UNIVERSITY OF CALICUT
SCHOOL OF DISTANCE EDUCATION

B.Sc MATHEMATICS
(CORE COURSE)

SIXTH SEMESTER
(2011 Admission)

COMPLEX ANALYSIS

Question Bank

Module- I

(1) An analytic function with constant modulus is :

A) zero B) a constant
C) identity map D) None of these.

(2) Real part of $f(z) = \log z$ is:

A) $\frac{1}{2}\log(x^2 + y^2)$ B) $\log(x^2 + y^2)$
C) $\log(x + iy)$ D) None of these

(3) If n is a positive integer, then $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n$ is equal to :

A) $2^n \sin \frac{n®}{2}$ B) $2^{n+1} \cos \frac{n®}{3}$
C) $2^{n+1} \sin \frac{n®}{3}$ D) None of these.

(4) Value of $(1 - i)^{10} + (1 + i)^{10}$ equals:

A) $1 + i$ B) i
C) 0 D) None of these.

(5) Real part of $f(z) = z^3$ is:

A) $x^3 - 3xy^2$ B) $x^3 + 3xy^2$
C) $x^3 - 3x^2 y$ D) None of these

(6) If $f(z) = (x^2 + ay^2 - 2xy) + i(bx^2 - y^2 + 2xy)$ is analytic, then value of a and b is :

A) -1, 1 B) 1, -1
C) 1, 0 D) None of these

(7) If G is an open set in complex plane and $f : G \to C$ is differentiable, then on G, f is:

A) Analytic B) not analytic
C) Discontinuous D) bounded

(8) Value of $(1 + i)^{24}$ is:

A) 2^{24} B) $\left(\sqrt{2}\right)^{24} e^{\frac{i®}{4}}$
C) 2^{12} D) None of these

(9) For any complex number Z , $|e^z| \le 1$ if:

A) $\operatorname{Re} z \ge 0$ B) $\operatorname{Re} z \le 0$
C) $\operatorname{Im} z \le 0$ D) None of these

(10) For any complex number z , $e^{z+® i}$ equals:

A) e^{z} B) e^{iz}

C) $-e^{z}$ D) None of these

(11) For any real number ¯ , $\frac{e^{i¯} - e^{-i¯}}{i(e^{i¯} + e^{-i¯})} =$ ....

A) cot ¯ B) tan ¯

C) cos ec¯ D) None of these

(12) If S and T are domains in the complex plane , which of the following need *NOT* be true?

A) $S \cup T$ is a domain if $S \cap T \neq$ ¤ B) $S \cup T$ is an open set

C) $S \cap T$ is an open set D) $S \cap T$ is a domain

(13) The derivative of $f(z) = \left(\frac{z^2 - 1}{z^2 + 1}\right)^{100}$ $at\ z \neq \pm i$ **is :**

A) $z\left(\frac{z^2 - 1}{z^2 + 1}\right)^{99}$ B) $400\, z \frac{(z^2 - 1)^{99}}{(z^2 + 1)^{101}}$

C) $400 \frac{(z^2 - 1)^{99}}{(z^2 + 1)^{101}}$ D) None of these

(14) If $f(z)$ is a real valued analytic function in a domain D, then :

A) $f(z)$ is a constant B) $f(z)$ is identically zero

C) $f(z)$ has modulus 1 D) None of these

(15) If v is a harmonic conjugate for u , then a harmonic conjugate for v is:

A) **u** B) **-u**

C) ***i* u** D) None of these

(16) If f(z) is analytic and non zero in a domain D , then in D , $\ln|f(z)|$ is:

A) Analytic B) a constant

C) zero D) harmonic

(17) The function e^{iz} has period:

A) $2®i$ B) $2®$

C) ® D) $3®$

(18) For real numbers x and y , $\sin(x + iy)$ equals :

A) $SinxCoshy + i \cos xSinhy$ B) $CosxCoshy - iSinxSinhy$

C) $SinxCoshy - i \cos xSinhy$ D) $CosxCoshy + iSinxSinhy$

(19) The function Log z is analytic at:

A) all points in the complex plane

B) all non zero complex numbers

C) all complex numbers except that on the non positive real axis

D) all complex numbers except that on the non positive imaginary axis

(20) Let G be a region not containing 0. Which of the following functions is *NOT* harmonic in G?

