School of Distance Education
University of Calicut
B.Sc. Mathematics - VIth Semester - Real Analysis

1. Let A be the open interval $(-1,1)$. Then, the cluster points of A are
   (a) 1 and -1 only (b) All the points of $(-1,1)$
   (c) All the points of $[-1,1]$.

2. Let I be a closed and bounded interval and $f: I \to R$ be continuous on I. Then $f(I)$ is
   (a) closed but not bounded (b) bounded but not closed (c) closed and bounded.

3. Which of the following have no cluster points?
   (a) $(3,4]$ (b) $[2,3]$ (c) $\{2,3,4\}$

4. The function $f(x) = \frac{1}{x}$ is not continuous at
   (a) $x=1$ (b) $x=0$ (c) all $x \in \mathbb{N}$.

5. Which among these functions are continuous on $A = \{x \in \mathbb{R}, x \geq 0\}$?
   (a) $f(x) = \sin x$ (b) $f(x) = \sin\left(\frac{1}{x}\right)$ (c) $f(x) = \frac{1}{x}$

6. Let $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then $\lim_{x \to 0} f(x)$ is
   (a) 1 (b) -1 (c) None of these.

7. Value of the limit $\lim_{x \to 0} \frac{e^{1/x}}{e^{1/x}+1}$ equals
   (a) ∞ (b) e (c) Does not exist.

8. The limit of the sequence $\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right)$ is

(a) 1 (b) -1 (c) 0

9. given a, b satisfy $0 < a < 1$, $b > 1$. Then which of the following sequences is convergent?

(a) $\frac{ab^n}{2^n}$ (b) $\frac{2^{3n}}{3^{2n}}$ (c) $\frac{n}{b^n}$

10. Function $f(x) = \frac{1}{2x}$ defined on $A = (0, \infty)$ is an example for

(a) a continuous function that is not bounded.

(b) a bounded function that is continuous.

(c) an unbounded function that is not continuous.

11. Let $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ such that for every $\delta > 0$ there exist at least one point $x \in A$, $x \neq c$ such that $|x - c| < \delta$. Then c is a ______ of A.

(a) isolated point (b) cluster point (c) None of these.

12. Let $f : A \to \mathbb{R}$ and let L be a real number such that for every $\epsilon > 0$, there exist a $\delta(\epsilon) > 0$ such that if $x \in A$ and $0 < |x - c| < \delta(\epsilon)$, then $|f(x) - L| < \epsilon$. Then L is a ______ of 'f' at c.

(a) derivative (b) limit (c) None of these.

13. Let $f : A \to \mathbb{R}$ and c be a cluster point of A. Then which of the following is correct?

(a) 'f' can have atleast 1 limit at 'c'.

(b) 'f' can have atmost 1 limit at 'c'.

(c) None of these

14. Let $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, $L, M \in \mathbb{R}$. Then which of the following need not hold true?

(a) $\lim_{x\to c} [f(x) \cdot g(x)] = L \cdot M$ (b) $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M}$

(c) $\lim_{x\to c} [f(x)]^{m/n} = L^{m/n}$, $L^{m/n} \in \mathbb{R}$, $m, n \in \mathbb{Z}$.

15. When the graph of a function 'f' in $\mathbb{R}$ has a break such that the function value and the limit (does not exist) aren't the same, then 'f' is said to have a __________ discontinuity at that point.

(a) removable (b) Jump (c) None of these.

16. When the graph of a function 'f' in $\mathbb{R}$ has a hole in the graph such that the function value and the limit aren't the same, then 'f' has a __________ discontinuity at that point.

(a) removable (b) Jump (c) None of these.

17. Let $A \subseteq \mathbb{R}$ and let $f : A \to \mathbb{R}$. If there exist a point $x' \in A$ such that $f(x') \geq f(x)$, for all $x \in A$, then 'f' has an __________ on A.

(a) absolute maximum (b) absolute minimum

(c) None of these.

18. $\lim_{x\to 0} e^{\sin x} =$

(a) 0 (b) 1 (c) None of these

19. Let $h(x) = k$, the constant function for $x \in \mathbb{R}$. Then every point of $\mathbb{R}$ is an __________ point for 'h'.

(a) absolute maximum (b) absolute minimum

(c) both absolute maximum and absolute minimum.

20. Let $f: I \to \mathbb{R}$ be continuous. Then 'f' has an absolute maximum and absolute minimum on I if

(a) I is closed not bounded (b) I is closed and bounded

(c) I is bounded but not closed.

