
UNIVERSITY OF CALICUT

SCHOOL OF DISTANCE EDUCATION

BSc(MATHEMATICS)-VI SEMESTER-QUESTION BANK

MAT6B12-NUMBER THEORY AND LINEAR ALGEBRA

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1. If V is a vector space over the set of real numbers then $(V, +)$ is
- (a) a group
 - (b) an abelian group
 - (c) need not be an abelian group
 - (d) not a group

Answer : (b)

2. A subset W of a vector space V is a subspace if
- (a) the sum of elements of W belongs to W
 - (b) the sum and scalar multiples of elements of W belongs to W
 - (c) the scalar multiples of elements of W belongs to W
 - (d) the sum of elements of V belongs to W

Answer : (b)

3. A subspace of a real vector space \mathbb{R} is
- (a) $\{0\}$
 - (b) $\{1\}$
 - (c) $\{0, 1\}$
 - (d) $\{1, 1\}$

Answer : (a)

4. A subspace of a real vector space \mathbb{R}^2 is
- (a) $\{(1, x); x \in \mathbb{R}\}$
 - (b) $\{(x, 1); x \in \mathbb{R}\}$
 - (c) $\{(x, 0); x \in \mathbb{R}\}$
 - (d) $\{(1, 1 + x); x \in \mathbb{R}\}$

Answer : (c)

5. A line in \mathbb{R}^3 is a subspace of \mathbb{R}^3 if
- (a) it passes through the origin
 - (b) it does not pass through the origin
 - (c) it passes through $(1, 0)$
 - (d) it passes through $(-1, 0)$

Answer : (a)

6. The intersection of any set of subspaces of a vector space V is
- (a) not a subspace of V
 - (b) a subspace of V
 - (c) need not be a subspace of V
 - (d) a proper subspace of V

Answer : (b)

7. The union of any two subspaces of a vector space V is
- (a) a subspace of V
 - (b) not a subspace of V
 - (c) need not be a subspace of V
 - (d) a proper subspace of V

Answer : (c)

8. The span of subset $\{(1, 0), (0, 1)\}$ of the real vector space \mathbb{R}^2 is
- (a) \mathbb{R}^2
 - (b) \mathbb{R}^3

(c) \mathbb{R}^4 (d) \mathbb{R} **Answer : (a)**

9. If $e_i = (0, 0, \dots, 1, 0, \dots, 0)$, where 1 is in the i^{th} position. Then the span of subset $\{e_1, e_2, \dots, e_n\}$ of the real vector space \mathbb{R}^n is

(a) \mathbb{R}^2 (b) \mathbb{R}^3 (c) \mathbb{R}^n (d) \mathbb{R} **Answer : (c)**

10. The subspace of the real vector space \mathbb{R}^3 spanned by $\{(1, 0, 0), (0, 0, 1)\}$ is

(a) $\{(x, 0, z); x, z \in \mathbb{R}\}$ (b) $\{(x, y, z); x, y, z \in \mathbb{R}\}$ (c) $\{(0, 0, z); z \in \mathbb{R}\}$ (d) $\{(x, 0, 0); x \in \mathbb{R}\}$ **Answer : (a)**

11. The subset $\{(1, 0), (0, 1)\}$ of the real vector space \mathbb{R}^2 is

(a) linearly independent

(b) linearly dependent

(c) neither linearly independent nor dependent

(d) not a basis

Answer : (a)

12. A linearly independent subset of a vector space V does not contain

(a) 3

(b) 1

(c) 2

(d) 0_V **Answer : (d)**

13. The vectors $\{(1, 1), (2, 2)\}$ of real vector space \mathbb{R}^2 is

(a) linearly independent

(b) linearly dependent

(c) neither linearly independent nor dependent

(d) a basis

Answer : (b)

14. The set $Mat_{m \times n} \mathbb{R}$ of all $m \times n$ matrices with real entries under the usual operations of addition of matrices and multiplication by scalars is

(a) a real vector space

(b) a complex vector space

(c) not a real vector space

(d) not a complex vector space

Answer : (a)

15. The subset $\left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix} \right\}$ of $Mat_{2 \times 2} \mathbb{R}$ is

(a) linearly independent

(b) linearly dependent

- (c) neither linearly independent nor dependent
- (d) not a basis

Answer : (a)

