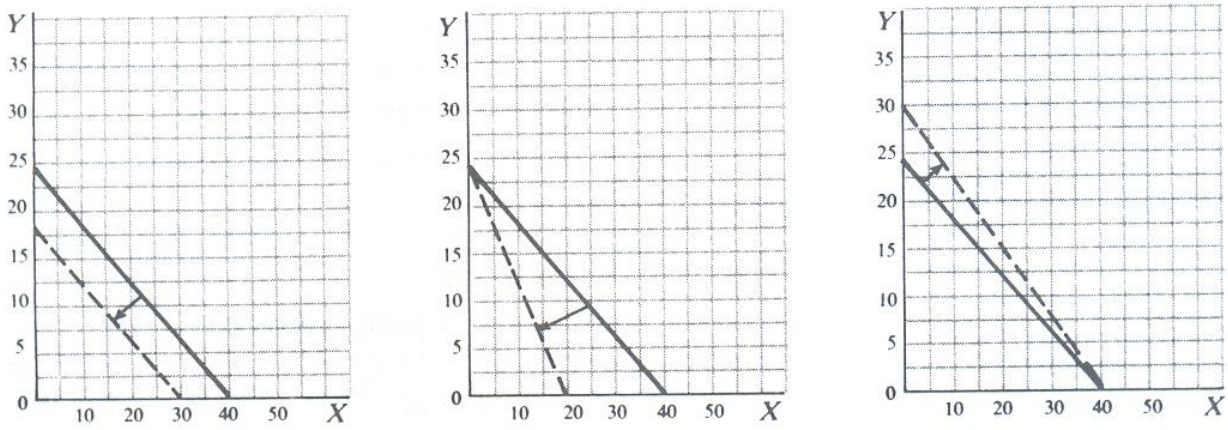
Solving for Y in terms of X in order to graph the function, $y = 24 - \frac{3}{5}x$

The graph is the solid line in figure (a)

(b) If the budget falls by 25 percent, the new budget is 90. The equation for the new budget line will be 3X + 5Y = 90.

Solving for Y in terms of X in order to graph the function, $y = 18 - \frac{3}{5}x$

The graph is the dashed line in figure (a). The lowering of budget causes the budget line to shift to the left.



(a) Reduced budget (b) Increase in P_X (c) Reduction in P_Y

(c) If Px doubles, the equation becomes 6X + 5Y = 120

Solving for Y in terms of X in order to graph the function, $y = 24 - \frac{6}{5}x$

The graph is the dashed line in figure (b). With a higher price for X, less X can be bought with the given budget.

(d) If Py now equals 4,

$$3X + 4Y = 120$$

$$y = 30 - \frac{3}{4}x$$

The graph is the dashed line in figure (c) in the panel.

Supply and Demand Analysis

A graphical analysis of the market combines the demand curve, which captures the demand side, with a corresponding supply curve, which illustrates the supply side. Equilibrium in supply and demand analysis occurs when Qs = Qd. We can determine equilibrium price and quantity by equating the supply and demand functions.

Example 1

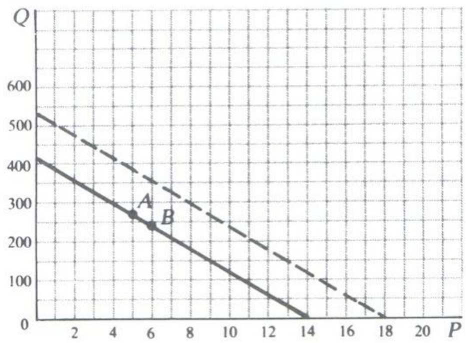
A demand function is given by the equation $Q_d = -30P + 0.05Y + 2Ps + 4T$ where P is the price good, Y is income, Ps is the price of a substitute and T is taste. You are further given the data that Y = 5000, Ps = 25 and T = 30. (a) draw the demand function (b) What happens if income increases to 7400.

(a) Let us re write the demand function including all the available information

$$Q_d = -30P + 0.05Y + 2Ps + 4T = -30P + 0.05(5000) + 2(25) + 4(30)$$

$$= -30P + 420$$

This is graphed in the following figure.



(b) If income increases to 7400, we have

$$Q_d = -30P + 0.05(7400) + 2(25) + 4(30) = -30P + 540$$

This is graphed as the dashed line in the above figure.

<u>Problem 1</u>

Find the equilibrium price and quantity for a market represented by the supply function Qs = −20 + 3P and the demand function Qd = 220 – 5P.

Equilibrium is the point where

Qs = Qd

−20 + 3P = 220 – 5P

3P + 5P = 220 + 20

$8P = 240$

$P = 30$

So the equilibrium price is Rs. 30. Substitute P = 30 in supply function or demand function to find equilibrium quantity.

$Qs = -20 + 3(30) = -20 + 90 = 70$ = equilibrium quantity

Equilibrium price is Rs. 30 and equilibrium quantity is 70.

Problem 2

Find the equilibrium price and quantity for a market represented by the supply function $Qs + 32 - 7P = 0$ and the demand function $Qd - 128 + 9P = 0$.

$Qs = 7P - 32$

$Qd = 128 - 9P$

$7P - 32 = 128 - 9P$

$16P = 160$

$P = 10$

So the equilibrium price is Rs. 10. Substitute P = 10 in supply function or demand function to find equilibrium quantity.

$Qs = 7(10) - 32 = 70 - 32 = 38$ = equilibrium quantity.

Equilibrium price is Rs. 10 and equilibrium quantity is 38.

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