21. Consider $f(x) = 6x^3 - 3x^2 + 2$. Then which of the following is true?

(a) 'f' has a zero in $(0,1)$

(b) 'f' has no zero.

(c) 'f' has a zero in $(-1,1)$.

22. Which of the following statements is true?

(a) every continuous function is uniformly continuous.

(b) every uniformly continuous function is continuous.

(c) None of these.

23. Let $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$. If for every $\epsilon > 0$ there exist a $\delta(\epsilon) > 0$ such that for $x, u \in A$ satisfying $|x-u| < \delta(\epsilon)$; $|f(x) - f(u)| < \epsilon$, then

(a) 'f' is continuous and uniformly continuous on A.

(b) 'f' is continuous but not uniformly continuous on A

(c) None of these.

24. Let $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$. Which of these statements are not equivalent to "f is not uniformly continuous on A"?

(a) there exist $\epsilon_0 > 0$ s.t. for every $\delta > 0$ there are points x_δ, u_δ in A such that $|x_\delta - u_\delta| < \delta$ and $|f(x_\delta) - f(u_\delta)| \geq \epsilon_0$.

(b) there exist $\epsilon_0 > 0$ and two sequences (x_n) and (u_n) in A s.t. $\lim(x_n - u_n) = 0$ and $f(x_n) - f(u_n) \geq \epsilon_0$ $\forall n \in \mathbb{N}$.

(c) there exist $\epsilon_0 > 0$ s.t. for every $\delta > 0$ there are points x_δ, u_δ in A such that $|x_\delta - u_\delta| < \delta$ and $|f(x_\delta) - f(u_\delta)| < \epsilon_0$.

25. Let $f: [a,b] \to \mathbb{R}$. If there exist a $L \in \mathbb{R}$ such that for every $\epsilon > 0$, there exist $\delta_\epsilon > 0$ such that for any tagged partition $\dot{P}$ of $[a,b]$ with $\|\dot{P}\| < \delta_\epsilon$; $|S(f;\dot{P}) - L| < \epsilon$, then 'f' is

(a) Riemann integrable (b) Lesbegue integrable

(c) None of these.

26. Let $f \in R[a,b]$. Then $\int_a^b f$ is

(a) not unique (b) always unique (c) None of these.

27. Let $g : [0,4] \to \mathbb{R}$ be defined by $g(x) = 3$ for $0 \leq x \leq 2$ and $g(x) = 2$ for $2 < x \leq 4$. Then $\int_0^4 g =$

(a) 5 (b) 7 (c) 10

28. Suppose f and g are in $R[a,b]$ such that $f(x) \leq g(x)$ for all $x \in [a,b]$, then

(a) $\int_a^b f(x) < \int_a^b g(x)$ (b) $\int_a^b f(x) \leq \int_a^b g(x)$

(c) None of these.

29. Which of the following is true?

(a) $f \in R[a,b] \implies f$ is bounded.

(b) $f \in R[a,b] \implies f$ is continuous.

(c) $f \in R[a,b] \implies f$ is monotone.

30. Let $I = [2,9]$. Which of the following partitions of I has the largest norm?

(a) $P_1 = (2, 4, 6, 8, 9)$ (b) $P_2 = (2, 6, 9)$

(c) $P_3 = (2, 5, 7, 9)$.

31. Let $f(x) = 2$ if $0 \leq x < 1$ and $f(x) = 1$, $1 \leq x \leq 2$. then

(a) $f \in R[0,2]$ and $\int_0^2 f = 3$

(b) $f \in R[0,2]$ and $\int_0^2 f = 2$

(c) $f \notin R[0,2]$

32. Let $x \in [0,1]$ and 'g' be such that

$$g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ \frac{1}{x}, & \text{if } x \text{ is irrational} \end{cases}$$

Then

(a) $g \in R[0,1]$ (b) $g \notin R[0,1]$ (c) None of these

33. Let 'f' be bounded on $[a,b]$. Let there exist two sequences of tagged partitions of $[a,b]$ such that $\|\dot{P}_n\| \to 0$ and $\|\dot{Q}_n\| \to 0$, $\lim_n S(f;\dot{P}_n) \neq \lim_n S(f;\dot{Q}_n)$. Then

(a) $f \in R[a,b]$ (b) $f \notin R[a,b]$ (c) None of these.

34. Let $x \in [0,1]$ and 'f' be such that

$$f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational.} \end{cases}$$ Then

(a) $f \in R[0,1]$ (b) $f \notin R[0,1]$ (c) None of these.