16. A basis of a vector space V is a subset of V which is

- (a) linearly independent and spans V
- (b) linearly dependent and spans V
- (c) linearly independent only
- (d) linearly dependent

Answer : (a)

17. A basis of a vector space $\mathbb{R}_3[X]$ of all polynomials of degree atmost three is

- (a) $1, X, X^2, X^3, X^4$
- (b) $1, X, X^2$
- (c) $1, X$
- (d) $1, X, X^2, X^3$

Answer : (d)

18. A basis of real vector space \mathbb{R}^3 is

- (a) $\{(1, 1, 1), (2, 2, 2), (1, 0, 0)\}$
- (b) $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$
- (c) $\{(1, 1, 1), (1, 1, 1), (1, 1, 0)\}$
- (d) $\{(1, 1, 1), (2, 2, 2), (3, 3, 3)\}$

Answer : (b)

19. If B_1 and B_2 are any two finite bases of a vector space V then

- (a) number of elements in $B_1 >$ number of elements in B_2
- (b) number of elements in $B_1 <$ number of elements in B_2
- (c) number of elements in $B_1 =$ number of elements in B_2
- (d) number of elements in $B_1 \neq$ number of elements in B_2

Answer : (c)

20. The dimension of real vector space \mathbb{R}^n is

- (a) 2
- (b) 4
- (c) $n + 1$
- (d) n

Answer : (d)

21. The dimension of the vector space $Mat_{m \times n} \mathbb{R}$ is

- (a) $m + n$
- (b) mn
- (c) m
- (d) n

Answer : (b)

22. The dimension of the vector space \mathbb{C} over \mathbb{C} is

- (a) 2
- (b) 4
- (c) 1
- (d) 3

Answer : (c)

23. The dimension of the vector space \mathbb{C} over \mathbb{R} is

- (a) 2
- (b) 4
- (c) 1
- (d) 3

Answer : (a)

24. If a vector space V is of dimension n then every linearly independent set containing n elements is

- (a) a basis of V
- (b) not a basis of V
- (c) need not be a basis of V
- (d) linearly dependent

Answer : (a)

25. If a vector space V is of dimension n then every subset containing more than n elements is

- (a) linearly independent
- (b) linearly dependent
- (c) neither linearly independent nor dependent
- (d) a basis

Answer : (b)

26. If W is a subspace of a finite dimensional vector space V then

- (a) $\dim V = \dim W$
- (b) $\dim V \leq \dim W$
- (c) $\dim V \geq \dim W$
- (d) $\dim V \neq \dim W$

Answer : (c)

27. The map $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by $f(a, b) = (a + b, a - b, b)$ is

- (a) linear
- (b) not linear
- (c) not injective
- (d) not surjective

Answer : (a)

28. The subset $\{(1, 1), (1, -1)\}$ of real vector space \mathbb{R}^2 is

- (a) linearly independent only
- (b) linearly dependent
- (c) neither linearly independent nor dependent
- (d) a basis

Answer : (d)

29. A subset of a linearly independent set is

- (a) linearly independent
- (b) linearly dependent
- (c) neither linearly independent nor dependent
- (d) need not be linearly independent

Answer : (a)

30. Let S_1 and S_2 be non empty subsets of a vector space such that $S_1 \subset S_2$. If S_1 is linearly dependent then S_2 is

- (a) linearly independent
- (b) linearly dependent
- (c) neither linearly independent nor dependent
- (d) need not be linearly dependent

Answer : (b)

31. The differentiation map, $D : \mathbb{R}_n[X] \longrightarrow \mathbb{R}_n[X]$ given by $D(a_0 + a_1X + \dots + a_nX^n) = a_1 + 2a_2X + \dots + na_nX^{n-1}$ is
- (a) linear
 - (b) not linear
 - (c) injective
 - (d) surjective

Answer : (a)

32. A map $f : \mathbb{R} \longrightarrow \mathbb{R}$ which is linear is

- (a) $f(x) = x^2$
- (b) $f(x) = 3x$
- (c) $f(x) = x^3$
- (d) $f(x) = \sqrt{x}$

Answer : (b)

33. If the map $f : V \longrightarrow W$ is linear then $f(0_V) = \dots$

- (a) 0_V
- (b) 0
- (c) 0_W
- (d) 1

Answer : (c)