A) $x + y$ B) $x^2 + y^2$

C) $x^2 - y^2$ D) $\log(x^2 + y^2)$.

(21) Which of the following is not harmonic?

A) $u = 2x(1 - y)$ B) $u = 2xy + 3y^2 - 2y^3$

C) $u = 3x^2 y + 2x^2 - y^3 - 2y^2$ D) **None of these**

(22) If v is the imaginary part of an analytic function f, an analytic function with real part v is given by:

A) $\frac{1}{f}$ B) $-f$

C) if D) $-if$

(23) Value of the limit $\lim_{z\to 0} \bar{z}/z$ equals:

A) 1 B) -1

C) 0 D) Not exists

(24) Real part of the function $f(z) = |z|^2$ equals:

A) $2xy$ B) $x^2 - y^2$

C) $x^2 + y^2$ D) None of these

(25) For any complex number z, exp(z + 2® i) equals:

A) $\exp(z)$ B) $-\exp(z)$

C) $\exp(\frac{1}{z})$ D) None of these

Module – II

(1) An arc $z = z(t);\ a \le t \le b$ is simple if :

A) $z(t)$ is continuous B) $z(t)$ is a one to one function

C) $z(t)$ is such that $z(a) = z(b)$ D) None of these

(2) Which of the following is not a simply connected region?

A) circular disk B) half planes

C) an annulus region D) a parallel strip

(3) Which of the following subset of C is a simply connected region?

A) $\{z \mid ; 0 < |z| < 1\}$ B) $\{z \mid ; 0 < |z| \le 4\}$

C) $\{z \mid ; 1 < |z| < 2\}$ D) $\{z \mid ; 0 \le |z| < 3\}$

(4) Let Γ be any circle enclosing the origin and oriented counter clockwise. Then the value of the integral $\int_\Gamma \frac{\cos z\, dz}{z^2}$ is :

A) 2® i B) 0

C)-2® i D) undefined

(5) The integral $\int_{|z|=2®} \frac{Sin\ z}{(z-®)^2} dz$ where the curve is taken anti-clockwise, equals :

A) $-2\pi i$. B) $2\pi i$.

C) 0. D) $4\pi i$.

(6) The value of the integral $\int_C \frac{dz}{(z-a)^{10}}$, where C is |z - a| = 3 is :

A) 0 B) 2® i

C) ® i D) None of these

(7) The only bounded entire functions are:

A) Real valued functions B) harmonic functions

C) Constant functions D) Exponential function

(8) Suppose $f(z)$ **is analytic inside and on unit circle. If** $|f(z)| \leq 1, \quad \forall z$ with $|z| = 1$.
Then an upper bound for $|f^{n}(0)|$ **is :**

A) $n!$ B) $\frac{1}{n!}$

C) n D) None of these

(9) The integral $\oint_C \frac{dz}{z^2 - 1}$ **around a closed curve containing -1 but not 1, has the value :**

A) $-®i$ B) 0

C) $2®i$ D) None of these

(10) The value of the integral $\int_C \frac{e^{5z}}{z^3} dz$ **, where C is |z | = 3 is :**

A) $10®i$ B) $2®i$

C) $25®i$ D) None of these

(11) Value of the integral $\int_0^{®} e^{it} dt$ **is:**

A) $2i$ B) 0

C) $2®i$ D) None of these

(12) Value of $\oint_C \frac{3z - 2}{z^2 - z} dz$ **where C is** $|z| = 2$ **; is:**

A) $2®i$ B) 0

C) $6®i$ D) $-2®i$

(13) If n is any non zero integer, then $\int_0^{2®} e^{in¯} d¯$ **equals :**

A) **0** B) $2®$

C) 1 D) None of these

(14) The value of the integral $\int |z| \, dz$ **evaluated and the semi circle** $|z| = 1 \; ; 0 \leq \arg z \leq ®$
starting at $z = 1$ **is:**

A) $®i$ B) $-®i$

C) -2 D) None of these

(15) Converse of Cauchy's integral theorem is known as:

A) Liouville's theorem B) Goursat's theorem

C) Morera's theorem D) Euler's theorem.