35. Suppose $f:[a,b] \to \mathbb{R}$, and $f(x)=0$ except for finite number of points $c_1, c_2, \ldots c_n$ in $[a,b]$. Then

(a) $f \notin R[a,b]$ (b) $\int_a^b f = 0$ (c) None of these.

36. Let $g \in R[a,b]$ and $f(x) = g(x)$ except for finite number of points in $[a,b]$. Then

(a) f need not be Riemann integrable on $[a,b]$

(b) $\int_a^b f = \int_a^b g$.

(c) $f \notin R[a,b]$

37. Let $0 \leq a < b$ and $g(x) = x^2$, $x \in [a,b]$.
Let $P = \{[x_{i-1}, x_i]\}_{i=1}^{n}$ be a partition of $[a,b]$.
Let q_i be the positive square root of $\frac{1}{3}(x_i^2 + x_i x_{i-1} + x_{i-1}^2)$
Then which of the following is true?
(a) $g \in R[a,b]$ (b) $\int_a^b g = \int_a^b x^2\,dx = \frac{1}{3}(b^3 - a^3)$

(c) All the above.

38. Let $f \in R[a,b]$ and $c \in \mathbb{R}$. Define g on $[a+c, b+c]$ by $g(y) = f(y-c)$. Then which of these holds?
(a) $\int_a^b f = \int_a^b g$ (b) $\int_a^b f = \int_{a+c}^{b+c} g$ (c) $g \in R[a,b]$

39. Which of the following is true?
(a) Every convergent sequence is Cauchy.
(b) Every Cauchy sequence is convergent.
(c) Both a and b.

40. Let $\phi : [a,b] \to \mathbb{R}$ such that $c \leq d$ and c, d are points in $[a,b]$. Let $\phi(x) = \alpha > 0$ for $x \in [c,d]$ and $\phi(x) = 0$ elsewhere in $[a,b]$. Then, which of the following is not true?
(a) $\phi \in R[a,b]$ (b) $\int_a^b \phi = \alpha(d-c)$
(c) $\int_a^b \phi = \alpha(b-c)$

41. Suppose J is a subinterval of $[a,b]$ having endpoints $c<d$ and $\phi_J(x)=1$ for $x \in J$, $\phi_J(x)=0$ elsewhere in $[a,b]$. Then

(a) $\int_a^b \phi = (b-a)$ (b) $\int_a^b \phi = (d-c)$

(c) $\int_a^b \phi = (b-d)+(c-a)$.

42. Let $f:[a,b] \to \mathbb{R}$. Then which of the following is true?

(a) f is continuous on $[a,b] \Rightarrow f \in R[a,b]$.

(b) $f \in R[a,b] \Rightarrow f$ is continuous on $[a,b]$.

(c) $f \in R[a,b] \Rightarrow f$ is uniformly continuous on $[a,b]$.

43. Let $f:[a,b] \to \mathbb{R}$. Then which of the following is true?

(a) f is monotone on $[a,b] \Longrightarrow f \in R[a,b]$

(b) $f \in R[a,b] \Longrightarrow f$ is monotone on $[a,b]$.

(c) None of these.

44. Consider 'h' defined on $[0,1]$ by

$$h(x) = \begin{cases} x+1, & x \text{ is rational} \\ 0, & x \text{ is irrational.} \end{cases}$$

Then which of these is true?

(a) $h \in R[0,1]$ (b) $h \notin R[0,1]$ (c) h is continuous on $[0,1]$

45. Let H be defined on $[0,1]$ such that $H(x) = \begin{cases} K, & \text{for } x = \frac{1}{K}, \ K \in \mathbb{R} \\ 0, & \text{elsewhere} \end{cases}$. Then which of these is true?

(a) $H \in R[0,1]$ (b) $H \notin R[0,1]$ (c) H is continuous on $[0,1]$.

46. Let f be continuous on $[a,b]$ and $f(x) \geq 0$ for all $x \in [a,b]$. If $\int_a^b f = 0$, then which of the following is true?

(a) $f(x) = 0$ for all $x \in [a,b]$

(b) $f(x) = 0$ for all but finitely many points of $[a,b]$.

(c) None of these.

47. If f is bounded on $[a,b]$ and if 'f' restricted to $[c,b]$, $c \in (a,b)$ is Riemann integrable, then which of the following does not hold true?

(a) $f \in R[a,b]$ (b) $f \notin R[a,b]$ (c) $\lim_{c \to a^+} \int_c^b f = \int_a^b f$

48. Define 'g' in $[0,1]$ as $g(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$.

Then which of these is true?