34. If the map $f : V \longrightarrow W$ is linear then for all $x \in V$, $f(-x) = \dots$

- (a) $-f(x)$
- (b) $f(x)$
- (c) $(f(x))^2$
- (d) 1

Answer : (a)

35. If the map $f : V \longrightarrow W$ is linear. If X is a subspace of V then $f^{\rightarrow}(X)$ is a

- (a) a subspace of V
- (b) not a subspace of W
- (c) a subspace of W
- (d) a subset of V

Answer : (c)

36. If the map $f : V \longrightarrow W$ is linear. If Y is a subspace of W then $f^{\leftarrow}(Y)$ is a

- (a) a subspace of V
- (b) not a subspace of V
- (c) a subspace of W
- (d) a subset of W

Answer : (a)

37. If the map $f : V \longrightarrow W$ is linear. Then the image or range of f is

- (a) $f^{\rightarrow}(W)$
- (b) $f^{\rightarrow}(V)$

- (c) $f^{\leftarrow}(V)$
 - (d) $f^{\leftarrow}(W)$
- Answer : (b)**

38. If the map $f : V \longrightarrow W$ is linear. Then the kernel or null space of f is

- (a) $f^{\rightarrow}(W)$
- (b) $f^{\rightarrow}(\{0_V\})$
- (c) $f^{\leftarrow}(\{0_W\})$
- (d) $f^{\leftarrow}(W)$

Answer : (c)

39. The Im D of differentiation map, $D : \mathbb{R}_n[X] \longrightarrow \mathbb{R}_n[X]$ is

- (a) $\mathbb{R}_n[X]$
- (b) $\mathbb{R}_{n-2}[X]$
- (c) $\mathbb{R}_{n-1}[X]$
- (d) $\mathbb{R}_{n+1}[X]$

Answer : (c)

40. The Ker D of differentiation map, $D : \mathbb{R}_n[X] \longrightarrow \mathbb{R}_n[X]$ is

- (a) $\mathbb{R}_n[X]$
- (b) \mathbb{R}^2
- (c) $\mathbb{R}_{n-1}[X]$
- (d) \mathbb{R}

Answer : (d)

41. The i^{th} projection $pr_i : \mathbb{R}^n \longrightarrow \mathbb{R}$ defined by $pr_i(x_1, \dots, x_n) = x_i$ is

- (a) linear only
- (b) not linear
- (c) linear and surjective
- (d) linear and injective

Answer : (c)

42. The image of i^{th} projection $pr_i : \mathbb{R}^n \longrightarrow \mathbb{R}$ defined by $pr_i(x_1, \dots, x_n) = x_i$ is

- (a) \mathbb{R}
- (b) \mathbb{R}^2
- (c) \mathbb{R}^3
- (d) \mathbb{R}^n

Answer : (a)

43. The kernel of i^{th} projection $pr_i : \mathbb{R}^n \longrightarrow \mathbb{R}$ defined by $pr_i(x_1, \dots, x_n) = x_i$ is

- (a) \mathbb{R}^n
- (b) \mathbb{R}
- (c) \mathbb{R}^3
- (d) the set of all n-tuples whose i -th component is zero

Answer : (d)

44. The map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ given by $f(a, b) = (b, 0, a)$ is

- (a) linear only
- (b) not linear
- (c) linear and surjective
- (d) linear and injective

Answer : (d)

45. If the map $f : V \longrightarrow W$ is linear and injective then $\text{Ker } f$ is

- (a) $\{0\}$
- (b) V
- (c) W
- (d) $V + W$

Answer : (a)

46. Let V and W be vector spaces of finite dimension over a field F . If $f : V \longrightarrow W$ is linear then $\dim \text{Im } f + \dim \ker f$ is

- (a) $\dim W$
- (b) $\dim V$
- (c) $\dim(V + W)$
- (d) $\dim(V \cap W)$

Answer : (b)

47. The rank of 1st projection $pr_1 : \mathbb{R}^3 \longrightarrow \mathbb{R}$ defined by $pr_1(x, y, z) = x$ is

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Answer : (a)

48. If $f : V \longrightarrow W$, where V and W are vector spaces over a field F . Then f is a linear isomorphism if f is