(16) The value of the integral $\frac{1}{2®i} \int_{|z|=1} \frac{\cos z}{z^3}$ **dz is :**

A) 0 B) 1

C) 1/2 D) -1/2

(17) The parametric equation of the semi circle of radius 1 with center at 0, lying in the upper half plane from the point 1 to -1 is:

A) $z(t) = e^{it}; \; 0 \leq t \leq ®$ B) $z(t) = e^{it}; \; -® \leq t \leq ®$

C) $z(t) = e^{i2t}; \; 0 \leq t \leq ®$ D) $z(t) = e^{it}; \; 0 \leq t \leq 2®$.

(18) The integral $\int_C \frac{ze^z}{z^2 + 9} dz$ **has non zero value if** C **is :**

A) $|z| = 1$ B) $|z| = 2$

C) $|z - 1| = 1$ D) $|z| = 4$

(19) If f is continuous in a domain D and if $\int_C f(z)dz = 0$ for every simple closed positively oriented contour C in D, then:

A) f is a constant in D
B) f is analytic in D
C) f is real valued in D
D) f is purely imaginary in D

(20) If f is a analytic within and on a simple closed, positively oriented contour C and if z_0 is a point interior to C, then $\int_c \frac{f(z)}{(z - z_0)^{n+1}}dz$ equals :

A) $\frac{2\text{®} i}{n!} f^n(z_0)$
B) $\frac{n!}{2\text{®} i} f^n(z_0)$
C) $\frac{2\text{®} i}{n+1} f^n(z_0)$
D) $\frac{2\text{®} i}{(n+1)!} f^n(z_0)$

Module – III

(1) A Maclaurin series is a Taylor series with centre

A) $z_0 = 1$
B) $z_0 = 0$
C) $z_0 = 2$
D) None of these

(2) Let *f* be an analytic function and let $f(z) = \sum_{n-0}^{\infty} a_n (z-2)^{2n}$ be its Taylor series in some disc. Then:

$A) f^{(n)}(0) = (2n)!a_n$
$B) f^{(n)}(2) = n!a_n$
$C) f^{(2n)}(2) = (2n)!a_n$
$D) f^{(2n)}(2) = (n)!a_n$

(3) The radius of convergence of the power series of the function f(z) = $\frac{1}{1-z}$ about z= 1/4 is :

A) 1
B)¼
C) ¾
D) 0

(4) The coefficient of 1/z in the Laurent series expansion of $f(z) = \frac{1}{z(z-2)}$ in the region 2<$|z|$<∞ is :

A) 0
B) ½
C)2
D) 4

(5) If $f(z)$ is entire, then $f(z) = \sum_{n=1}^{\infty} a_n z^n$ has radius of convergence:

A) 0
B) e
C) ∞
D) None of these

(6) A power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ always converges for:

A) at least one point z.
B) all complex numbers z
C) at all z which are either real or purely imaginary
D) at all z with $|z - z_0| < R$ for some R > 0.

(7) If $f(z)$ admits a Laurent series expansion $\sum_{j=-\infty}^{\infty} a_j (z-a)^j$ in an annulus region. Then a_j is given by:

A) $\frac{1}{2\text{®}i}\int_C \frac{f(\text{¸})}{(\text{¸} - a)^{j+1}} d\text{¸}$
B) $\frac{j!}{2\text{®}i}\int_C \frac{f(\text{¸})}{(\text{¸} - a)^{j+1}} d\text{¸}$
C) $\frac{2\text{®}i}{j!}\int_C \frac{f(\text{¸})}{(\text{¸} - a)^{j+1}} d\text{¸}$
D) None of these

(8) If $f(z)=\sum_{n=-\infty}^{\infty} a_n (z-z_0)^n$ **is represented as the Laurent series, then** z_0 **is a removable singularity of** $f(z)$ **if:**

A) $a_n = 0$, for $n > 0$ B) $a_n = 0$, for $n < 0$

C) $a_n = 0$, for $n \geq 0$ D) $a_n = 0$, for $n \leq 0$

(9) A function $f(z)$ **given by a power series is analytic at :**

A) Every point of its domain B) every point inside its circle of convergence

C) every point on its circle of convergence D) every point in the complex plane

(10) The Taylor series expansion of $f(z)=e^z$ **in the region** $|z|<\infty$ **is:**

A) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ B) $\sum_{n=0}^{\infty} n!\,z^n$

C) $\sum_{n=1}^{\infty} \frac{z^n}{n!}$ D) None of these

(11) The singular points of the function $f(z)=\frac{1}{4z-z^2}$ **are:**

A) $z=0$ and $z=-4$ B) $z=0$ and $z=4$

C) $z=4$ and $z=-4$ D) $z=2$ and $z=-2$

(12) The singular points of the function $f(z)=\frac{e^z}{z(z^2+1)}$ **that lies inside** $|z-i|=\frac{3}{2}$ **are :**

A) $z=0$ and $z=-i$ B) $z=0$ and $z=i$

C) $z=i$ and $z=-i$ D) $z=0$ and $z=1$

(13) The constant term in the Laurent series expansion of $f(z)=\frac{e^z}{z^2}$ **in the region** $0<|z|<\infty$ **is :**

A) 0 B) $\frac{1}{2}$

C)2 D) None of these

(14) The Laurent series expansion of $f(z)=e^{1/z}$ **in the region** $0<|z|<\infty$ **is:**

A) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ B) $\sum_{n=0}^{\infty} n!\,z^n$

C) $\sum_{n=1}^{\infty} \frac{1}{n!\,z^n}$ D) None of these

(15) The power series $b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots\ldots$ **converges:**

A) inside of some circle $|z|=R$ B) on the circle $|z|=1$

C) on some circle $|z|=R$ D) outside of some circle $|z|=R$

Module- IV

(1) The residue of $\frac{z^3}{z^2-1}$ at $Z=\infty$ is:

A) 0 B) -1
C) 1 D) ∞ .

(2) Value of $\int_0^\infty \frac{Sin\ x}{x}dx$ is :

. A) $\frac{\text{®}}{2}$ B) 0
C) ∞ D) 2®

(3) If $f(z)$ has a zero of order m at z_0 and $g(z)$ has a pole of order n at z_0 and n $\le$ m, then the product $f(z)g(z)$ has at z_0 :

A) An essential singularity B) a pole of order m - n
C) A removable singularity D) a pole of order m - 1.

(4) If $f(z)$ has a pole of order m at z_0 , then $g(z)=\frac{f^{/}(z)}{f(z)}$, at z_0 has:

A) a simple pole B) a pole of order m
C) a pole of order m + 1 D) a pole of order m - 1.

(5) If $f(z)$ has a pole of order m at z_0 , then at z_0 , $\frac{1}{f(z)}$ has :

A) A removable singularity B) an essential singularity
C) A pole of order m D) none of these

(6) Zeros of $Sin\,(1-z^{-1})$ are :

A) $\frac{1}{1+n\text{®}}$; $n \in Z$ B) $\frac{1}{1-n\text{®}}$; $n \in Z$
C) $1+n\text{®}$; $n \in Z$ D) 0.

(7) For $f(z)=\frac{\tan z}{z}$, $z=0$ is a :

A) Essential singularity B) simple pole
C) Removable singularity D) double pole

(8) If $f(z)$ has a pole of order m at $z=0$, then $f(z^2)$ has a pole of order ….. at $z=0$:

A) m B) m²
C)2 m D) m + 1

(9) Which of the following function has a simple zero at $z=0$ and an essential singularity $z=1$?

A) $ze^{\frac{1}{z-1}}$ B) $ze^{\frac{1}{1+z}}$
C) $(z-1)e^{\frac{1}{z}}$ D) $(z-1)e^{\frac{1}{z-1}}$

(10) If $f(z)=\sum_{n=-\infty}^{\infty} a_n(z-z_0)^n$ is represented as the Laurent series, then z_0 is a removable singularity of $f(z)$ if:

A) $a_n=0$, for n > 0 B) $a_n=0$, for n < 0
C) $a_n=0$, for n $\ge$ 0 D) $a_n=0$, for n $\le$ 0