(a) $g \in R[0,1]$ (b) $g \notin R[0,1]$ (c) None of these.

49. Define 'g' such that 'g' is bounded and is continuous at every point of $[a,b] \setminus E$, where E is a finite set. Then which of the following is true?

(a) $f \in R[a,b]$ (b) $f \notin R[a,b]$ (c) None of these.

50. Suppose $a>0$ and $f \in R[-a,a]$. If 'f' is even for all $x \in [0,a]$, then

(a) $\int_{-a}^{a} f = 0$ (b) $\int_{-a}^{a} f = 2\int_{0}^{a} f$ (c) None of these

51. Suppose $a>0$ and $f \in R[-a,a]$. If 'f' is odd for all $x \in [0,a]$, then

(a) $\int_{-a}^{a} f = 0$ (b) $\int_{-a}^{a} f = 2\int_{0}^{a} f$ (c) None of these.

52. If 'f' is continuous on $[-1,1]$ then $\int_{0}^{\pi/2} f(\cos x)\cdot dx$ is not equal to which of the following?

(a) $\int_{0}^{\pi/2} f(\sin x)\cdot dx$ (b) $\frac{1}{2}\int_{0}^{\pi} f(\sin x)dx$ (c) $\int_{0}^{\pi} f(\sin x)dx$

53. If 'f' is continuous at every point of $[a,b]$ and F is any antiderivative of 'f' on $[a,b]$, then $\int_{a}^{b} f(x)\cdot dx =$

(a) $(b-a)(F(b)-F(a))$ (b) $F(b)-F(a)$ (c) None of these.

54. Let $f(x) = \begin{cases} 3x-1, & x<0 \\ 0, & x=0 \\ 2x+c, & x>0 \end{cases}$ and

$g(x) = \begin{cases} \frac{1}{2}-x, & 0<x<1/2 \\ \frac{3}{2}-x, & \frac{1}{2}<x<1 \end{cases}$, then

(a) neither $\lim_{x\to 0} f$ nor $\lim_{x\to 1/2} g$ exists.

(b) $\lim_{x\to 0} f$ and $\lim_{x\to 1/2} g$ both exists.

(c) $\lim_{x\to 0} f$ exists but $\lim_{x\to 1/2} g$ does not exist.

55. If $g(x) = \begin{cases} x, & |x| \geq 1 \\ -x, & |x| < 1 \end{cases}$ and if $G(x) = \frac{1}{2}|x^2 - 1|$,

then $\int_{-2}^{3} g(x) \cdot dx =$

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$

56. Let $B(x) = \begin{cases} -\frac{1}{2}x^2, & x < 0 \\ \frac{1}{2}x^2, & x \geq 0 \end{cases}$. Then $\int_a^b |x| \cdot dx =$

(a) $(b-a)[B(b) - B(a)]$ (b) $B(b) - B(a)$ (c) None of these.

57. If $f : [0,1] \to \mathbb{R}$ is continuous and $\int_0^x f = \int_x^1 f$ for all $x \in [0,1]$, then

(a) $f(x) = 0, \forall x \in [0,1]$ (b) $f \in R[0,1]$

(c) All the above

58. The sequence of functions (x^n) is pointwise convergent to 1 at

(a) $x = 1$ (b) all x such that $|x| > 1$

(c) $0 \leq x < 1$.

59. Let $a_n = \frac{x^2 + nx}{n}$, $x \in \mathbb{R}$. Then $\lim(a_n) =$

(a) 0 (b) x (c) x^2.

60. Which among the following sequences $X = (x_n)$ is convergent?

(a) $x_n = \frac{(-1)^n \cdot n}{n+1}$ (b) $\frac{n^2}{n+1}$ (c) $\frac{n}{n+1}$

Answer key

1.	c	16.	a	31.	a	46.	a
2.	c	17.	a	32.	b	47.	b
3.	c	18.	b	33.	b	48.	a
4.	b	19.	c	34.	b	49.	a
5.	a	20.	b	35.	b	50.	b
6.	c	21.	c	36.	b	51.	a
7.	c	22.	b	37.	c	52.	c
8.	a	23.	a	38.	b	53.	b
9.	b	24.	c	39.	c	54.	a
10.	a	25.	a	40.	c	55.	b
11.	b	26.	b	41.	b	56.	b
12.	b	27.	c	42.	a	57.	c
13.	b	28.	b	43.	a	58.	a
14.	b	29.	a	44.	b	59.	b
15.	b	30	b	45.	b	60.	c