- (a) linear only
- (b) linear and injective
- (c) linear and surjective
- (d) linear and bijective

Answer : (d)

49. Let V be a vector space of dimension $n \geq 1$ over a field F . Then V is isomorphic to the vector space

- (a) \mathbb{R}^{n-1}
- (b) \mathbb{R}^n
- (c) \mathbb{R}^5
- (d) \mathbb{R}^4

Answer : (b)

50. A linear mapping is completely and uniquely determined by its action on

- (a) domain
- (b) range
- (c) a basis
- (d) kernel

Answer : (c)

51. The quotient and remainder when 10 is divided by 3 is

- (a) 3 and 1
- (b) 2 and 1
- (c) 3 and 2
- (d) 4 and 1

Answer : (a)

52. If a, b, c are any integers and $a|b$ and $b|c$ then

- (a) $c|a$
- (b) $c|b$
- (c) $b|a$
- (d) $a|c$

Answer : (d)

53. Suppose a and b are integers with $a \neq 0$, then $a|b$ if

- (a) $a = bc$, c is some integer
- (b) $b = ac$, c is some integer
- (c) $c = ab$, c is some integer
- (d) $ab = 1$

Answer : (b)

54. If a and b are two non zero integers and $\gcd(a, b) = d$, then d is the greatest positive integer with

- (a) $d|a$ and $d|b$
- (b) $d|a$ only
- (c) $d|b$ only
- (d) $a|d$ and $b|d$

Answer : (a)

55. $\gcd(-5, 5) = \text{---}$

- (a) 3
- (b) 1
- (c) 5
- (d) -5

Answer : (c)

56. a is an integer, $a|1$ if

- (a) $a = \pm 1$
- (b) $a = \pm 2$
- (c) $a = 2$
- (d) $a = \pm 3$

Answer : (a)

57. $\gcd(-8, 36) = \text{---}$

- (a) 36
- (b) -8
- (c) 8
- (d) 4

Answer : (d)

58. Let a and b be integers, not both zero. Then a and b are relatively prime if and only if there exists integers x and y such that

- (a) $1 = ax + by$
- (b) $2 = ax + by$
- (c) $ab = ax + by$
- (d) $a - b = ax + by$

Answer : (a)

59. $\gcd(39, 42, 54) = \text{---}$

- (a) 8

- (b) 39
- (c) 3
- (d) 6

Answer : (c)

60. Let a and b be integers. If $a|b$ and $b \neq 0$, then

- (a) $|a| \leq |b|$
- (b) $|b| < |a|$
- (c) $|a| \neq |b|$
- (d) $|a| \leq -|b|$

Answer : (a)

61. The Euclidean algorithm is used for finding the

- (a) lcm of two integers
- (b) gcd of two integers
- (c) prime numbers
- (d) composite numbers

Answer : (b)

62. If a, b, c are integers with $a|b$ and $b|a$, then

- (a) $ab = \pm 1$
- (b) $a = \pm b$
- (c) $ab = 1$
- (d) $a/x + b/y = 1$, x and y are integers

Answer : (b)

63. A linear combination of integers a and b is

- (a) ab
- (b) $a/x + b/y$, x and y are integers
- (c) $ab = 1$
- (d) $ax + by$, x and y are integers

Answer : (d)

64. The product of any three consecutive integers is divisible by

- (a) 36
- (b) 9
- (c) 6
- (d) 8

Answer : (c)

65. If $\gcd(a, b) = 1$ then $\gcd(a + b, a - b) = \text{---}$

- (a) 3 or 4
- (b) 1 or 2
- (c) 5 or 6
- (d) 7 or 8

Answer : (b)

66. If p is a prime, then $p^\#$ is the

- (a) sum of all primes that are less than or equal to p
- (b) product of all primes that are less than or equal to p
- (c) sum of squares of all primes that are less than or equal to p
- (d) product of all primes that are greater than or equal to p

Answer : (b)

67. $31^\# + 1$ is

- (a) prime
- (b) composite
- (c) even integer
- (d) irrational

Answer : (a)

68. If a and b are non zero integers with $a|b$, then $\gcd(a, b) = - - -$

- (a) $|a|$
- (b) b
- (c) ab
- (d) a

Answer : (d)