(11) The function $f(z)=\frac{Sin\ z}{(z^2-1)^2}$ at z = 1 has :

A) pole of order 4 B) pole of order 1
C) pole of order 2 D) essential singularity

(12) If a rational function has a pole of order m at z_0, then its derivative has a pole of order ------ at z_0

A) m - 1
B) m + 1
C) m
D) m²

(13) Residue of $f(z)=\dfrac{z^3+4z+9}{(2z+2)(z-3)^5}$ **at** $z=-1$ **is:**

A) $\dfrac{-1}{64}$
B) $\dfrac{-1}{32}$
C) $\dfrac{-1}{16}$
D) None of these

(14) The function $f(z)=\dfrac{z^2+2iz+3}{(z-i)^2(z+i)}$ **at** $z=i$ **has:**

A) Regular point
B) Simple pole
C) double pole
D) removable singularity

(15) Singularities of a rational function are:

A) poles
B) essential
C) non isolated
D) removable

(16) If $f(z)=\dfrac{1-e^z}{1+e^z}$**, then at** $z=\infty$**, f(z) has:**

A) Isolated singularity
B) non-isolated singularity
C) Pole
D) a zero

(17) If a function f(z) has an isolated singularity at z = a, **then it is a removable singularity if :**

A) $\lim_{z\to a}(z-a)f(z)\neq 0$
B) $\lim_{z\to a}(z-a)f(z)=0$
C) $\lim_{z\to a}f(z)\neq 0$
D) $\lim_{z\to a}f(z)=0$

(18) The singularity of the function $\dfrac{\sin z}{z}$ **at z = 0 is :**

A) Essential singularity
B) simple pole
C) Removable singularity
D) double pole

(19) Residue of $\dfrac{1}{z^m(1-z)^n}$**, m and n are natural numbers at** z **= 0 is :**

A) $\dfrac{(m+n-1)!}{(m-1)!(n-1)!}$
B) $\dfrac{(m+n-1)!}{m!n!}$
C) $\dfrac{(m+n-2)!}{(m-1)!(n-1)!}$
D) $\dfrac{(m+n-2)!}{m!\,n!}$

(20) At z = 1 , the residue of $f(z)=\dfrac{z^2+1}{z(z-1)}$ **is:**

A) -1
B) 0
C) 1
D) 2

(21) The residue of $f(z)=\dfrac{2z+3}{z(z-1)}$ **at** $z=1$ **is :**

A) - 5
B) 0
C) 2
D) None of these

(22) The function f(z) = 1 / tan ($\dfrac{®}{z}$ **) , at z = 0 has a :**

A) isolated singularity
B) a non- isolated singularity
C) simple pole
D) None of these

(23) The zero of first order is known as

A) Complex zero
B) Simple zero
C) Singularity
D) None of these

(24) The poles of the function f(z) = sin z / cos z are at

A) (2n+1) π\2, n any integer
B) 2n π \3 ; n any integer
C) n π, n any integer
D) None of these

(25) For the function $f(z)=e^{1/z}$, the point $z=0$ is a:

A) removable singularity
B) simple pole
C) essential singularity
D) None of these

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B.Sc DEGREE PROGRAMME MATHEMATICS (CORE COURSE)

SIXTH SEMESTER - MM6B010 - COMPLEX ANALYSIS

Answer Key

Module – I

1. B	2. A	3. B	4. C	5. A	6. A	7. A
8. C	9. B	10. C	11. B	12. D	13. B	14. A
15. B	16. D	17. B	18. A	19. C	20. B	21. B
22. D	23. D	24. C	25. A			

Module – II

1. B	2. C	3. D	4. B	5. A	6. A	7. C
8. A	9. A	10. C	11. A	12. C	13. A	14. C
15. C	16. D	17. A	18. D	19. B	20. A	

Module – III

1. B	2. C	3. C	4. A	5. C	6. A	7. A
8. B	9. B	10. A	11. B	12. B	13. B	14. C
15. D						

Module – IV

1. C	2. A	3. C	4. A	5. A	6. B	7. C
8. C	9. A	10. B	11. C	12. B	13. D	14. B
15. A	16. B	17. B	18. C	19. C	20. D	21. D
22. B	23. B	24. A	25. C			