69. The number of primes is

- (a) finite
- (b) infinite
- (c) uncountable
- (d) 1729

Answer : (b)

70. Two integers a and b , not both of which are zero, are said to be relatively prime if

- (a) $\gcd(a, b) = a$
- (b) $a|b$
- (c) $\gcd(a, b) = 1$
- (d) $b|a$

Answer : (c)

71. If $\gcd(a, b) = d$, then $\gcd(a/d, b/d) = - - -$

- (a) 1
- (b) b
- (c) a
- (d) d

Answer : (a)

72. If a, b, c are any integers with $a|bc$ and $\gcd(a, b) = 1$, then

- (a) $b|a$
- (b) $a|c$
- (c) $c|a$
- (d) $c|b$

Answer : (b)

73. If a is an odd integer then $\gcd(3a, 3a + 2) = - - -$

- (a) 3
- (b) 5
- (c) 1
- (d) 2

Answer : (c)

74. The $\gcd(12378, 3054) = - - -$

- (a) 6
- (b) 1

(c) 3

(d) 2

Answer : (a)75. If $k > 0$, then $\gcd(ka, kb) = \dots$ (a) $a \gcd(k, b)$ (b) $b \gcd(a, k)$ (c) kab (d) $k \gcd(a, b)$ **Answer : (d)**76. For any integer $k \neq 0$, then $\gcd(ka, kb) = \dots$ (a) $|k| \gcd(a, b)$ (b) $b \gcd(a, k)$ (c) $k \gcd(a, b)$ (d) $a \gcd(k, b)$ **Answer : (a)**77. The $\text{lcm}(a, b)$ is the least positive integer m with(a) $a|m$ and $b|m$ (b) $a|m$ only(c) $b|m$ only(d) $m|a$ and $m|b$ **Answer : (a)**78. For positive integers a and b , $\gcd(a, b)\text{lcm}(a, b) = \dots$ (a) $a + b$ (b) ab (c) $a - b$ (d) a/b **Answer : (b)**79. The $\text{lcm}(3054, 12378) = \dots$

(a) 6300402

(b) 6678

(c) 6300400

(d) 300402

Answer : (a)80. If a and b are non zero integers and $k > 0$, then $\text{lcm}(ka, kb) = \dots$ (a) $a \text{lcm}(k, b)$ (b) $b \text{lcm}(a, k)$ (c) $k \text{lcm}(a, b)$ (d) kab **Answer : (c)**81. The linear Diophantine equation $ax + by = c$ has a solution if and only if —————(a) $\gcd(a, c)|b$ (b) $\gcd(a, b)|c$ (c) $\gcd(c, b)|a$ (d) $c|\gcd(a, b)$ **Answer : (b)**

82. Which of the following Diophantine equation cannot be solved ?

- (a) $6x + 51y = 22$
- (b) $33x + 14y = 115$
- (c) $14x + 35y = 93$
- (d) $11x + 13y = 21$

Answer : (c)

83. An integer $p > 1$ is called a prime, if its only positive divisors are

- (a) 1 and p
- (b) 1 and 2
- (c) 2 and p
- (d) 3 and p

Answer : (a)

84. Every positive integer $n > 1$ can be expressed as a product of

- (a) composite numbers
- (b) prime numbers
- (c) even numbers
- (d) odd numbers

Answer : (b)

85. If n is an odd pseudoprime, then $2^n - 1$ is

- (a) pseudoprime
- (b) prime
- (c) irrational
- (d) not pseudoprime

Answer : (a)

86. If p is a prime and $p|ab$, then

- (a) $p|a$ only
- (b) $p|b$ only
- (c) $p|a$ or $p|b$
- (d) $p|a$ and $p|b$

Answer : (c)

87. The *Sieve of Eratosthenes* is used for finding,

- (a) all primes below a given integer
- (b) all even numbers below a given integer
- (c) all odd numbers below a given integer
- (d) all composite numbers below a given integer

Answer : (a)

88. If p is any prime then \sqrt{p} is

- (a) prime
- (b) an integer
- (c) irrational
- (d) rational

Answer : (c)

89. If m and n are positive integers and $n|m$, then

- (a) $R_n|R_m$
- (b) $R_m|R_n$

(c) $R_n|m$

(d) $R_n|n$

Answer : (a)

90. If a and b are integers. Then $a \equiv b \pmod{n}$ is equivalent to

(a) a is a multiple of n

(b) b is a multiple of n

(c) $a - b$ is a multiple of n

(d) $a + b$ is a multiple of n

Answer : (c)

91. $-- \equiv 24 \pmod{7}$

(a) 1

(b) 2

(c) 4

(d) 3

Answer : (d)

92. If a is an odd integer, then $a^2 \equiv --- \pmod{8}$

(a) 1

(b) 2

(c) 3

(d) 4

Answer : (a)

93. Let $n > 1$ be fixed and a, b, c are integers. If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then

(a) $ac \equiv 1 \pmod{n}$

(b) $1 \equiv bc \pmod{n}$

(c) $a \equiv c \pmod{n}$

(d) $ab \equiv -1 \pmod{n}$

Answer : (c)

94. $(1001111)_2 = ----$

(a) 79

(b) 89

(c) 69

(d) 99

Answer : (a)

95. If a is a solution of $P(x) \equiv 0 \pmod{n}$ and $a \equiv b \pmod{n}$, then

(a) ab is also a solution

(b) $a + b$ is also a solution

(c) $a - b$ is also a solution

(d) b is also a solution

Answer : (d)

96. Give an example for $a^2 \equiv b^2 \pmod{n}$ need not imply $a \equiv b \pmod{n}$

(a) $a = 2, b = 3, n = 5$

(b) $a = 4, b = 9, n = 5$

(c) $a = 8, b = 13, n = 5$

(d) $a = 16, b = 11, n = 5$

Answer : (a)

97. If a is an odd integer then $a^2 - 1$ is

- (a) a multiple of 7
- (b) a multiple of 8
- (c) a multiple of 11
- (d) a multiple of 9

Answer : (b)

98. The International Standard Book Number (ISBN) consists of nine digits a_1a_2, \dots, a_9 followed by a tenth check digit a_{10} , which satisfies

- (a) $a_{10} \equiv \sum_{k=1}^9 ka_k \pmod{9}$
- (b) $a_{10} \equiv \sum_{k=1}^9 ka_k \pmod{11}$
- (c) $a_{10} \equiv \sum_{k=1}^9 ka_k \pmod{10}$
- (d) $a_{10} \equiv \sum_{k=1}^9 ka_k \pmod{7}$

Answer : (b)

99. The system of linear congruences $ax + by \equiv r \pmod{n}$, $cx + dy \equiv s \pmod{n}$ has a unique solution modulo n whenever

- (a) $\gcd(a - c, n) = 1$
- (b) $\gcd(a - d, n) = 1$
- (c) $\gcd(ad - bc, n) = 2$
- (d) $\gcd(ad - bc, n) = 1$

Answer : (d)

100. Any *palindrome* with even number of digits is divisible by

- (a) 5
- (b) 2
- (c) 11
- (d) 12

Answer : (c)

101. The solution of $25x \equiv 15 \pmod{29}$ is

- (a) $x \equiv 18 \pmod{29}$
- (b) $x \equiv 29 \pmod{29}$
- (c) $x \equiv 18 \pmod{19}$
- (d) $x \equiv 17 \pmod{19}$

Answer : (a)

102. The number of primes of the form $n^2 - 2$ is

- (a) finite
- (b) infinite
- (c) 1729
- (d) 1

Answer : (b)

103. Let p be a prime and suppose that $p \nmid a$ then $a^{p-1} - 1$ is

- (a) a multiple of p^2
- (b) a multiple of p
- (c) a multiple of $2p$

(d) a multiple of $p - 1$

Answer : (b)

104. If p is a prime and a is any integer then $a^p - a$ is

(a) a multiple of p^2

(b) a multiple of $p - 1$

(c) a multiple of $2p$

(d) a multiple of p

Answer : (d)

105. If p and q are distinct prime with $a^p \equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$, then

(a) $a^{p+q} \equiv a \pmod{pq}$

(b) $a^{pq} \equiv a \pmod{pq}$

(c) $a^{p-q} \equiv a \pmod{pq}$

(d) $a^{p/q} \equiv a \pmod{pq}$

Answer : (b)

106. An integer n is called a pseudoprime if

(a) $n \mid 2^n - 2$

(b) n is composite and $n \mid 2^n - 2$

(c) n is prime and $n \mid 2^n - 2$

(d) n is composite and $n \mid 2^n - 1$

Answer : (b)

107. A composite number n for which $a^n \equiv a \pmod{n}$ is called

(a) a pseudoprime

(b) a prime

(c) a pseudoprime to the base a

(d) an absolute pseudoprime

Answer : (c)

108. The composite numbers n that are pseudoprime to every base a are called

(a) a pseudoprime

(b) a prime

(c) a pseudoprime to the base a

(d) absolute pseudoprimes

Answer : (d)

109. If p is a prime, then $(p - 1)! + 1$ is

(a) a multiple of p

(b) a multiple of $p - 1$

(c) a multiple of $p + 1$

(d) a multiple of p^2

Answer : (a)

110. The remainder when $12!$ is divided by 13 is

(a) 12

(b) 13

(c) 2

(d) 3

Answer : (a)

111. $\tau(12) = - - -$
 (a) 5
 (b) 6
 (c) 7
 (d) 8

Answer : (b)

112. $\sigma(12) = - - -$
 (a) 28
 (b) 26
 (c) 23
 (d) 32

Answer : (a)

113. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ is the prime factorization of $n > 1$, then $\tau(n) = - - -$
 (a) $k_1 + k_2 + \dots + k_r$
 (b) $(k_1 + 1)(k_2 + 1) \dots (k_r + 1)$
 (c) $k_1 k_2 \dots k_r$
 (d) $k_1 - k_2 - \dots - k_r$

Answer : (b)

114. A number theoretic function f is said to be *multiplicative* if whenever $\gcd(m, n) = 1$, then
 (a) $f(mn) = f(m) + f(n)$
 (b) $f(mn) = f(m)f(n)$
 (c) $f(mn) = f(m) - f(n)$
 (d) $f(mn) = f(m) / f(n)$

Answer : (b)

115. The domain of number theoretic function is the
 (a) set of prime numbers
 (b) set of negative integers
 (c) set of rational numbers
 (d) set of positive integers

Answer : (d)

116. If m and n are relatively prime integers then $\tau(mn) = - - -$
 (a) $\tau(m) + \tau(n)$
 (b) $\tau(m) - \tau(n)$
 (c) $\tau(m)\tau(n)$
 (d) $\tau(m) / \tau(n)$

Answer : (c)

117. If m and n are relatively prime integers then $\sigma(mn) = - - -$
 (a) $\sigma(m) + \sigma(n)$
 (b) $\sigma(m) - \sigma(n)$
 (c) $\sigma(m) / \sigma(n)$
 (d) $\sigma(m)\sigma(n)$

Answer : (d)

118. The largest integer less than or equal to π , $[\pi] = - - -$
 (a) 3

- (b) 4
- (c) π
- (d) $\pi - 1$

Answer : (a)

119. If n and r are positive integers with $1 \leq r < n$, then the binomial coefficient $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ is

- (a) an integer
- (b) a prime
- (c) irrational
- (d) an even integer

Answer : (a)

120. If n is a prime, then $\phi(n) = - - -$

- (a) n
- (b) $n - 1$
- (c) $n + 1$
- (d) $n - 2$

Answer : (b)

121. If m and n are relatively prime integers then $\phi(mn) = - - -$

- (a) $\phi(m) + \phi(n)$
- (b) $\phi(m) / \phi(n)$
- (c) $\phi(m) - \phi(n)$
- (d) $\phi(m)\phi(n)$

Answer : (d)

122. For $n > 2$, $\phi(n)$ is

- (a) an odd integer
- (b) an even integer
- (c) irrational
- (d) prime

Answer : (b)

123. $\phi(360) = - - -$

- (a) 96
- (b) 98
- (c) 86
- (d) 90

Answer : (a)

124. If $n \geq 1$ and $\gcd(a, n) = 1$, then $a^{\phi(n)} - 1$ is

- (a) a multiple of n^2
- (b) a multiple of n
- (c) a multiple of $n - 1$
- (d) a multiple of $n + 2$

Answer : (b)

125. For any integer a , $a^{37} \equiv - - - \pmod{1729}$

- (a) a
- (b) $2a$
- (c) a^2
- (d) $a - 1$

Answer : (a)
